1080/211
MATHEMATICS
Oct./Nov. 2010
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

HIGHER DIPLOMA IN MECHANICAL ENGINEERING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;
Mathematical tables/scientific calculator;
Tables of Laplace transforms and Normal Distribution functions are attached.

*Answer any **FIVE** of the **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) The production schedule of two products A and B is given by:

$$A=\begin{bmatrix}2 & 1 & 0\\ 2 & 5 & 0\\ 0 & -2 & 3\end{bmatrix},\ B=\begin{bmatrix}2 & 1 & -1\\ 1 & -2 & 3\\ -2 & 1 & 2\end{bmatrix},$$

Determine:
(i) $A+B$;
(ii) AB;
(iii) B^TA. (8 marks)

(b) Three forces F_1, F_2 and F_3 acting on a rigid body are such that:

$F_1 + F_2 + 3F_3 = 110N$

$2F_1 + 3F_2 + 4F_3 = 170N$

$4F_1 + 3F_2 + 13F_3 = 460N$

Use Cramer's rule to determine the magnitude of F_1, F_2 and F_3. (12 marks)

2. (a) Derive from first principles, the Laplace transform of $f(t) = t^2 \sin 4t$. (9 marks)

(b) Use Laplace transforms to solve the equation:

$\frac{d^2x}{dt^2} + 9x = \cos 2t$, given that when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 3$. (11 marks)

3. (a) Find the half - range Fourier cosine series of the function $f(x) = x$ on the interval $(0, \pi)$. (8 marks)

(b) A function f(x) is defined by:

$f(x) = x(\pi^2 - x^2)$, $\quad 0 \leq x\ \pi$;

(i) find the Fourier sine series representation of f(x).

(ii) by setting $x = \frac{\pi}{2}$ in the result in (b)(i) above, show that:

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots\ldots$$ (12 marks)

4. (a) (i) Using De Moivre's theorem, prove that

$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.

(ii) by putting $\theta = \frac{\pi}{8}$, show that:

$$\cos\frac{\pi}{8} = \left(\frac{2+\sqrt{2}}{4}\right)^{1/2}$$ (9 marks)

(b) (i) By writing z = tan w in terms of exponential, show that,

$$\tan^{-1}z = \frac{1}{2j}\,ln\left(\frac{1+jz}{1-jz}\right)$$

(ii) Hence prove that $\tan^{-1}\left(\frac{2\sqrt{3}-3j}{7}\right) = \frac{\pi}{6} - \frac{j}{2}\,ln\,2$

(11 marks)

5. (a) Show that the solution of the differential equation

$x^2\frac{dy}{dx} = xy + y^2\frac{dy}{dx}$ may be expressed in the form $y = Ae^{-x^2/2y^2}$ where A is an arbitrary constant. (9 marks)

(b) Use the D-operator method to find the general solution of the differential equation $\frac{d^2y}{dx^2} + y = x\cos x$. (11 marks)

6. (a) Find the equation of the tangent to the curve $x^2y - xy^2 + x^2 + y^2 = 10$ at the point (1,2), using partial differentiation. (6 marks)

(b) Locate the stationary points of the function $z^3 = x^2 + y^2 - 2xy + 5$, and hence determine their nature. (14 marks)

7. (a) Use partial fractions to show that:

$$\int_0^1 \frac{dx}{1+x^3} = \frac{1}{3}\ln 2 + \frac{\pi}{3\sqrt{3}}$$

(12 marks)

(b) Use integration to find the area of the region bounded by the curve $y = 4 - (x-3)^2$ and the line $y = x - 5$. (8 marks)

8. A continuous random variable x has a probability density function f(x) given by

$$f(x) = \begin{cases} k(\tfrac{1}{2}+c) & 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Given that the mean of the distribution is $\frac{10}{9}$, determine the:

(a) values of the constants k and c; (12 marks)

(b) standard deviation; (8 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^n}\right] = \dfrac{1}{(n-1)!}\,t^{n-1}$

$\mathcal{L}[e^{at}] = \dfrac{1}{s-a}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s-a}\right] = e^{at}$

$\mathcal{L}[\sin at] = \dfrac{a}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^2+a^2}\right] = \dfrac{1}{a}\sin at$

$\mathcal{L}[\cos at] = \dfrac{s}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{s}{s^2+a^2}\right] = \cos at$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \dfrac{1}{s}\,\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$\mathcal{L}[e^{at}f(t)] = F(s-a)$ $\qquad$ $\mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$

Second Differentiation Formula

$\mathcal{L}[t^nf(t)] = (-1)^n\dfrac{d^n}{ds^n}\,\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$

Second Shift Formula

$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)]$ $\qquad$ $\mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$

NORMAL DISTRIBUTION FUNCTION

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2}\, dt$$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	9.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998