Name: ....MARKING SCHEME..........Class: .... Adm.No........

School: .... Date: ....

Sign:....................

121/2
MATHEMATICS
PAPER 2
TIME: 2 ½ HOURS

SUKELLEMO EXAMINATION - 2021
Kenya Certificate to Secondary Education
MATHEMATICS (PAPER 2)
TIME: 2 ½ HOURS

Instructions

- *Write your name, class, admission number, school, date and signature in spaces provided above.*
- *The paper contains **two** sections **I** and **II.***
- *Answer **all** questions in section **I** and **any five** questions from section **11** in the spaces provided below each question.*
- *Show all the steps in your calculations giving your answers at each stage in the spaces below each question.*
- *Non-programmable silent electronic calculator and mathematical tables may be used except where stated otherwise.*

For Examiner's Use Only

SECTION A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION B

17	18	19	20	21	22	23	24	TOTAL

PERCENTAGE SCORE

SECTION I (50 MARKS)

*Answer **all** the questions from this section*

1. Use logarithms to evaluate (4 Marks)

$\frac{0.5249^2 \times 83.58}{\sqrt[3]{0.3563}}$

NO	Log
0.5249^2	$\bar{1}.7201 \times 2$
	$\bar{1}.4402$
83.58	1.9221 +
	1.3623
$0.3563^{1/3}$	$\bar{1}.5518 \div 3$
	$\bar{1}.8506$ −
3.249×10^1	1.5117

32.49

All logs correct M1

+, − M1

÷ × M1

A1

2. Make n the subject of the formula $\frac{r}{p} = \sqrt{\frac{m}{n^2-1}}$ (3 Marks)

$\frac{r^2}{p^2} = \frac{M}{n^2-1}$ M1

$r^2n^2 - r^2 = Mp^2$

$\frac{r^2n^2}{r^2} = \frac{Mp^2 - r^2}{r^2}$

$n^2 = \frac{Mp^2 + r^2}{r^2}$ M1

$n = \pm\sqrt{\frac{Mp^2+r^2}{r^2}}$ A1

3. Solve for x in the equation $2\sin^2x - 1 = \cos^2x + \sin x$ for $0^\circ \le x \le 360^\circ$ (4 Marks)

$2\sin^2x - 1 = 1 - \sin^2x + \sin x$ M1

Let $\sin x = y$

$3y^2 - y - 2 = 0$

$3y^2 - 3y + 2y - 2 = 0$

$3y(y-1) + 2(y-1) = 0$

$(3y+2)(y-1) = 0$ M1

$y = -\frac{2}{3}$ or $y = 1$

$\sin x = -\frac{2}{3}$ or $\sin x = 1$ M1

$x = 90^\circ, 221.8^\circ, 318.2^\circ$ A1

4. The image of a scalene triangle under the transformation given by the matrix $\begin{bmatrix} x+1 & 1 \\ 2 & x \end{bmatrix}$ is a straight line, find the possible value of x (3 Marks)

$x(x+1) - 2 = 0$ M1

$x^2 + x - 2 = 0$

$x^2 - x + 2x - 2 = 0$

$x(x-1) + 2(x-1) = 0$

$(x+2)(x-1) = 0$ M1

$x = 2$ or $x = 1$ A1

5. Evaluate without using tables or a calculator (4 Marks)

$\frac{1+Sin60°}{1-Cos30°}+\frac{1-Sin\,60°}{1+Cos30°}$

$\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}+\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}$ M1

$\frac{\left(1+\frac{\sqrt{3}}{2}\right)\left(1-\frac{\sqrt{3}}{2}\right)+\left(1-\frac{\sqrt{3}}{2}\right)\left(1+\frac{\sqrt{3}}{2}\right)}{\left(1+\frac{\sqrt{3}}{2}\right)\left(1-\frac{\sqrt{3}}{2}\right)}$ M1

$\frac{1+\frac{2\sqrt{3}}{2}+\frac{3}{4}+1-\frac{2\sqrt{3}}{2}+\frac{3}{4}}{1-\frac{3}{4}}$ M1

$=\frac{3\frac{1}{2}}{\frac{1}{4}}=\frac{7}{2}\times 4$

$=14.$ A1

6. The equation of a circle is $x^2 + 4x + y^2 - 2y - 4 = 0$. Determine the centre and radius of the circle (3 Marks)

$x^2+4x+(2)^2+y^2-2y+(-1)^2=4+4+1$ M

$(x+2)^2+(y-1)^2=9.$ M1

Centre $(-2, 1)$ A1

$r = 3$ units.

7. Use the expansion of $(x - y)^5$ to evaluate $(9.8)^5$ correct to 1 decimal place (3 Marks)

$x^5-x^4y+x^3y^2-x^2y^3+xy^4-y^5$

$x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$ B1

$(x-y)^5=(9.8)^5$

$x-y=10-0.2$

$10^5-5(10)^4(0.2)+10(10)^3(0.2)^2-10(10)^2(0.2)^3+5(10)(0.2)^4-(0.2)^5$ M1

$=90392.1$ A1

8. If $\int_1^a 3(x+1)^2dx = a^3 + 11$ find the possible values of a (4 Marks)

$3(x^2+2x+1)$

$\int_1^a (3x^2+6x+3)dx$

$=\left[x^3+3x^2+3x+c\right]_1^a$ M1

$=(a^3+3a^2+3a+c)-(1+3+3+c)$ M1

$=a^3+3a^2+3a-7$

$=a^3+3a^2+3a-7=a^3+11$

$3a^2+3a-18=0$

$a^2+a-6=0$

$a^2+3a-2a-6=0$

$a(a+3)-2(a+3)=0$

$(a+3)(a-2)=0$ M1

$a=-3$ or $a=2.$ A1

9. The measurement of the radius and height of a cylinder are given as 8cm and 9.5cm respectively. Calculate the percentage error in volume of a cylinder (Take π=3.142)

(3 Marks)

Max $V = 3.142 \times 8.5^2 \times 9.55$

$= 2167.94 cm^3$

Min $V = 3.142 \times 7.5^2 \times 9.45$

$= 1670.17 cm^3$

Actual $V = 3.142 \times 8^2 \times 9.5$

$= 1910.34 cm^3$

$A.E = \frac{1}{2}(2167.94 - 1670.17)$ M1

$= 248.885$

$\frac{248.885}{1910.34} \times 100$ M1

$= 13.03\%$ A1

10. Find x if, log(x-1) + 2 = log (3x+2) + log 25

(3 Marks)

$\log(x-1) + \log 100 = \log(3x+2) + \log 25$

$\log(x-1)100 = \log(3x+2)25$ M1

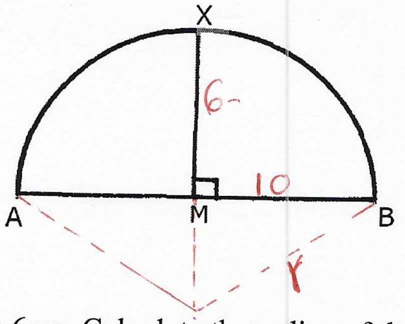
$100(x-1) = 25(3x+2)$ M1

$100x - 100 = 75x + 50$

$25x = 150$

$x = 6.$ A1

11. The figure below is a segment of a circle cut-off by chord AB. Line MX is perpendicular Bisector of chord AB.



If AB is 20cm and MX is 6cm. Calculate the radius of the circle from which the chord was cut.

(2 Marks)

$10^2 + (r-6)^2 = r^2$ M1

$100 + r^2 - 12r + 36 = r^2$

$12r = 136$

$r = 11\frac{1}{3}$ or 11.333 cm A1

12. a) Find the inverse of the matrix (1 Marks)

$$\begin{pmatrix} 4 & 3 \\ 3 & 5 \end{pmatrix}$$

Det $= 20 - 9$
$= 11$

$\frac{1}{11}\begin{pmatrix} 5 & -3 \\ -3 & 4 \end{pmatrix}$

$= \begin{pmatrix} \frac{5}{11} & -\frac{3}{11} \\ -\frac{3}{11} & \frac{4}{11} \end{pmatrix}$ B1

b) hence solve the following simultaneous equation using matrix method (2 Marks)

4x + 3y = 6

3x + 5y =5

$$\begin{pmatrix} \frac{5}{11} & \frac{-3}{11} \\ \frac{-3}{11} & \frac{4}{11} \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{11} & -\frac{3}{11} \\ -\frac{3}{11} & \frac{4}{11} \end{pmatrix}\begin{pmatrix} 6 \\ 5 \end{pmatrix} \text{ M1}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{11}\times 6 + \frac{-3}{11}\times 5 \\ \frac{-3}{11}\times 6 + \frac{4}{11}\times 5 \end{pmatrix}$$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1\frac{4}{11} \\ \frac{2}{11} \end{pmatrix}$

$x = 1\frac{4}{11}$ A1

$y = \frac{2}{11}$.

13. Point A is at (10°S, 20°E); when it is 1p.m in A, it is 9 p.m at B. Find the position of point B if both points lie in the same latitude. (2 Marks)

8 hours difference

1 hour = 15°

8hrs = ?

8x15 M1

= 120°

(10°S, 140°E) A1

14. A soda manufacturing company supplies two types of drinks, fanta and coke the total number of crates must not be more than 400. The company must supply more crates of fanta than coke. However, the number of crates of fanta must not be more than 300 and the number of crates of coke must be less than 80 let x represent fanta and y coke. Write down all the inequalities representing the given instructions (4 Mark)

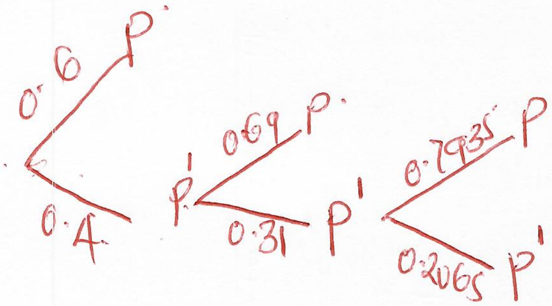
$x + y \leq 400$ B1

$x > y$ B1

$x \leq 300$ B1

$y < 80$ B1

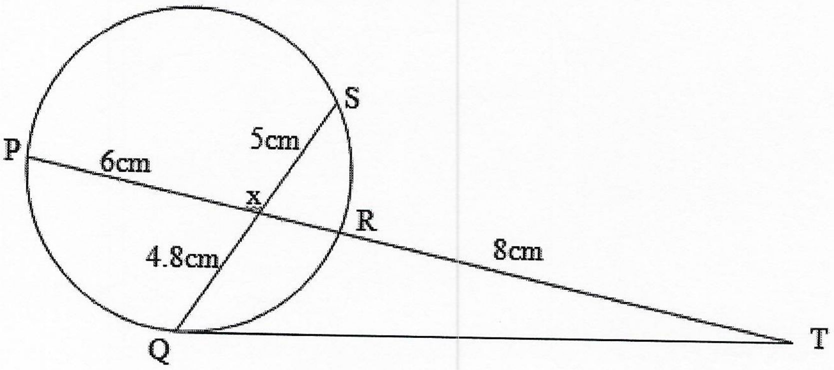
15. A student at a certain college has 60% chance of passing an examination at the first attempt. Each time a student fails and repeats the examination his chances of passing are increased by 15%. Calculate the probability that a student in the college passes an examination at the third attempt. (2 Marks)



$0.4 \times 0.31 \times 0.7935$ M1

$= 0.098394$ A1

16. In the figure below QT is a tangent to the circle at Q. PXRT and QXS are straight lines. PX = 6cm, RT = 8cm, QX = 4.8cm and XS = 5cm.



Find the length of QT (3 Marks)

$PX \cdot XR = QX \cdot XS$

$6 \cdot y = 4.8 \times 5$

$6y = 24$

$y = 4.$ B1

$QT^2 = 18 \times 8$ M1

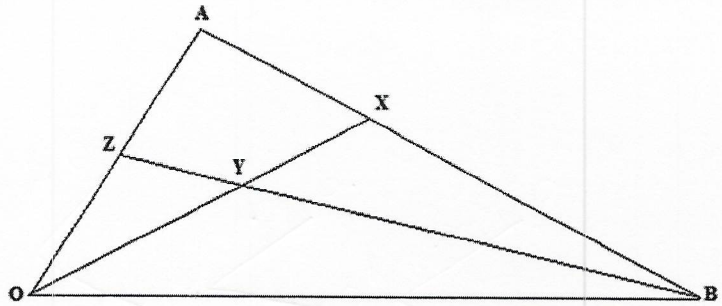
$= 144.$

$QT = 12cm$ A1

SECTION II (50 MARKS)

*Answer **FIVE** questions **ONLY** from this section*

17.



In the figure above O is the origin. **OA** = **a**, **OB** = **b**. The point x is on AB such that 2**AX** = **XB** and Y is the mid point of OX.

a) Find in terms of **a** and **b**, the vectors.

i) **OX** (2 Marks)

$a + \frac{1}{3}(b - a)$ M1

$= \frac{2}{3}a + \frac{1}{3}b$ A1

ii) **BY** (2 Marks)

$-b + \frac{1}{2}\left(\frac{2}{3}a + \frac{1}{3}b\right)$ M1

$= -b + \frac{1}{6}b + \frac{1}{3}a$

$= \frac{1}{3}a - \frac{5}{6}b$ A1

b) BY produced meets OA at Z. Given that **OZ** = h**a** and **BZ** = k**BY** where h and k are constants, find the values of h and k. (4 Marks)

$\vec{OZ} = ha$

$\vec{OZ} = b + k\left(\frac{1}{3}a - \frac{5}{6}b\right)$ M1

$= \left(1 - \frac{5}{6}k\right)b + \frac{1}{3}ka$

$ha = \left(1 - \frac{5}{6}k\right)b + \frac{1}{3}ka$ M1

$h = \frac{1}{3}k$

$0 = 1 - \frac{5}{6}k$

$-1 = -\frac{5}{6}k$

$k = \frac{6}{5}$ A1

$h = \frac{1}{3}\left(\frac{6}{5}\right)$

$= \frac{2}{5}$ B1

c) Find the position vector, **OZ** and obtain the ratio **OZ**: **ZA**. (2 Marks)

$\vec{OZ} = \frac{2}{5}a$ B1

$\vec{OZ} : \vec{ZA} = 2 : 3$ B1

18. The second, third and fourteenth terms of Arithmetic progression are the three consecutive terms of a geometric progression. The 10th term of the arithmetic progression is 18. Find;

a) The first term and the common difference of the progression. (5 Marks)

$a+d \quad a+2d \quad a+13d$

$r = \frac{a+2d}{a+d} = \frac{a+13d}{a+2d}$ M1

$a^2+4ad+4d^2 = a^2+14ad+13d^2$ M1

$10ad+9d^2=0$

$10a+9d=0$

$a+9d=18$

$10a+9d=0$

$-9a=18$ M1

$a=-2$ A1

$-2+9d=18$

$9d=20$

$d=\frac{20}{9}=2\frac{2}{9}$ B1

b) The sum of the first 10 terms of the progression. (3 Marks)

$S_{10} = \frac{10}{2}\{(-2\times2)+(9\times\frac{20}{9})\}$ M1

$= 5(-4+20)$ M1

$= 5\times16$

$= 80$ A1

c) The sixth term of the A.P (2 Marks)

$a+5d$

$= -2+\frac{5\times20}{9}$ M1

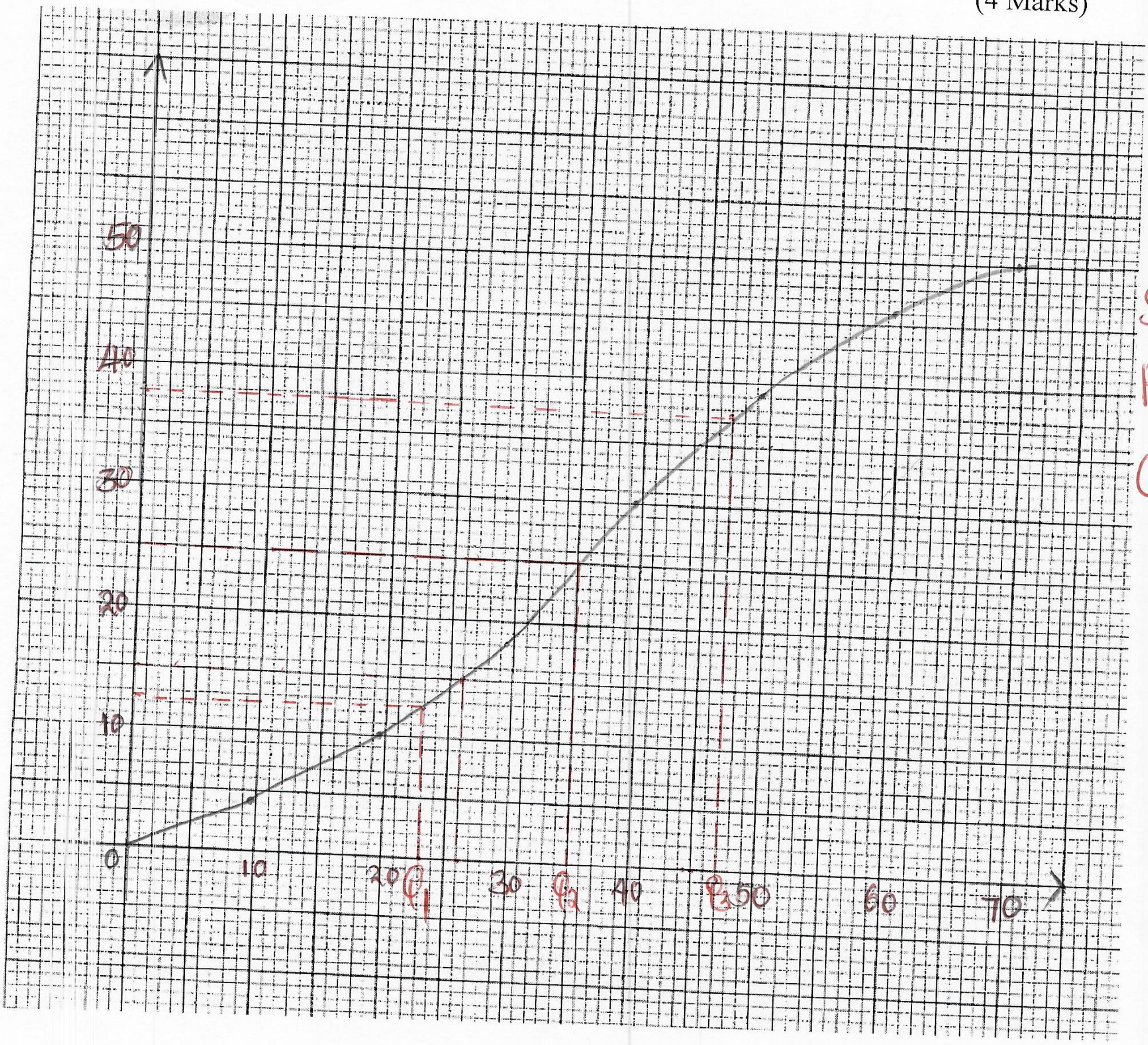
$= 9\frac{1}{9}$ or 9.111 A1

19. Fifty candidates sat for an exam. The following results were obtained

Marks	0-9	10-19	20-29	30-39	40-49	50-59	60-69
No. of candidates	4	6	8	12	9	7	4
CF	4	10	18	30	39	46	50.

B1

Draw a cumulative frequency curve (4 Marks)



S_1

P_1

C_1

b) use your graph to estimate

i) Median (1 Mark)

35 ± 1 B1

ii) Upper Quartile (1 Mark)

47 ± 1 B1

i) Lower Quartile (1 Mark)

23 ± 1 B1

b) The pass mark if 30% of the candidates passed (3 Marks)

$\frac{30}{100} \times 50 = 15$ students A1

M1

P.M = 46% B1

20. Alice and Cate can dig a shamba in 12 and 15 days respectively. Alice did the work for three days alone and she was joined by Cate on the 4^{th} day and both of them worked for the next two days. Find

a) The proportion of the work Alice had done before she was joined by Cate. (2 Marks)

1 day Alice $= \frac{1}{12}$

Cate $= \frac{1}{15}$

$\frac{1}{12} \times 3$ ~~M~~1

$= \frac{1}{4}$ A1

b) The work done in the first 5 days (3 Marks)

$\left(\frac{1}{12} + \frac{1}{15}\right)2 + \frac{1}{4}$ M1

$\frac{3}{10} + \frac{1}{4}$ M1

$= \frac{11}{20}$ A1

c) The work remaining by the end of the first five days (1 Mark)

$1 - \frac{11}{20}$

$= \frac{9}{20}$ B1

d) After five days they were joined by John who can dig the shamba a lone in 10 days. How long will the three take to clear the remaining portion of the work (4 Marks)

$\frac{1}{12} + \frac{1}{15} + \frac{1}{10}$ M1

$= \frac{15}{60} = \frac{1}{4}$ A1

1 day $\Rightarrow \frac{1}{4}$

$? = \frac{9}{20}$

$\frac{9}{20} \times 4$ M1

$= 1\frac{4}{5}$ days A1

21. The table below shows income tax for a certain year

Monthly Income in Kenya Shillings (Ksh)	Tax Rate
0 - 10164	10%
10165 - 19740	15%
19741 - 29316	20%
29317 - 38892	25%
Over 38892	30%

A tax relief of Ksh 1162 per month was allowed. In a certain month of the year, an employee's taxable income in the fifth band was Ksh.2108.

a) Calculate

i) Employees total taxable income in that month (2 Marks)

38892 + 2108 M1

= 41000 A1

ii) The tax payable by the employee in that month (5 Marks)

$\frac{10}{100} \times 10164 = 1016.40$

$\frac{15}{100} \times 9576 = 1436.40$ M1

$\frac{20}{100} \times 9576 = 1915.20$

$\frac{25}{100} \times 9576 = 2394.00$ M1

$\frac{30}{100} \times 2108 = 634.40$ M1

7394.40

7394.40
1162.00 − M1

6232.40 A1

b) The employee's income include a house allowance of Ksh 15,000 per month. The employee contributed of the 5% basic salary to a co-operative society. Calculate the employee's net pay for that month (3 Marks)

41000 − 15000 = 26000

$\frac{5}{100} \times 26000 + 6232.40$ M1

= 7532.40

41000 − 7532.40 M1

= 33467.60 A1

22. a) complete the table below for the functions y=Sin x and y=2Sin(x+30°) for 0°≤ x≤360°

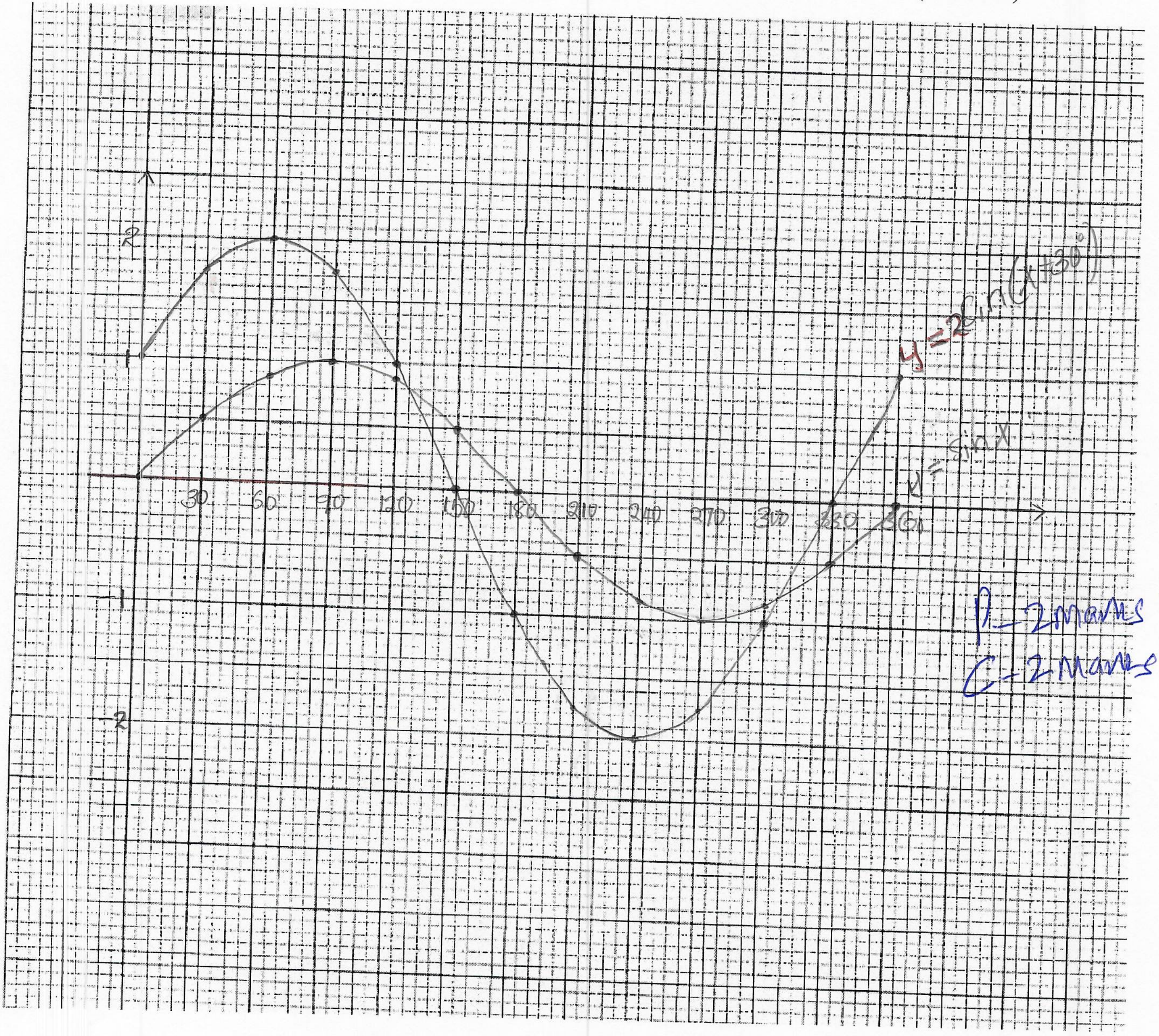
(2 Marks)

x°	0	30	60	90	120	150	180	210	240	270	300	330	360
Sin x	0	0.5	0.87	1	0.87	0.5	0	−0.5	-0.87	-1	−0.87	-0.5	0
2Sin(x+30°)	1	1.73	2	1.73	1	0	-1	-1.73	−2	-1.73	-1	0	1

all points 2mks
4 points 1mk

On the same axis, draw the graphs of y=Sin x and y=2Sin (x +30°) for 0°≤x≤360°

(4 Marks)



P - 2marks
C - 2marks

c) i) State the amplitude of the graph y= 2Sin(x+30) (1 Mark)

2 B1

ii)State the period of the graph y = Sin x (1 Mark)

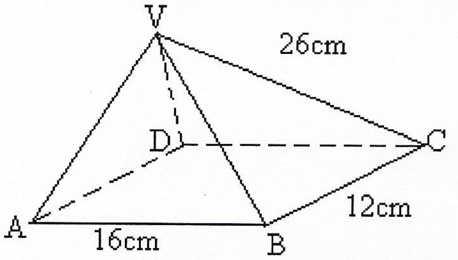
360 B1

d) use your graph to solve the equation Sin x – 2Sin (x +30°) = 0 (2 Marks)

$X = 216^{0}$ or $X = 306^{0}$

B1 B1

23. The figure below shows a right pyramid ABCDV with a rectangle base.



Given that AB = 16cm, BC = 12cm and VA = VB = VC = VD = 26cm.

Find

a) The height of the pyramid. (3 Marks)

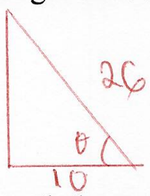
$AC = \sqrt{16^2 + 12^2}$

$= 20cm$

$Height = \sqrt{26^2 - 10^2}$

$= 24cm$

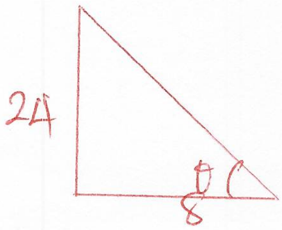
b) The angle between line DV and the base ABCD. (2 Marks)



$\cos\theta = \frac{10}{26}$ M1

$\theta = 67.38^\circ$ A1

c) The angle between plane VBC and the base ABCD. (2 Marks)

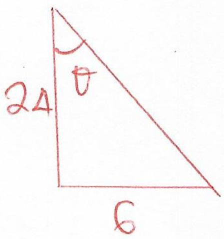


$\tan\theta = \frac{24}{8}$ M1

$\tan\theta = 3$

$\theta = 71.57^\circ$ A1

d) The angle between planes AVB and CVD. (3 Marks)

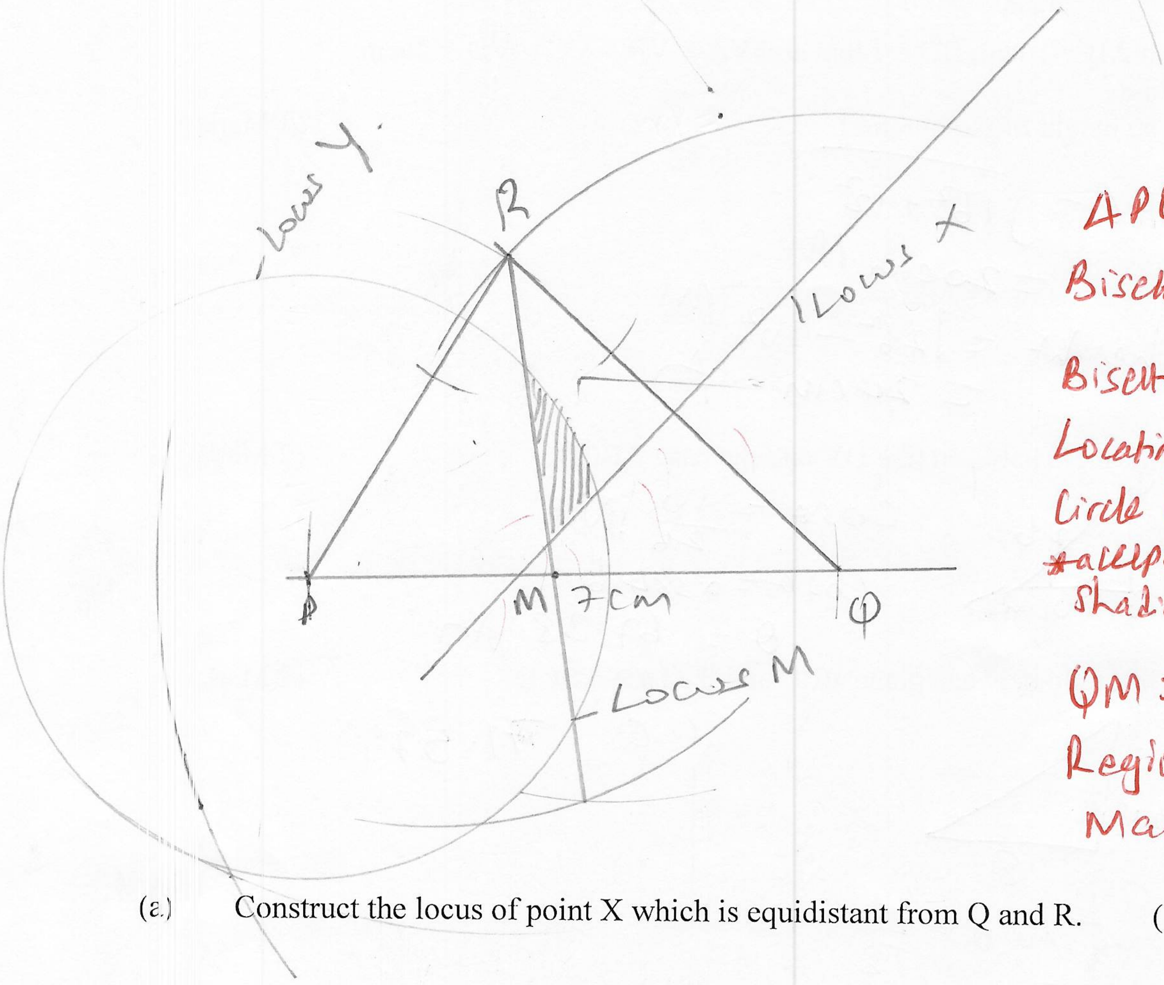


$\tan\theta = \frac{6}{24} = 0.25$

$\theta = 14.04 \times 2$

$= 28.07^\circ$

24. Construct triangle PQR such that PQ = 7cm, QR = 6cm and RP = 5cm. (1 Mark)



ΔPQR — 1 Mk B1
Bisector of QR – 1 Mk B1
Bisector $\angle PRQ$ — B1
Locating M — B1
Circle Centre P — B1
*accept arc in ΔPQR
Shading
QM = 3.8cm ± 0.1 B1
Region T correctly Marked — 4 Marks B4

(a) Construct the locus of point X which is equidistant from Q and R. (1 Mark)

(b) Construct the locus of M which is equidistant from PR and QR.
Mark with letter M the point where this locus meets PQ. Measure QM. (3 Marks)

QM = 3.8cm

(c) Construct the locus of Y such that PY = 4cm. (1 Mark)

(d) Shade the region in which T lies given that QT ≥ TR and ∠PRT ≥ ∠QRT and PT ≤ 4cm (4 Marks)