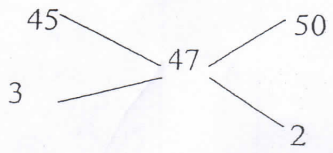
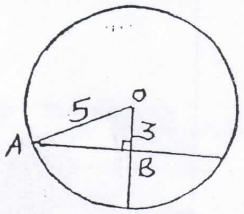
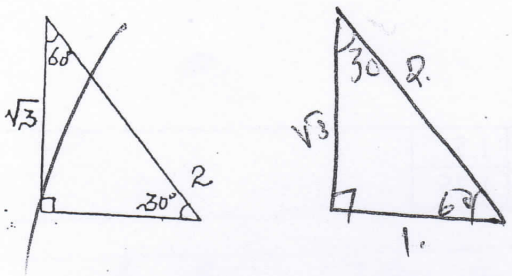
MOMANICHE 4 CYCLE 8

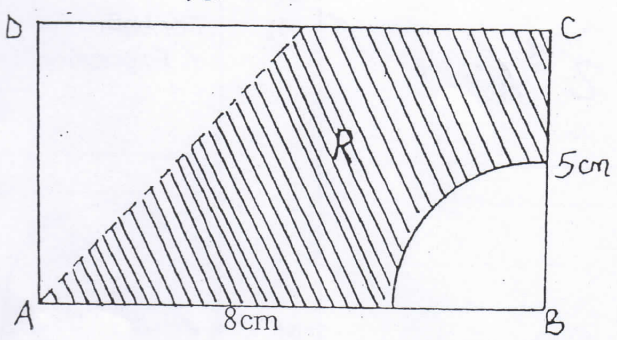
Kenya Certificate of Secondary Education (K.C.S.E.)

121/2
MATHEMATICS
PAPER 2
~~APRIL 9TH~~

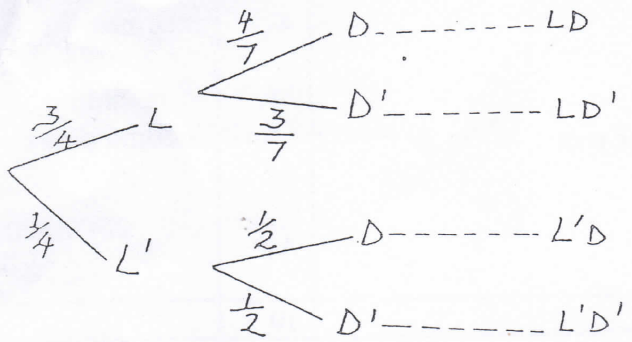
1	No	log		
	0.5249^2	$\bar{1}.7200$ X2	M_1	All logs✓
	83.58	$\bar{1}.4400$ $1.9221+$	M_1	✓addition and subtraction
	$3\sqrt{0.3563}$	1.3621 $\frac{\bar{1}.5518}{3} = \frac{\bar{3}}{3} + \frac{2.5518}{3}$	M_1	✓division by 3
	$10^1 \times 3.247$ ←	1.8505 1.5115	A_1	CAO
	$= 32.5$			
			04	
2	$n^2 = \frac{4x^2}{g^2} \times \frac{H-K}{3y}$ $n^2 = \frac{4x^2H - 4kx^2}{3yg^2}$ $n^2 3yg^2 = 4x^2H - Kx^2$ $n^2 3yg^2 + 4kx^2 = 4x^2H$ $\frac{4kx^2}{4x^2} = \frac{4x^2H - 3n^2 3yg^2}{4x^2}$ $K = \frac{4x^2H - 3n^2 3yg^2}{4x^2}$		M_1 M_1 A_1 ✓ Answer	simplification ✓ grouping
			03	
3	$2\cos 2\theta + 1 = 0$ $\cos 2\theta = -0.5$ $\Rightarrow 2\theta = 120^0, 240^0, 480^0, 600^0$ $\theta = 60^0, 120^0, 240^0$		B_1 B_1	For all For all

	$\frac{\pi^c}{3}, \frac{2}{3}\pi^c \frac{4}{3}\pi^c$	B_1	
		03	
4	$4\frac{7}{15} = 4.66667$ Actual $= 4.667$ Truncation $= 4.666$ 4.466 Error $= 4.667 - 4.666$ $4.467 - 4.466 = 0.001$ $= 0.001$ % Error $= \frac{0.001}{4.667} \times 100$ $\frac{0.001}{4.467} \times 100 = 0.022386389$ $= 0.02143\%$	M_1 M_1 A_1	✓ Expression for error C.A.O
		03	
5	$\frac{40X3 + 60X1}{4} = \frac{120+60}{4} = 45/ = perkg$  $= 3 : 2$	M_1 M_1 A_1	**ALT$_1$** Let the ratio be 1: n $\frac{45x1 + 50xn}{1+n} = 47$ $50n - 47n = 47 - 45$ $3n = 2$ $n = \frac{2}{3}$ $1 : \frac{2}{3}$ $\Rightarrow 3 : 2$
		03	
6	 AB $= \sqrt{5^2 - 3^2}$ $= 4$	B_1	

	Length = 4x2 = 8cm	B_1	
		02	
7	AB = AO + OB $= (2i - j + 8k) + (-3i + 2j - 2k)$ = -i + j + 6k $(AB) = \sqrt{(-1)^2 + (1)^2 + (6)^2}$ $= \sqrt{38}$ = 6.164	B_1 M_1 A_1	✓ Attempt to find magnitude.
		03	
8	 1 $\frac{\frac{1}{2} - \frac{1}{2}}{\sqrt{3 - \frac{1}{2}}} = \frac{-1}{2\sqrt{3-1}}$ $= \frac{-1}{(2\sqrt{3}-1)} \times \frac{(2\sqrt{3}+1)}{(2\sqrt{3}+1)}$ $= \frac{-2\sqrt{3}-1}{12-1}$ $= \frac{-2\sqrt{3}-1}{11}$	B_1 M_1 M_1 A_1	✓substitution of trig ratio ✓ Attempt to rationalize denomination
		03	
9	Total monthly installment = 3200x12 = 38400 = Principal = 30000 – 10000 = 20000 $38400 = 20000\,(1+\frac{r}{100})^{12}$ $\text{Log } 1.92 = 12 \log (1+\frac{r}{100})$ $1.059 = 1+\frac{r}{100}$ r = 5.9%	B_1 M_1 M_1 A_1	For both ✓ Expression ✓ simplification C.A.O
		04	
10 a	$(1-3x)^4 = 1 - 12x + 54x^2 - 108x^3$	M_1	✓ Expression

	$(a+\frac{x}{2})^4 = 9^4 + 2a^3x + 2ya^2x^2 + \frac{1}{2}ax^3$ $\Rightarrow -108x^3 + \frac{1}{2}ax^3 = -107x^3$ $-108 + \frac{1}{2}a = -107$ $a = 2$ b) $2^4 + 2(2^3)x + 24(2^2)x^2 + \frac{1}{2}(2)x^3$ $= 16 + 16x + 96x^2 + x^3$	M_1 A_1 B_1	✓ identification of co-efficient of x^3 and equation
		04	
11	Det = 0 $x(x+1) - 2 = 0$ $x^2 + x - 2 = 0$ $(x+1)(x-2) = 0$ $x = -1$ or 2	M_1 A_1	**Both values** ✓
12	X: -1.5, -0.5, 0.5, 1.5 Y: 1.75, 3.75, 3.75, 1.75 Area = 1(1.75 + 3.75+3.75+1.75) = 11sq.units	B_1 M_1 A_1	for all values
		03	
13	Log a = -0.35 a= 0.45 $\log b = \frac{0.35}{2}$ log b = 0.175 b = 1.496 (Accept 1.5) $M = ab^T$ When T=4 $M = 0.45 \times 1.496^4$ = 2.254 (Accept 2.278)	B_1 M_1 A_1 B_1	Expression of grad For b For m
		04	
14	1 cm rep 1m  	B_1 B_1 B_1	✓ arc constructed Bisecting<BAD Shading the region
		04	

15



M1

A1

b) $P(LD^1 \text{ or } L^1D^1)$

$= P(LD^1) + P(L^1D^1)$

$= \left(\frac{3}{4} \times \frac{3}{7}\right) + \left(\frac{1}{4} \times \frac{1}{2}\right)$

$= \frac{25}{56}$

03

16

a, ar, ar^2

a, a + 3d, a+9d

$\Rightarrow$ ar = a + 3d

6r = 6+3d

$r = 1 + \frac{1}{2}d$

$a(1+d+\frac{1}{4}d^2) = a + 9d.$

$\frac{1}{4}d^2 - \frac{1}{2}d = 0$

d = 2

$r = \frac{8}{4} = 2$ B1

$S_n = \frac{a(r^n-1)}{r-1} = \frac{4(2^8-1)}{1}$ M1

4 x 255

1020 A1

M1

M1

A1

03

✓ expressions of G.P and A.P

For equation

SECTION II

a) Gross tax = Net tax + Deductions (reliefs)

= 2336 + 1056

= Shs. 3392

b) Gross tax p.a = 339 x 12

= 40,704/=

Income p.a	Tax rate	Tax
121,968	10%	12196.80
114,912	15%	17236.80
56,352	20%	11270.40
293,232		40704.00

$\frac{20}{100} X = 11270.40$

$X = 11270.40 \ \frac{10}{2}$

= 56,352/=

M1

A1

M1

M1

M1

M1

✓ Attempt to find x

24. 17

WORKING

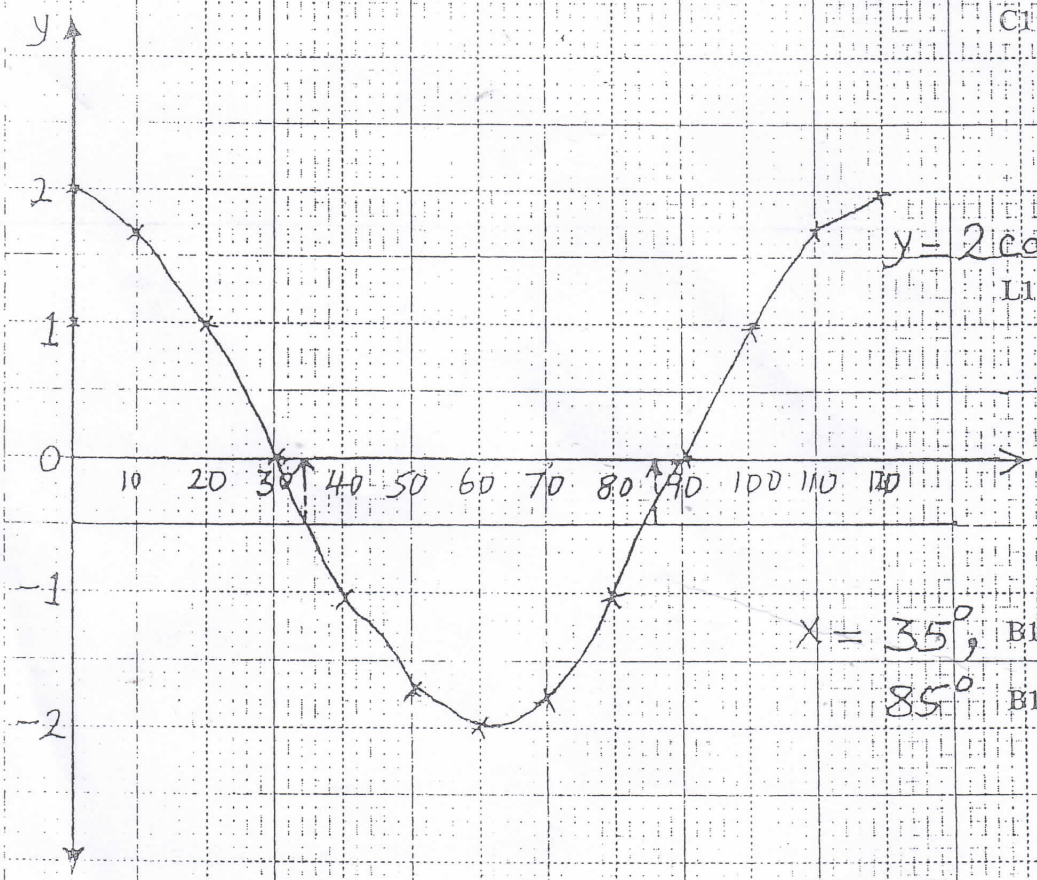
X	0	10	20	30	40	50	60	70	80	90	110	120
Cos 3x	0.8660		0.0000		-0.8660			0.5000	0.0000		0.8660	
y	1.73		0.00		-1.73			1.00	0.00		1.73	

B2 all
B1 at least 8

S1 Scale

P1

C1 Curve drawn



L1 Line drawn

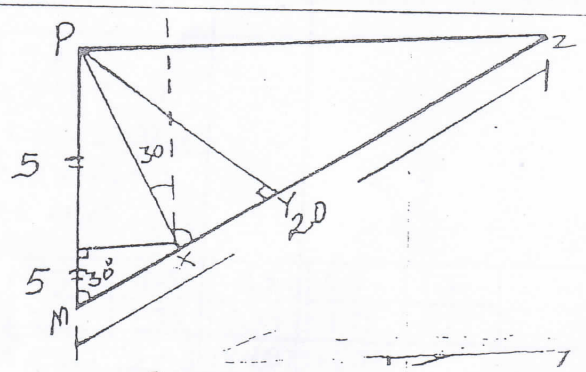
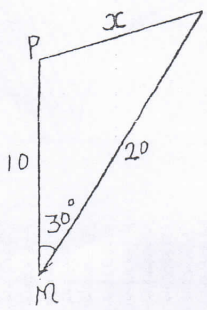
$X = 35^0$, B1

85^0 B1

Amplitude = 2 units B1

Period = 120^0 B1

10 Marks

	Taxable income = 293,232/= p.a = 293,232/= p.a = 24,436/ p.m. c). Allowance = $\frac{15.}{100}$ x 24,436 = 3635.40 Monthly basic salary = taxable – Allowance income p.m = 24436 – 3635. 40 = Shs. 20,800.60	M_1 A_1 M_1 A_1	Addition of incomes ✓ taxable income p.m
		10	
18	 a). Cos 30 = $\frac{5}{MX}$ mx = $\frac{5}{\cos 30}$ = 5.774 b) $360^0 - 30^0$ $= 330^0$ c) $\cos 30^0 = \frac{MY}{10}$ my = 10 cos 30^0 = 10 x 0.8660 = 8.66m d) By cosine rule $x^2 = 10^2 + 20^2 - 2 \times 200\cos 30^0$ $= 500 - 400\cos 30^0$ = 500 – 346. 41 = 153.59 x = 12.39 	M_1 A_1 M_1 A_1 M_1 M_1 A_1 M_1 M_1 A_1	
		10	

19. The height of a number of orange trees in an orchard were measured to the nearest (cm) and recorded in the table below.

Height (cm)	Frequency	c.f
131 - 140	13	13
141 – 150	23	36
151 – 160	36	72
161 – 170	50	122
171 – 180	35	157
181 – 190	28	185
191 - 200	15	200

Using an assumed mean of 165.5, calculate

a) The mean height (3mks)

x	$x-A$	fd
135.5	−30	−390
145.5	−20	−460
155.5	−10	−360
165.5	0	0
175.5	10	350
185.5	20	560
195.5	30	450
		$\Sigma fd = 150$

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$165.5 + \frac{150}{200}$$

$$165.5 + 0.75$$

$$166.25$$

b) The standard deviation of the distribution (4mks)

d^2	fd^2
900	11700
400	9200
100	3600
0	0
100	3500
400	11200
900	13500
	$\Sigma = 52700$

$$S = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$\sqrt{\frac{52700}{200} - (0.75)^2}$$

$$\sqrt{263.5 - 0.5625}$$

$$\sqrt{262.9375} = 16.215$$

$$16.22$$

c) The quartile deviation (3mks)

$$\frac{Q_3 - Q_1}{2}$$

$$Q_1 = \frac{1}{4} \times 200 = 50^{th}$$

$$150.5 + \left(\frac{50 - 36}{36}\right) \times 10 = 154.3888$$

$$170.5 + \left(\frac{150 - 122}{35}\right) 10 = 178.5$$

$$\frac{178.5 - 154.3888}{2} = 17.055$$

20. The points A (5,-1) B (1,-2) and C (x ,y) of a triangle are mapped onto A^1 (1,5) B^1(2,1) and C^1 (4,2) by a matrix $\mathbf{N} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find

a) Matrix **N** of the transformation. (4mks)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 1 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 5 & 1 \end{pmatrix}$$

$5a - b = 1$ $\quad 5c - d = 5$

$a - 2b = 2$ $\quad c - 2d = 1$

$10a - 2b = 2$ $\quad 10c - 2d = 10$

$a - 2b = 2$ $\quad c - 2d = 1$

$9a = 0$ $\quad 9c = 9$

$a = 0$ $\quad c = 1$

$b = -1$ $\quad d = 0$

$N = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

b) Co-ordinates of C (2mks)

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$x = 2$

$y = -4$

C (2, -4)

c) $A^{"} B^{"} C^{"}$ are the image of $A^{'} B'C'$ under a transformation represented by matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$.

Write down the co-ordinates of $A^{"} B^{"} C^{"}$ (2mks)

$$\begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 5 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

A^{11} (-3, 0)

B^{11} (3, 0)

C^{11} (6, 0)

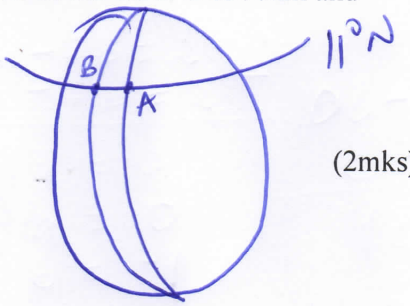
d) A transformation **N** followed by **M** can be represented by a single transformation **K**. Determine K (2mks)

MN

$$\begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix}$$

M_1 $\qquad$ A_1

21. A ship left port A (11^0 N, 32^0 W) and sailed due west to another port B. The journey took 160 hours at an average speed of 28 knots. Given that radius of the earth is 6370km and $\pi = 22/7$



a) Calculate the distance between A and B

i) In nautical miles (2mks)

$D = S \times T$

$28 \times 160 = 4480$ nm

M_1 A_1

ii) In km (2mks)

$4480 \times 1.853 = 8301.44$ km

M_1 A_1

b) Calculate the average speed of the ship in km/h correct to 2d.p (2mks)

$S = \frac{D}{T} = \frac{8301.44}{160}$ M_1 $= 51.88$ km/h A_1

c) Calculate to the nearest whole number the longitude of port B and hence state its position (4mks)

$4480 = \theta \times 60 \times \cos 11$ M_1

$\theta = \frac{4480}{60 \times 0.9816}$

$= 76.06$

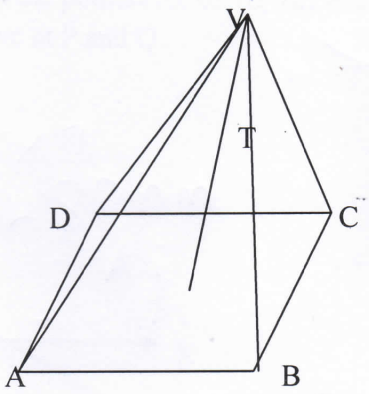
$= 76^\circ$ A_1

Longitude diff $= x - 32 = 76$ M_1

$x = 108^\circ$

$(11^\circ N, 108^\circ W)$ A_1

22.



The right pyramid above (not drawn to scale) has AB = 12 cm and BC = 16 cm. O is the centre of the base with OV = 15 cm.
Calculate, giving your answer to four significant figures.

a) The length of the slant edge (2mks)

$AC = \sqrt{12^2 + 16^2}$

$= 20\text{ cm}$

$AV = \sqrt{10^2 + 15^2}$

$\sqrt{325}$

$= 18.03\text{ cm}$

b) The angle between the lines VA and VC (2mks)

$\tan\theta = \frac{10}{15} = 0.6667$

$\theta = \tan^{-1} 0.6667$

$= 33.69^\circ$

c) The angle between the plane ABV and the base ABCD (3mks)

$VX = \sqrt{18.03^2 - 6^2}$

$= 17.00$

$\cos\beta = \frac{8}{17.00}$

$\beta = \cos^{-1}\frac{8}{17.00}$

$\beta = \cos^{-1} 0.4706$

$\beta = 61.93^\circ$

d) The pyramid is chopped at point T to form a frustum such T divides OV in the ration 2:1.
Find the volume of the frustum. (3mks)

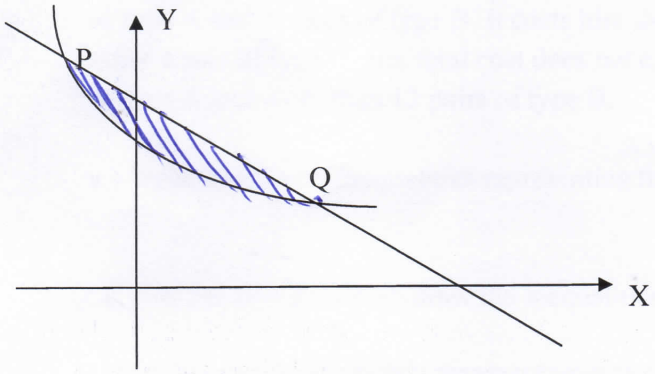
$\frac{1}{3}Ah = \frac{1}{3} \times 16 \times 12 \times 15 = 960\text{ cm}^3$

$\frac{1}{3}Ah = \frac{1}{3} \times 4 \times \frac{16}{3} \times 5 = 35.56\text{ cm}^3$

V. of frustrum $= 960 - 35.56$

$= 924.44\text{ cm}^3$

23. The figure below shows the sketch of the curve $y = x^2 - 2x + 4$ and a straight line PQ which cuts the x-axis and the y-axis at the points (10, 0) (0, 10) respectively.
The line also intersects the curve at P and Q



Find.

a i.) The equation of the straight line in the form $y = m x + c$ (2mks)

$y = x^2 - 2x + 4$

$a = \frac{\Delta y}{\Delta x}$

$a = \frac{10-0}{0-10}$

$a = -1$ M_1

$\frac{-1}{1} = \frac{y-10}{x-0}$

$y - 10 = -x$

$y = -x + 10$ A_1

ii.) The co-ordinates of P and Q (4mks)

$y = x^2 - 2x + 4$ ... (1)

$y = -x + 10$

$-x + 10 = x^2 - 2x + 4$ M_1

$x^2 - 2x + x + 6 = 0$

$x^2 - x + 6 = 0$

$x^2 - x - 6 = 0$ M_1

sum $= -1$ } 2
prod $= -6$ } -3

$x^2 + 2x - 3x - 6 = 0$

$x(x+2) - 3(x+2) = 0$

$x_1 = 3$

$x_2 = -2$ A_1

$P = (-2, 12)$ B_1

$Q = (3, 7)$

B Use integration to find the area of the shaded part. (4mks)

A. of Trapezium $= \int_{-2}^{3} (-x + 10)\,dx$

$\left[-\frac{x^2}{2} + 10x\right]_{-2}^{3}$

$\left(-\frac{(9)}{2} + 30\right] - \left[\frac{-4}{2} - 20\right]$ M_1

$25.5 + 22$

$= 47.5$ square Units

Area enclosed by curve

$= \int_{-2}^{3} (x^2 - 2x + 4)\,dx$

$\left[\frac{x^3}{3} - \frac{2x^2}{2} + 4x\right]_{-2}^{3}$ M_1

$(9 - 9 + 12) - \left[-\frac{8}{3} - 4 - 8\right]$

$12 + 14\frac{2}{3} = 26\frac{2}{3}$ S. Units

A. of shaded region

$47.5 - 26\frac{2}{3} = 30\frac{1}{6}$ square units A_1

(Q24)

Let no. of type A dresses be x

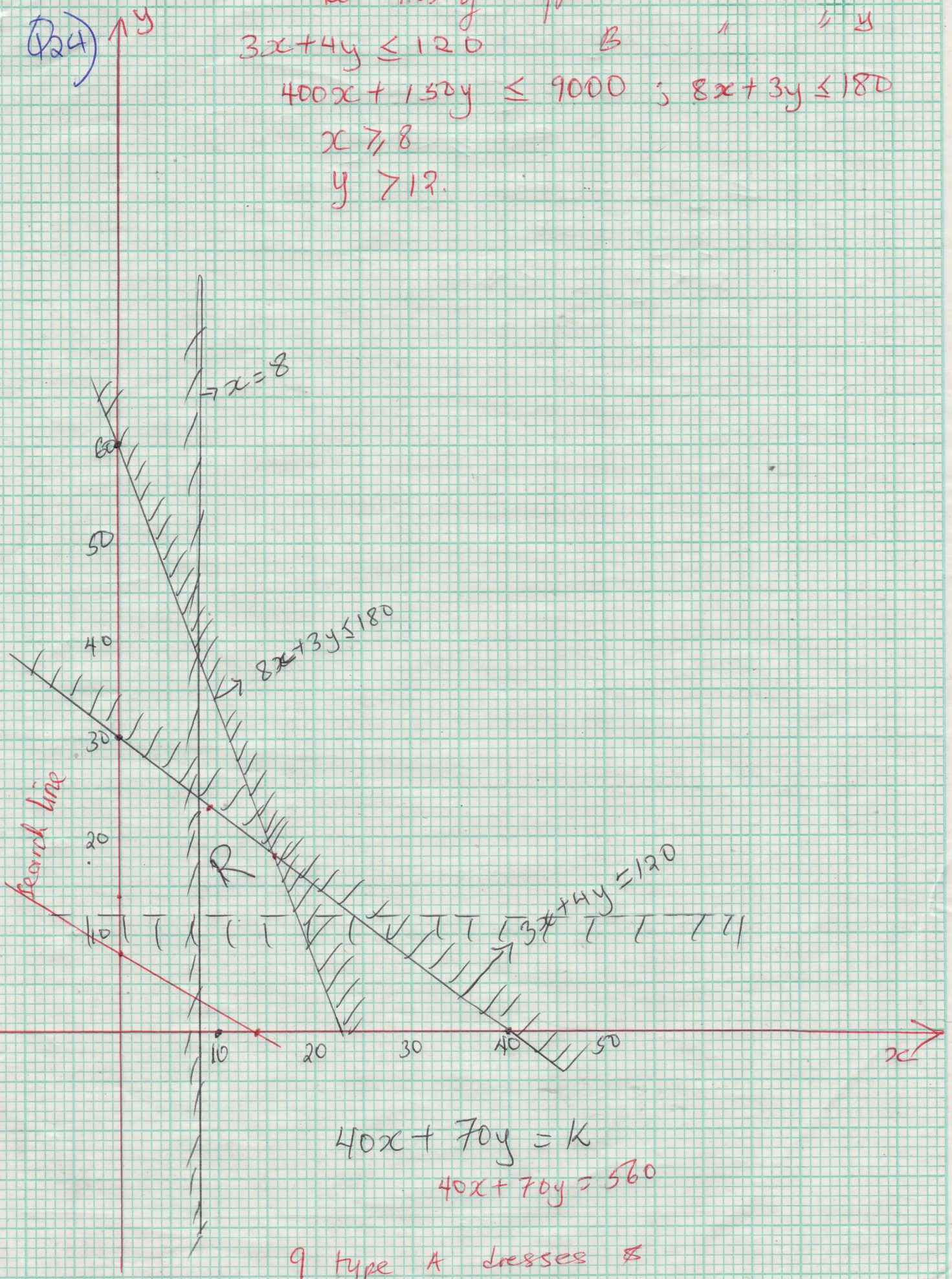
" " " B " " y

$3x + 4y \leq 120$

$400x + 150y \leq 9000$; $8x + 3y \leq 180$

$x \geq 8$

$y \geq 12$.



$40x + 70y = K$

$40x + 70y = 560$

9 type A dresses &

23 type B dresses!