GATITU SECONDARY SCHOOL P.O BOX 327- 01030 GATUNDU.

Form 4 2014

END OF TERM 1 MATHEMATICS PP 2

TIME 2 ½ HRS

NAME---------- MARKING SCHEME ----------ADM----------

INSTRUCTION:

1 Answer all the questions in section A and ANY 5 in section B.

2 Show all your working below each question.

Section A (50 marks)

1 Use logarithims to evaluate $\sqrt[3]{1.23 \times 0.0468}$ (4mks)

No	S.F	No
1.23	1.23×10^{0}	0.0899
0.0468	4.68×10^{-2}	$\bar{2}.6702$
		$\bar{2}.7601$
Log b	7.782×10^{-1}	$\bar{1}.8911$
		$\bar{2}.8690$

$\frac{1}{3}\times\bar{2}.8690$

Log 6.

$\frac{1}{3}\times-3+1.8690$

$\bar{1}.023$

$10^{\bar{1}}\times 4.1976$

0.4198

2 Solve the following equation, $\sin[2x - 30]= \frac{3}{4}$ for $0^0 \le X \le 180^0$ (3mks)

$\sin(2x-30)=0.75$

$2x-30=48.6^\circ$

$2x=48.6+30$

$2x=78.6$

$x=39.3$

$2x-30=131.4$

$2x=131.4+30$

$2x=161.4$

$x=80.7^\circ$

$x=39.3^\circ$

3 Find the centre and the radius of the circle whose equation is $X^2-6X+Y^2-10Y+30=0$ (3mks)

$x^2-\left(\frac{6}{2}\right)^2 \quad y^2-\left(\frac{10}{2}\right)^2$

$(x-3)^2+(y-5)^2=30$

$x=3 \quad -30+9+25$

$y=5$

$(3,5) \quad \sqrt{84}$

$C(3,5)$

$r=\sqrt{84}$ cm ≈ 2 cm

1

4 Simplify $\frac{12x^2 - 16x}{20 - 11x - 3x^2}$ (3mks)

$$\frac{4x(3x^2-4)}{3x^2+11x-20}$$

$$\frac{4x(3x-4)}{3x^2-4x+15x-20}$$

$$x(3x-4)+5(3x-4)$$

$$\frac{4x(3x-4)}{(x+5)(3x-4)}$$

$$\frac{-4x}{(x+5)}$$

5 Simplify $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (3mks)

$$\frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$\frac{3-\sqrt{6}-\sqrt{6}+2}{3-\sqrt{6}+\sqrt{6}-2}$$

$$\frac{5-2\sqrt{6}}{1}$$

$$= 5-2\sqrt{6}$$

6 Use matrix method to solve simultaneously $2x - 3y = 5$

$4x - 6y = 7$ (4mks)

Give a geometrical interpretation of your result

$$\begin{pmatrix} 2 & -3 \\ 4 & -6 \end{pmatrix} \quad \det = -12 - -12$$

$$\det = 0$$

No inverse

$-3y = 5 - 2x$

$y = -5 + \frac{2}{3}x$

$-6y = 7 - 4x$

$y = -\frac{7}{6} + \frac{4}{6}x$

$y = -\frac{7}{6} + \frac{2}{3}x$

|| Lines.

7 Three types of spirits X ,Y and Z are mixed in the ratio 5: 2 :3 respectively.The cost per litre of spirits X ,Y and Z are SH 105 , SH 80 and SH 60 . If the mixture is sold at a profit of 25%, find the selling price of the mixture per litre. (4 mks)

X	Y	Z
5	2	3
105	80	60

$5+2+3=10$

T.C $(5\times105)+(2\times80)+(60\times3)$

$525+160+180=$ sh. 865

$\frac{125}{100}\times865$

$=$ sh $\frac{108125}{10}$

108.125

sh 108.10

8 Solve for x $[\log_{27} X]^2 - 2/3[\log_{27} X] = 1/3$ (4mks)

Let $\log_{27} x = k$.

$k^2 - \frac{2}{3}k - \frac{1}{3} = 0$

$3k^2 - 2k + 1 = 0$

$3k^2 - 3k + k - 1 = 0$

$3k(k-1)+1(k-1)=0$

$3k+1=0$ or $k-1=0$

$k=-\frac{1}{3}$ or $k=1$

$\log_{27} x = -\frac{1}{3}$

$27^{-\frac{1}{3}} = x$

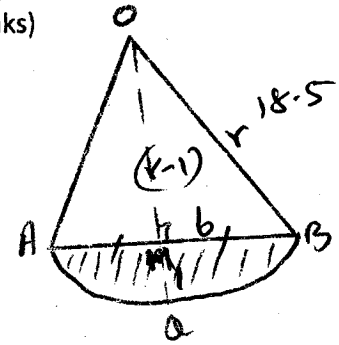
$x=-\frac{1}{3}$

$\log_{27} x = 1$

$27^1 = x$

$x=27$

9 In the figure below A,M,B is a chord of the circle centre O passing through A ,Q and B. OM is a perpendicular bisector of AB. AB is 12 cm and QM is 1cm. Calculate the area of the shaded region. (4mks)



$r^2 = 36 + r^2 - 2r + 1$

$0 = 37 - 2r$

$2r = 37$

$r = 18.5$ cm.

$\sin\theta = \frac{6}{18.5}$

$\theta = 37.85^\circ$

$\frac{37.85}{360}\times3.142\times(18.5)^2$

$= 113.06$cm^2

$\frac{1}{2}\times(18.5)^2\sin 37.85$

105.00cm^2

113.06cm^2

105.00 cm^2

8.06cm^2

10 Find the first five terms of the expansion $[2 - 1/x]^8$. (3mks)

$2^8 - 2^7\times\frac{1}{x} + 2^6\times\frac{1}{x^2} - 2^5\times\frac{1}{x^3} + 2^4\times\frac{1}{x^4}$

1, 8, 28, 56, 70, 56, 28, 8, 1

$256 - \frac{1024}{x} + \frac{1792}{x^2} - \frac{1856}{x^3} + \frac{1120}{x^4}$

$1.75 = (1 - \frac{1}{x})x = 1.3$

$0.25 = -\frac{1}{x}$

$x = \frac{1}{0.75}$

$(2 - \frac{1}{x}) = (1 + 75)$

$-\frac{1}{x} = -0.25$

$0.25x = 1$

$x = \frac{1}{0.25}$

$x = -0.04$

$x = 4$

(3mks)

(b) Hence evaluate $[1.75]^8$

$256 - \frac{1024}{x} + \frac{1792}{x^2} - \frac{1856}{x^3} + \frac{1120}{x^4}$

$256 - \frac{1024}{1.3} + \frac{1792}{(1.3)^2} - \frac{1856}{(1.3)^3} + \frac{1120}{(1.3)^4}$

$256 + 787.69 + 1060.36 + 844.79 + 392.14$

3340.98

11 S varies partly as v and partly as v^2. If S = 31 when v =20 and S = 58 when v = 30, find the value of S when v =25. (3mks)

$S = kv^2 + vc$

$31 = 400k + 20c$

$58 = 900k + 30c$

$93 = 1200k + 60c$

$116 = 1800k + 60c$

$23 = 600k$

$k = \frac{23}{600}$

$c = 0.7833$

43.54

12 A sewing machine valued at sh 25,000 can be bought by cash at a discount of 10% or by instalments whereby a deposit of sh 3,000 is paid followed by 15 monthly instalments of sh 1,500each. Find:

(a) The cash price of the machine. (3mks)

$\frac{90}{100} \times 25,000$

$= 22,500/=$

(b) The hire purchase price of the machine. (3mks)

$\Delta = 3,000$

$15 \times 1,500 = 22500$

$25,500/=$

13 Find the distance between the points 3j + k and 2i + j +k (3mks)

$$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$\sqrt{2^2+2^2+0^2}$

$\sqrt{4+4}$

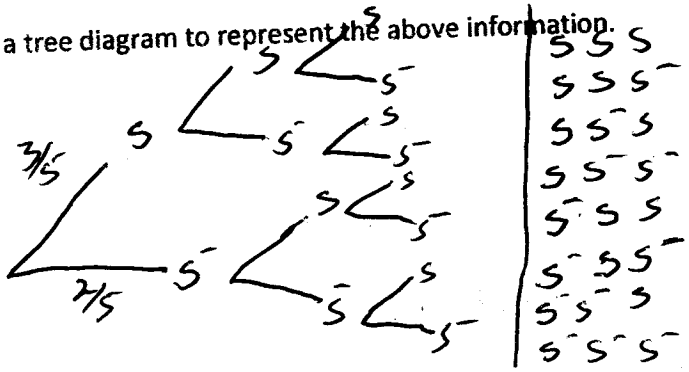
$\sqrt{8}$

2.83

SECTION B 50 MRKS (ANSWER ANY 5 questions)

14 In driving to work jane has to pass through three sets of traffic lights. The probability that she will have to stop at any of the lights is 3/5.

(a) Draw a tree diagram to represent the above information. (2mks)



SSS
SSS⁻
SS⁻S
SS⁻S⁻
S⁻SS
S⁻SS⁻
S⁻S⁻S
S⁻S⁻S⁻

(b) Using the diagram determine the probability that on any one journey she will have to stop at; (2mks)

---All the three sets.

$P(SSS)$

$\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$

(2mks)

--Only one of the sets

$P(SSS^-)$ or $P(SS^-S)$ $P(S^-SS)$

$\frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{18}{125} \times 3$

$\frac{54}{125}$

$\frac{36}{125}$

----Only two of the sets . (2mks)

$P(S\bar{S}\bar{S})$ or $P(\bar{S}S\bar{S})$ or $P(\bar{S}\bar{S}S)$

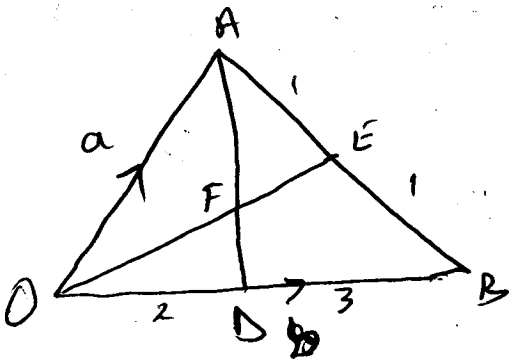
$\frac{3}{5}\times\frac{2}{5}\times\frac{2}{5} = \frac{12}{125}\times 3 = \frac{36}{125}$ $\frac{54}{125}$

---None of the sets. (2mks)

$P(\bar{S}\bar{S}\bar{S})$

$\left(\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}\right) = \frac{8}{125}$

15 In the figure below E is the midpoint of AB. OD : DB =2 : 3 and F is the point of intersection of OE and AD.



(a) Given that OA =a and OB= b, express OE and AD in terms of a and b. (2mks)

$\vec{OE} = \vec{OA} + \vec{AE}$

$\vec{OE} = a + \frac{1}{2}\vec{AB}$

$\vec{OE} = a + \frac{1}{2}(b-a)$

$\vec{OE} = \frac{1}{2}a + \frac{1}{2}b$

$\vec{AD} = \vec{AO} + \vec{OD}$

$\vec{AD} = -a + \frac{2}{5}b$

$\vec{AD} = \frac{2}{5}b - a$

(b) Given that AF =t AD and OF = sOE find the values of s and t . (5mks)

$\vec{OF} = s\vec{OE}$

$\vec{OF} = s(\frac{1}{2}a + \frac{1}{2}b)$

$\vec{OF} = \frac{1}{2}as + \frac{1}{2}sb$

$\vec{OF} = \vec{OA} + \vec{AF}$

$\vec{OF} = a + t\vec{AD}$

$\vec{OF} = a + t(\frac{2}{5}b - a)$

$\vec{OF} = a + \frac{2}{5}tb - at$

$\vec{OF} = a(1-t) + \frac{2}{5}tb$

$(1-t) = \frac{1}{2}s$

$\frac{2}{5}t = \frac{1}{2}s$

$\frac{4}{5}t = s$

$1 - t = \frac{1}{2}(\frac{4}{5}t)$

$1 - t = \frac{2}{5}t$

$1 = \frac{2}{5}t + t$

$1 = \frac{7}{5}t$

$t = \frac{5}{7}$

$s = \frac{4}{5}t$

$s = \frac{4}{8}\times\frac{8}{7}$

$s = \frac{4}{7}$

$s = \frac{4}{7}$

$t = \frac{5}{7}$

(c) Show that O, F and E are collinear. (3mks)

$OF = \frac{1}{2}a + \frac{1}{2}b$

$OF = \frac{1}{2}a \times \frac{4}{7} + \frac{1}{2}b \times \frac{4}{7}$

$OF = \frac{2}{7}a + \frac{2}{7}b$

$OE = \frac{1}{2}a + \frac{1}{2}b$

$\frac{2}{7}(a+b) = OF$

$\frac{1}{2}(a+b) = OE$

$\frac{4}{7}OE = OF$

or

$\frac{7}{4}OF = OE$

16. The table below shows the rates of taxation in a certain year.

Income in k£ p.a	Rate of taxation in SH per k£
1----------3,900	2.
3,901----------7,800	3
7,801----------11,700	4
11,701----------15,600	5
15,601----------19,500	7
Above 19,500	9

In that period Jane was earning a basic salary of ksh 21,000 per month. In addition she was entitled to a house allowance of ksh 12,000 p.m and a personal relief of ksh 1,056 p.m

(a) Calculate how much income tax Jane paid per month. (7mks)

21,000
+12,000
33,000 × 12/20

= 19,800 kf

3,900 × 2 = 7,800

3900 × 3 = 11,700

3900 × 4 = 15,600

3900 × 5 = 19,500

3900 × 7 = 27300

300 × 9 = 2700

sh 84,600

84,600
− 12,672
sh 71,928

71,928 / 12 = 5,994/=

(b) Janes other deductions per month were

--co-operative society contribution 2,000

--Loan repayment 2,500

Calculate his net salary per month (3mks)

$2000 + 2500 = 4500$

$4500 + 5994 = 10,494$

$33,000 - 10\,494 = 22,506$

Sh 22,506

17 The 1st, 5th and 6th terms of an A P are the first three terms of a G P. If the first term of the A .P is 328 find:

(a) The common difference of the A .P. (3mks)

$a, (a+4d), (a+5d)$

$(a+5d) - (a+4d) = c$

$\frac{a+4d}{a} = \frac{a+5d}{a+4d} = r$

$(a+4d)^2 = a(a+5d)$

$a^2 + 8ad + d^2 = a^2 + 5ad$

$3ad + d^2 = 0$

$d(3a+d) = 0$

$d = -984$

(b) The constant ratio of the G .P. (2mks)

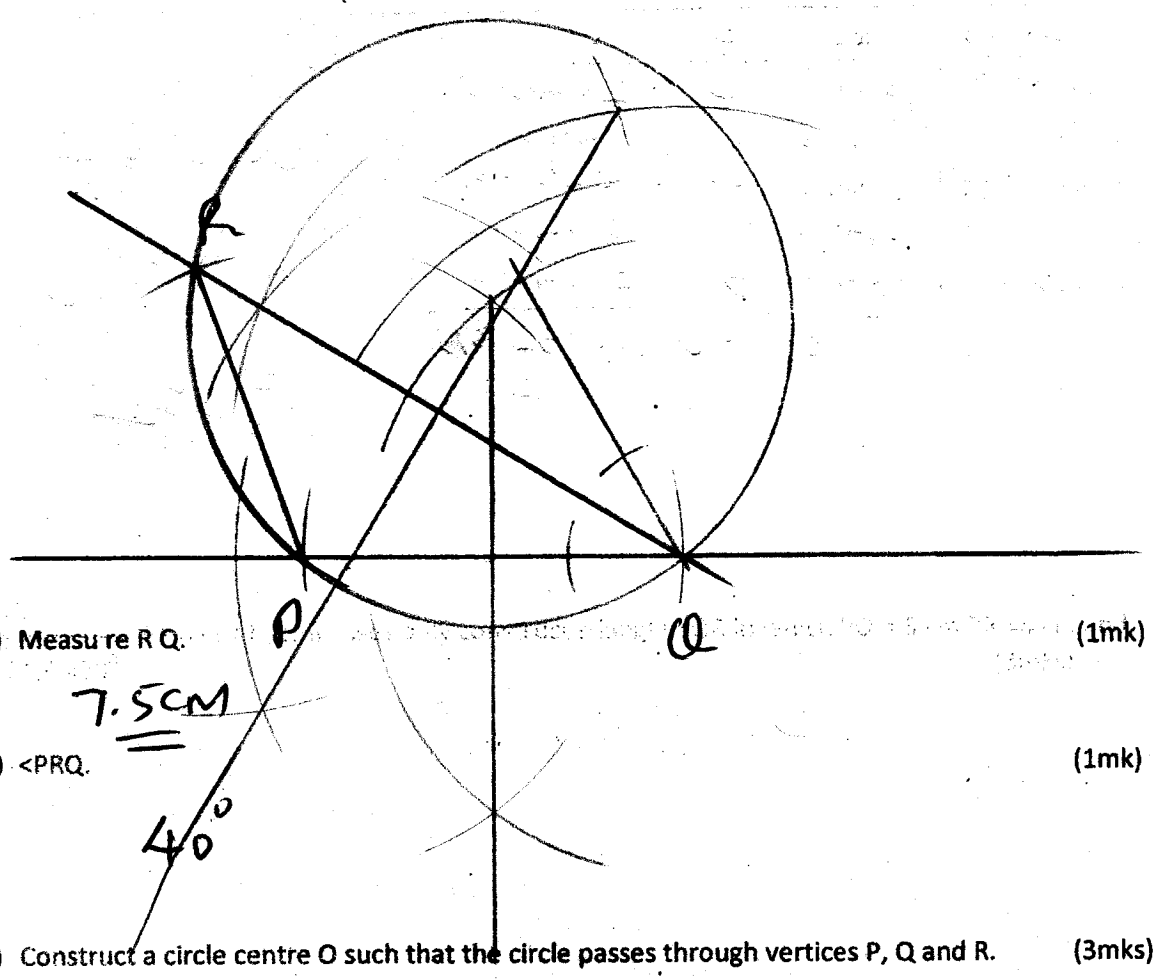
-11

(c) The nth term of the G .P. (2mks)

ar^{n-1}

(d) The sum of the first ten terms of the G .P. (3mks)

18 Using a ruler and a pair of compasses only construct triangle PQR in which PQ = 5 cm PR =4 cm and angle PQR =30^0 (3mks)



(a) Measure R Q. (1mk)

7.5CM

(b) <PRQ. (1mk)

40°

(c) Construct a circle centre O such that the circle passes through vertices P, Q and R. (3mks)

(d) Calculate the area of the circle. (2mks)

$A = \pi r^2$

$A = 3.142 \times 4 \times 4$

$50.272\ cm^2$

$= 50.27\ cm^2$

19 The table below shows the distribution of the wages in a week for 50 employees in a certain factory.

Wage (Ksh)	800---899	900---999	1,000—1,099	1,100—1,199	1,200—1,299
NO of workers	3	10	25	9	3

Using ks 104.5 per week as the assumed mean wage ,calculate

(a) The mean for the grouped wages. (3mks)

Class	x	f	fx
800 - 899	849.5	3	2548.5
900 - 999	949.5	10	9495.
1000 - 1099	1049.5	25	26237.5
1100 - 1199	1149.5	9	10345.5
1200 - 1299	1249.5	3	3748.5
		$\Sigma f = 50$	$\Sigma fx = 52,375$

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$\bar{x} = \frac{52375}{50}$

$= 1047.5$

(b) The standard deviation of the wages. (4mks)

30.71

(c) Estimate the median wage. (3mks)

$999.5 + \left(\frac{12}{25} \times 100\right) = 1047.5$

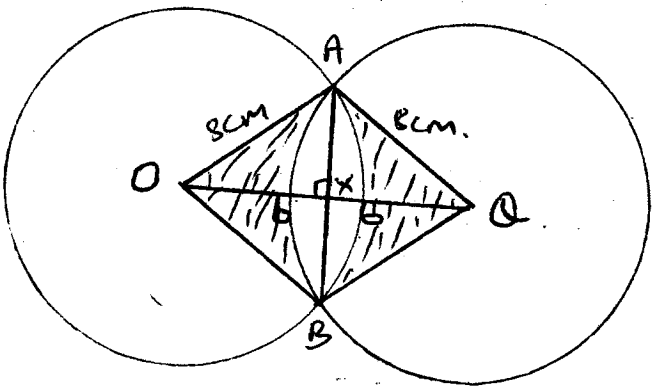
$999.5 + \left(\frac{13}{25} \times 100\right) = 1051.5$

$2,099$

$= 1,049.5$

20 Two equal circles with centres O and Q and a radius 8 cm intersect at points A and B as shown below.

Given that the distance between O and Q is 12 cm and that line A B meets OQ at X.



Find:

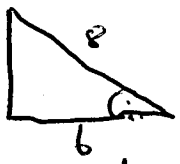
(a)The length of the chord A B. (2mks)

$\sqrt{8^2 - 6^2} = \sqrt{64-36}$

$\sqrt{28} = 5.3\text{cm}$

10.58cm

(c) The area of the shaded region (6mks)



$\cos\theta = \frac{6}{8}$

$\cos\theta = 0.75$

$41.4 \times 2 = 82.82^\circ$

$\frac{82.82}{360} \times 8 \times 8 \times 3.142$

$= 46.26\text{cm}^2$

$\frac{1}{2} \times 8 \times 8 \sin 82.82$

$4 \times 8 \times 0.9922$

$= 31.75\text{cm}^2$

$\times 2$

63.49

46.26

-31.75

14.51

$\times 2$

29.02

63.49

-29.02

34.47

(d) The reflex angle AOB (2mks)

$360 - 82.82$

277.18cm^2