5.0 MATHEMATICS (121)

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This Mathematics report is based on an analysis of performance of candidates who sat the year 2007 KCSE Mathematics examination. The KCSE Mathematics examination tested the candidates' abilities in two papers; ***paper 1 (121/1)*** and ***paper 2 (121/2)***. The two papers are equally weighted and each is marked out of one hundred percent. The two papers compliment each other to cover the entire syllabus; paper 1 (121/1) tests ***mainly*** forms 1 and 2 work, while paper 2 (121/2) tests ***mainly*** forms 3 and 4 work. It is hoped that this report will be helpful to teachers in the teaching/learning process and in preparing candidates for future examinations as well.

5.1 CANDIDATES' GENERAL PERFORMANCE

The table below shows the overall performance for both papers in the last four years.

Table 8: Candidates Overall Performance in Mathematics for the Last Four Years

Year	Paper	Candidature	Maximum Score	Mean Score	Standard Deviation
2004	1		100	14.57	15.42
	2		100	22.63	20.43
	Overall	**221,295**	**200**	**37.20**	**35.85**
2005	1		100	14.87	15.73
	2		100	17.04	16.74
	Overall	**259,280**	**200**	**31.91**	**31.00**
2006	1		100	22.71	20.09
	2		100	15.36	15.97
	Overall	**238,684**	**200**	**38.08**	**35.00**
2007	1		100	19.55	19.09
	2		100	19.91	20.74
	Overall	**273,504**	**200**	**39.46**	**39.83**

From the table above, the following observations can be made:

5.1.1 The overall mean in Mathematics showed a slight improvement in the year 2007 compared to the previous years.

5.1.2 The overall standard deviation also improved compared to the previous years.

5.2 INDIVIDUAL QUESTION ANALYSIS

Questions in which candidates' performance was poor have been identified and are analyzed in detail in the discussion that follows.

5.2.1 PAPER (121/1)

Question 6

Simplify the expression: $\dfrac{15a^2b-10ab^2}{3a^2-5ab+2b^2}$.

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This question tested the candidates' ability to factorize quadratic expressions.

Weaknesses

Majority of the candidates could not factorize the quadratic expression. Others could not identify common factors to be able to simplify the given expression.

Expected Responses

Candidates were expected to factorize the common factors for both the numerator and denominator and then simplify the factorized form as follows:

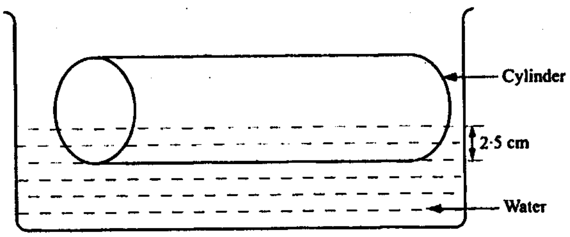
$$\frac{15a^2b-10ab^2}{3a^2-5ab+2b}=\frac{5ab(3a-2b)}{(3a-2b)(a-b)}=\frac{5ab}{a-b}$$

Advice to Teachers

Teachers are advised to be keen in teaching this topic in forms 1 and 2.

Question 9

A cylindrical solid of radius 5 cm and length 12 cm floats lengthwise in water to a depth of 2.5 cm as shown in the figure below.



Calculate the area of the curved surface of the solid in contact with water, correct to 4 significant figures.

This question tested the candidates' knowledge in measurement involving a subtended angle and surface area of a solid.

Weaknesses

Candidates saw the cylindrical solid as a test tube and hence failed to calculate the surface area under water.

Expected Responses

Candidates needed to work out the angle subtended at the centre by the arc that is submerged in the water. That angle will enable one to calculate the area of the curved surface of the solid in contact with water, correct to 4 significant figures, that is, using figure 1 below:

$$\text{Cos}\,\theta=\frac{2.5}{5}=0.5$$

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$\theta = 60^\circ$

Surface area under water:

$$= \frac{2 \times 60}{360} \times \pi \times 10 \times 12 = 125.7cm^2$$

Advice to Teachers

Teachers are advised to teach the topic of solids thoroughly.

Question 11

In fourteen years time, a mother will be twice as old as her son. Four years ago, the sum of their ages was 30 years. Find how old the mother was, when the son was born.

This question tested candidates' ability to form and solve algebraic equations from word problems.

Weaknesses

Candidates could not comprehend the given information and thus, were unable to interpret and form the expected equations.

Expected Responses

Letting m and s represent the current ages of the mother and son respectively, the candidates were required to form the following simultaneous equations,

$m + 14 = 2(s + 14)$

$(m - 4) + (s - 4) = 30$

$m = 2s + 14$

$\Rightarrow m + s = 38$

$2s + 14 + s = 38$

$s = 8$

$m = 30$

Mothers' age when son was born $= 30 - 8 = 22 years$

Advice to Teachers

Teachers are advised to thoroughly teach algebraic equations which are found in form one work.

Question 12

(a) Draw a regular pentagon of side 4 cm.

(b) On the diagram drawn, construct a circle which touches all the sides of the pentagon.

This question tested on construction skills and required the knowledge of regular polygons (pentagon), that is, equal angles and sides.

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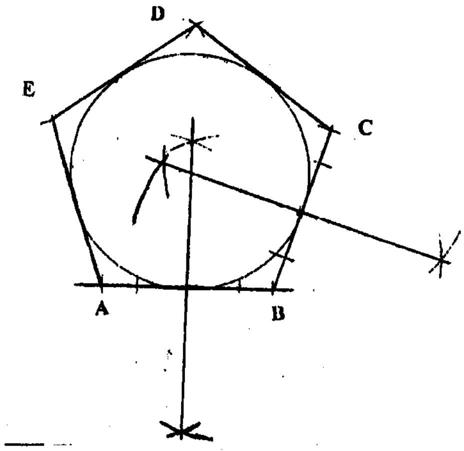
Weaknesses

Some candidates did not have any idea of what a pentagon is. Others could not determine the angle at the centre as 72° (from 108°).

Expected Responses

Candidates were expected to use the angle 108° to construct the pentagon. They were then expected to determine the centre of the incircle from the constructed pentagon. Lastly, the candidates were then required to construct the inscribed circle.



Advice to Teachers

Teachers are advised to give more examples in the area of plane figures.

Question 13

The sum of two numbers x and y is 40. Write down an expression, in terms of x, for the sum of the squares of the two numbers.
Hence determine the minimum value of $x^2 + y^2$.

This question tested candidates' knowledge on formation, subject making of a formula, minima and maxima, differentiation, formation and solving of equations.

Weaknesses

Most candidates found this question to be very difficult as they were unable to demonstrate all the above required skills.

Expected Responses

Given that $x + y = 40 \Rightarrow y = 40 - x$, then the expected sum of the squares in terms of x is:

$$s = x^2 + (40 - x)^2 = 2x^2 - 80x + 1600$$

Since $\frac{ds}{dx} = 0$

$\Rightarrow 4x - 80 = 0$

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$$x = 20$$

Thus, the sum of the squares

$$= 20^2 + (40 - 20)^2$$

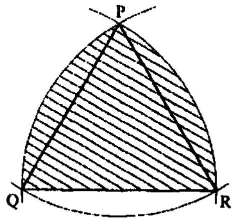
$$= 800$$

Advice to Teachers

Teachers are advised to use more examples during instruction.

Question 14

In the figure below, PQR is an equilateral triangle of side 6 cm. Arcs QR, PR and PQ are arcs of circles with centres at P, Q and R respectively.



Calculate the area of the shaded region to 4 significant figures.

This question tested candidates' knowledge on sectors, segments, triangles, relationship between them and area calculation.

Weaknesses

Candidates could not visualize that these were three sectors. Many could not therefore find the area of the segment.

Expected Responses

Candidates were expected to work out the area of each sector, first, as:

$$= \frac{60}{360} \times \pi \times 6^2 = 18.84955592$$

Then working out area of the triangle:

$$= \frac{1}{2} \times 6 \times 6 \times Sin60° = 15.58845727$$

Area of the shaded region:

$$= 15.58845727 + 2(18.84955592 - 15.5884527)$$

$$= 15.58845727 + 6.522197303$$

$$= 22.11065457$$

$$= 22.11$$

Advice to Teachers

Teachers are advised to offer more practice and exposure to students on calculation of areas of segments and sectors etc.

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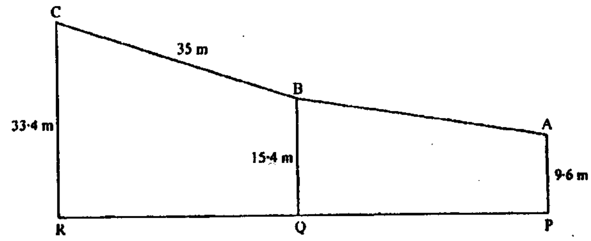
Question 18

In the diagram below, PA represents an electricity post of height 9.6 m. QB and RC represent two storey buildings of heights 15.4 m and 33.4 m respectively. The angle of depression of A from B is 5.5° while the angle of elevation of C from B is 30.5° and BC = 35 m.



(a) Calculate, to the nearest metre, the distance AB.

(b) By scale drawing find,

(i) the distance AC in metres.

(ii) ∠BCA and hence determine the angle of depression of A from C.

This question tested candidates' knowledge on angles of elevation and depression as well as their skills on scale drawing.

Weaknesses

Candidates were unable to understand the meaning of the phrase ***"to the nearest metre"***.

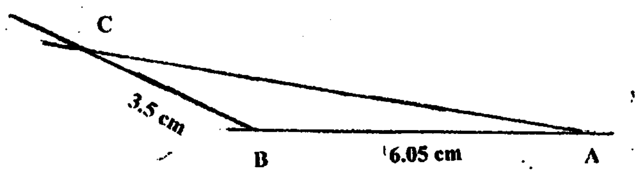
Expected Responses

(a) Candidates needed to find the angle ABQ before calculating the distance AB, that is, Angle ABQ $=180° - 95.5° = 84.5°$. Thus, distance AB = $\frac{5.8}{Cos84.5} = 61$(to the nearest metre.)

(b) (i) Candidates were required to find the distance AC and angle BCA as well as determine the angle of depression of A from C.
Working out angle ABC

$= 95.5° + (90 - 30.5)$
$= 155°$

Using the scale 1cm: 10m or any other, candidates were expected to come up with the figures below:



Measuring distance AC = 9.4 x 10 = 94cm.

(ii) In order to determine the angle of depression of A from C, candidates needed to measure angle BCA as 16°, thus, angle of depression of A from C

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$= \quad 30.5° - 16°$

$= \quad 14.5°$

Advice to Teachers

Teachers are advised to give students more practice on scale drawing, application of angle of depression, elevation etc.

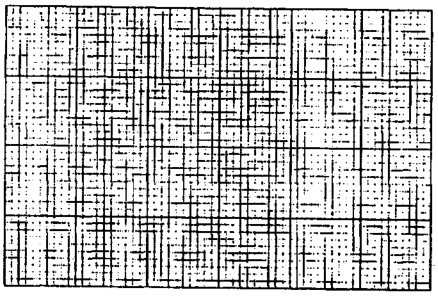
Question 19

A frequency distribution of marks obtained by 120 candidates is to be represented in a histogram. The table below shows the grouped marks, frequencies for all the groups and also the area and height of the rectangle for the group 30–60 marks.

Marks	0-10	10–30	30–60	60-70	70–100
Frequency	12	40	36	8	24
Area of rectangle			180		
Height of rectangle			6		

(a) (i) Complete the table.

(ii) On the grid provided below, draw the histogram.



(b) (i) State the group in which the median mark lies.

(ii) A vertical line drawn through the median mark divides the total area of the histogram into two equal parts.
Using this information or otherwise, estimate the median mark.

This question tested candidates' ability to draw a histogram on given data of unequal classes.

Weaknesses

Many candidates could not fill the data as required. Majority of the candidates completely avoided this question and as a result the question was very unpopular.

Expected Responses

(a) (i) Candidates were required to use the fact that in a histogram the area of each rectangle is proportional to the frequency of the class represented by that rectangle.

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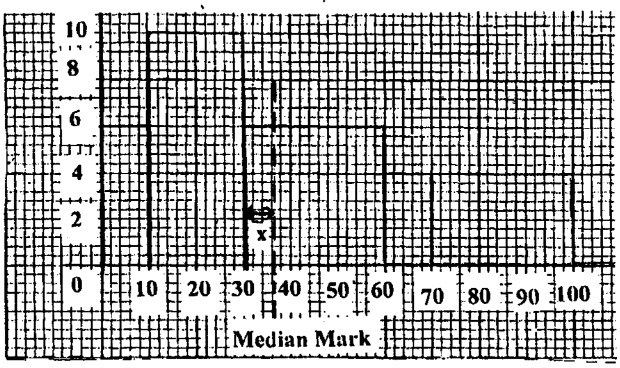
In order to obtain the area of rectangles, candidates needed to multiply the frequencies by 5. To obtain the heights of the rectangles, they were required to divide the area of each rectangle by its width.

Candidates should then have obtained values as indicated in the table below.

Marks	0 - 10	10 - 30	30 - 60	60 - 70	70 – 100
Frequency	12	40	36	8	24
Area of rectangle	**60**	**200**	180	**40**	**120**
Height of rectangle	**6**	**10**	6	**4**	**4**

(ii) Below is the drawing of the histogram that candidates were required to draw:



(b) The median group is (30 – 60) and candidates were required to work out the median as follows:

$$60 + 20 + 6x = \tfrac{1}{2}(60 + 200 + 180 + 40 + 120)$$

$$x = 6\tfrac{2}{3}$$

$$\text{Median} = 30 + 6\tfrac{2}{3} = 36\tfrac{2}{3}$$

Advice to Teachers

Teachers are advised to complete the syllabus and give candidates more practice using as many examples as possible to enable them comprehend the concepts involved.

5.2.2 PAPER 2 (121/2)

Question 3

Solve the equation $3 \cos x = 2 \sin^2 x$, where $0° \leq x \leq 360°$.

This question tested candidates' ability to solve a trigonometric equation involving cos x and $\sin^2 x$ in the interval $0° \leq x \leq 360°$.

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Weaknesses

Some candidates failed to get a correct substitution for $\sin^2 x$ in terms of cos x. Others were unable to factorize the quadratic expression which resulted after substitution.

Expected Responses

Substituting $1 - \cos^2 x$ for $\sin^2 x$,

$$3\cos x = 2(1-\cos^2 x)$$
$$2\cos^2 x + 3\cos x - 2 = 0$$
$$(2\cos x - 1)(\cos x + 2) = 0$$
$$\cos x = \tfrac{1}{2} \; or \; \cos x = -2$$
$$x = 60°, 300°$$

Advice to Teachers

Teachers are advised to give enough practice to students on solving trigonometric equations which involve cosine, sine and their powers.

Question 8

A rectangular block has a square base whose side is exactly 8 cm. Its height, measured to the nearest millimetre, is 3.1 cm.
Find in cubic centimetres, the greatest possible error in calculating its volume.

This question tested candidates' knowledge on calculation of errors. Candidates were required to find the greatest possible error, when calculating volume of a rectangular block whose base is a square and side is exact, while the height is measured to the nearest millimeter.

Weaknesses

Some candidates were unable to interpret an exact measurement. Other candidates could not find the error because they lacked the relevant knowledge.

Expected Responses

Absolute error =	0.05	
Actual volume =	8 x 8 x 3.1	= 201.6
Maximum volume=	8 x 8 x 3.15	= 201.6
Error in volume =	201.6 – 198.4	= 3.2cm3

Alternatively, the greatest possible error

$$= \frac{64(3.15-3.05)}{2}$$
$$= \frac{201.6-195.2}{2}$$
$$= 3.2cm^3$$

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Advice to Teachers

Teachers should guide learners to revise errors related to calculation of areas and volumes and to identify errors in measurement, when measurements are made to some given units.

Question 10

A carpenter wishes to make a ladder with 15 cross-pieces. The cross-pieces are to diminish uniformly in lengths from 67 cm at the bottom to 32 cm at the top.
Calculate the length, in cm, of the seventh cross-piece from the bottom.

This question tested candidates' ability to apply arithmetic progression in a real life situation.

Weaknesses

Candidates failed to identify that the question required the use of Arithmetic Progression (AP).

Expected Responses

Working out the common differences:

$$d = \frac{67-32}{14} = 2.5$$

The length of the seventh cross-piece from the bottom can then be worked out as:

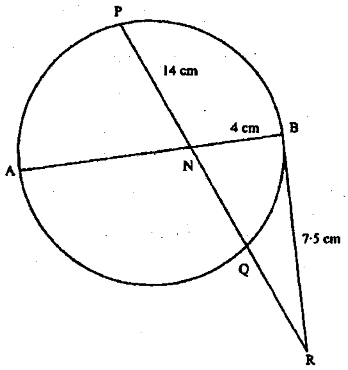
$$T7 = 67 - 6 \times 2.5$$
$$= 52\text{cm}$$

Advice to Teachers

Teachers are advised to guide students in doing simple problems involving Arithmetic Progression, and eventually give application questions.

Question 11

In the figure below AB is a diameter of the circle. Chord PQ intersects AB at N. A tangent to the circle at B meets PQ produced at R.



Given that PN = 14 cm, NB = 4 cm and BR = 7.5 cm, calculate the length of:

(a) NR

(b) AN.

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This question tested candidates' knowledge on calculations of lengths in a circle and tangent to the circle. Candidates were required to use Pythagoras theorem and the properties of intersecting chords internally and externally.

Weaknesses

Most candidates could not apply the properties of intersecting chords correctly. However, the use of Pythagoras theorem was well done generally.

Expected Responses

(a) To obtain the length of NR, candidates needed to realize that angle ABR is a right angle.
$NR = \sqrt{4^2 + 7.5^2} = 8.5cm$

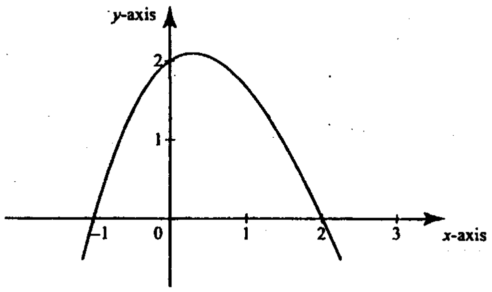
(b) To calculate AN, candidates' needed to use the properties of intersecting chords both internally and externally.

$QR(14 + 8.5) = 7.5^2$ (External property)
$QR = 2.5$
$4 \times AN = 14(8.5 - 2.5)$ (Internal property)
$AN = \frac{14 \times 6}{4} = 21cm$

Advice to Teachers

Teachers should derive the properties of intersecting chords with learners and give them adequate practice.
Question 14

The figure below is a sketch of the graph of the quadratic function $y = k(x + 1)(x - 2)$.



Find the value of k.

This question tested candidates' knowledge on graphical interpretation of a given quadratic function. Given a sketch of a graph of a quadratic function, candidates were required to find a constant k of the given function.

Weaknesses

Most candidates were unable to identify the values to substitute to be able to find k.

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Expected Responses

When $x = 0, y = 2$ $\quad$ $2 = k \times 1 \times {}^{-}2$

$k = 1$

Advice to Teachers

Teachers are advised to guide pupils to sketch quadratic functions and study the y and x intercepts.

Question 15

Simplify $\frac{3}{\sqrt{5}-2} + \frac{1}{\sqrt{5}}$ leaving the answer in the form $a + b\sqrt{c}$, where a, b and c are rational numbers.

This question tested candidates' ability to rationalize and simplify surds. Candidates' were required to rationalize and simplify surds that are added together.

Weaknesses

Candidates lacked the skills of rationalizing surds or adding the two surds. Several candidates were unable to simplify the surds to the required form, that is, $a + b\sqrt{c}$, where a, b and c are rational numbers.

Expected Responses

$$\frac{3}{\sqrt{5}-2} + \frac{1}{\sqrt{5}} = \frac{3(\sqrt{5}+2)}{5-4} + \frac{1\sqrt{5}}{5}$$

$$= 3\sqrt{5} + 6 + \frac{1\sqrt{5}}{5}$$

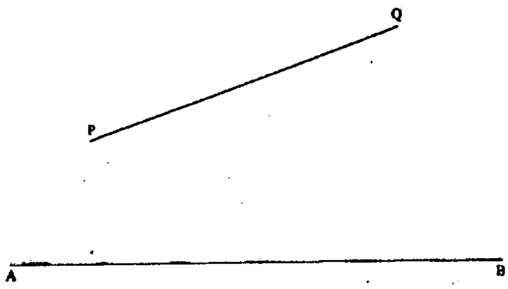
$$= 6 + \frac{16\sqrt{5}}{5}$$

Advice to Teachers

Teachers should try to equip learners with the technical know how of rationalizing and simplifying surds.

Question 21

In this question use a ruler and a pair of compasses only.
In the figure below, AB and PQ are straight lines.



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(a) Use the figure to:

(i) find a point R on AB such that R is equidistant from P and Q

(ii) complete a polygon PQRST with AB as its line of symmetry and hence measure the distance of R from TS.

(b) Shade the region within the polygon in which a variable point X must lie given that X satisfies the following conditions:

I: X is nearer to PT than to PQ

II: RX is not more than 4.5 cm

III: $\angle PXT > 90°$

This question tested candidates' skills in construction and knowledge of loci. Candidates were required to draw different loci as given in the question. Candidates were to undertake the following tasks:

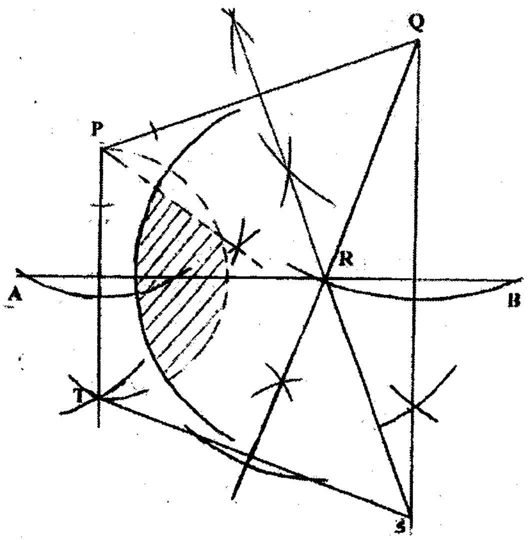
- Construct perpendicular bisector of PQ and mark point R.
- Drop the perpendicular from Q to AB or transfer angle QRB to angle BRS and mark RS equal to RQ.
- Construct the perpendicular from P to AB.
- Double the constructed perpendicular from P to AB to have PT and complete the polygon.
- Measure R from TS = 4.6 $\pm$ 0.1 cm.
- Draw the bisector of angle QPT dotted.
- Draw arc centre R with radius = 4.5cm.
- Draw a semi circle with PT as diameter dotted.
- Correctly shade the required region.

Weaknesses

Majority of the candidates failed to understand the question completely. Most did not shade the required regions, use a ruler and a pair of compasses to show evidence as required.

Expected Responses

Candidates were expected to produce the diagram below:



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Advice to Teachers

Teachers are advised to introduce this topic on loci by using simplified examples.

Question 24

Two bags A and B contain identical balls except for the colours. Bag A contains 4 red balls and 2 yellow balls. Bag B contains 2 red balls and 3 yellow balls.

(a) If a ball is drawn at random from each bag, find the probability that both balls are of the same colour.

(b) If two balls are drawn at random from each bag, one ball at a time without replacement, find the probability that;

(i) the two balls drawn from bag A or bag B are red,

(ii) all the four balls drawn are red.

This question tested candidates' knowledge on probability. Given balls in two bags candidates were required to find the probability that both balls are of the same color if drawn at random. They were required to find the probability that the two balls drawn from bag A or bag B are red (without replacement).

Weaknesses

Many candidates were unable to establish the required probability space. Many failed to address the issue of drawing one ball at a time without replacement.

Expected Responses

(a)

P (Both red) $=\frac{4}{6}\times\frac{2}{5}=\frac{8}{30}$

P (Both yellow) $=\frac{2}{6}\times\frac{3}{5}=\frac{6}{30}$

(P same color) $=\frac{8}{30}+\frac{6}{30}$

(b) (i)

P (the two balls from bag A are Red) $=\frac{4}{6}\times\frac{3}{5}=\frac{2}{5}$

P (the two balls from bag B are Red) $=\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$

P (Both Red for bag A or bag B) $=\frac{2}{5}+\frac{1}{10}=\frac{1}{2}$

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(b) (ii)

P(All the four balls one Red) $= \frac{2}{5} \times \frac{1}{10} = \frac{1}{25}$

Advice to Teachers

Tcachcrs arc adviscd to givc pupils practicc in rcal lifc probability qucstions.

5.3 GENERAL COMMENTS

5.3.1 The analysis of the year 2007 KCSE Mathematics examination papers revealed that candidates had difficulties in tackling questions that required ***drawing and construction*** skills just as has been the case in previous years. ***Question 18*** in paper 1(121/1) and ***question 21*** in paper 2 (121/2) attest to this assertion. Teachers should ensure students are provided with enough practice on these skills.

5.3.2 ***Sequential*** and ***logical*** display of pupils work is paramount at this level. Teachers are therefore advised to emphasize on this aspect during teaching. They should also emphasize on ***accuracy*** of calculations.

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