4.2 MATHEMATICS ALT. B (122)

4.2.1 Mathematics Alt. B Paper 1 (122/1)

SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1 Without using a calculator, evaluate:

$^{-}3(^{-}5 - {}^{+}7) \div {}^{+}2(^{-}3 + {}^{-}6)$. (3 marks)

2 The first four prime numbers are written in descending order to form a number.

(a) Write down the number. (1 mark)

(b) Find the total value of the hundreds digit in the number. (1 mark)

3 Without using a calculator evaluate:

$$\frac{\frac{2}{3} \text{ of } 5\frac{2}{5} - 2\frac{3}{10}}{\frac{3}{5} \div 4\frac{1}{2} + 1\frac{3}{5}}$$

(3 marks)

4 Tito owned Ksh 600 to Nekesa, Ksh 750 to Mwita and Ksh 650 to Auma. He had Ksh 1200 to repay to the three people in proportion to what he owed them. Calculate the amount of money Mwita received more than Nekesa. (3 marks)

5 Given that r = 2 and h = 3r − 1, evaluate $\frac{7r^2 + 2rh}{\sqrt{4h - 2r}}$. (3 marks)

6 The surface area of a cube is 1176 cm^2. Determine the length of one of its sides. (3 marks)

7 By construction, divide the line PQ below into six equal parts. (3 marks)

8 Given that $\tan x = \frac{3}{4}$ and x is an acute angle, without using mathematical tables or a calculator, find the value of $2 \sin x - \cos x$. (3 marks)

9 A box contains five shillings coins and ten shillings coins. The number of ten shillings coins are 6 times as many as the five shillings coins. The total value of all the coins in the box is Ksh 2600. Determine the total number of coins in the box. (4 marks)

10 Simplify $\frac{3^{-2} \times 81^{\frac{3}{2}}}{4^{-3} \div 8^{\frac{1}{3}}}$, leaving your answer in index form. Hence evaluate the expression. (4 marks)

11 A retailer bought a mobile phone for Ksh 5750. The marked price at the retailer's shop was 12% higher than the buying price. After allowing a certain discount, the retailer sold the mobile phone for Ksh 6118. Calculate the percentage discount. (3 marks)

12 Factorise $9a^2 - \frac{16}{b^2c^2}$ (2 marks)

13 Three types of books A, B and C were each piled on a table to attain the same height. The thickness of the books were 12 mm, 28 mm and 54 mm for types A, B and C respectively. Find:

(a) the least height attained; (3 marks)

(b) the number of type A books piled. (1 mark)

14 The sum of the interior angles of a regular polygon is 1260°. Find the size of each interior angle. (3 marks)

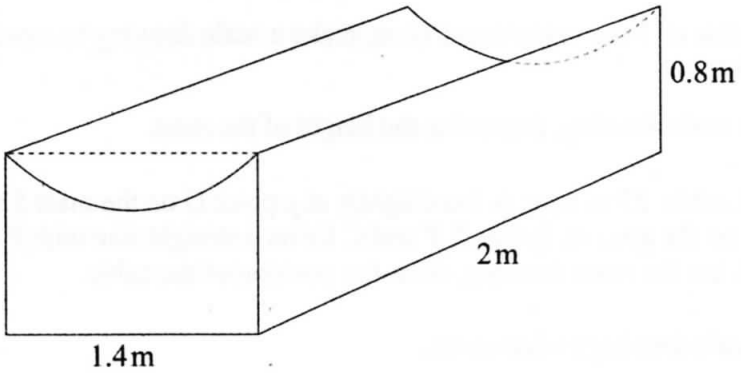
15 The corresponding lengths of two similar triangles are 5 cm and 7.5 cm. If the area of the larger triangle is 22.5 cm^2, calculate the area of the smaller triangle. (3 marks)

16 The area of a sector of a circle is 77 cm^2. The arc of the sector subtends an angle of 45° at the centre of the circle. Find the circumference of the circle. (Take $\pi = \frac{22}{7}$). (4 marks)

SECTION II (50 marks)

*Answer only **five** questions in this section in the spaces provided.*

17 The figure below represents a solid prism with a semi-circular groove. The dimensions are as shown.



(a) Calculate:

(i) the volume of the prism; (4 marks)

(ii) the total surface area of the prism. (4 marks)

(b) All the rectangular faces are painted. Calculate the percentage of the surface of the prism that is painted correct to 1 decimal place. (2 marks)

18 (a) Three vertices of a parallelogram ABCD are A(−7, 3), B(1, −1) and C(5, 1). On the grid provided, draw the parallelogram ABCD. (2 marks)

(b) Determine:

(i) the gradient of the line AB; (2 marks)

(ii) the equation of line AB in the form $y = mx + c$, where m and c are constants. (2 marks)

(c) Another line L is perpendicular to CD and passes through point (1, 3). Determine:

(i) the equation of L in the form $ax + by = c$ where a, b and c are constants; (3 marks)

(ii) the coordinates of the y-intercept of line L. (1 mark)

19 (a) The roots of a quadratic equation are $\frac{1}{2}$ and -1. Write down the quadratic equation in the form $ax^2 + bx + c = 0$, where a, b and c are integers. (3 marks)

(b) (i) Barasa bought $(2y + 1)$ mangoes at y shillings each. The total cost of the mangoes was Ksh 55. Find the cost of each mango. (4 marks)

(ii) Karau spent Ksh 95 more than Barasa to buy the same type of mangoes. For every 6 mangoes he bought, he was given one extra mango. Calculate the total number of mangoes Karau got. (3 marks)

20 The angle of elevation of the top T, of a vertical mast from a point P, 100 m away from the foot F, of the mast is 14°.

(a) Using a scale of 1 cm to represent 10 m, make a scale drawing to represent the above information. (3 marks)

(b) Using the scale drawing, determine the height of the mast. (2 marks)

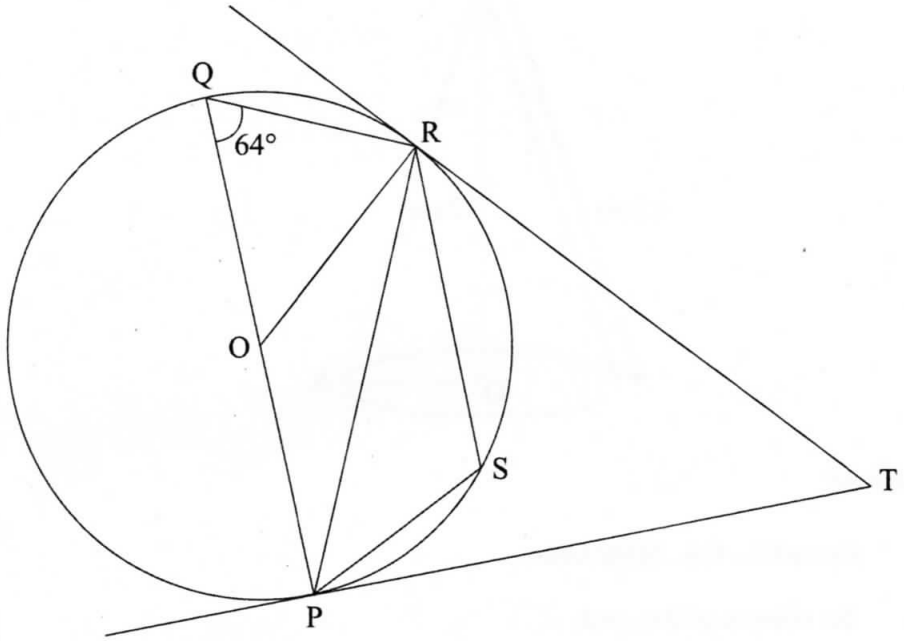
(c) A support cable, 27 m long, is fixed tightly at a point D on the mast 5 m below T and at a point C on the ground. Points P, F and C lie on a straight line with P and C on opposite sides of F. On the scale drawing, show the position of the cable. (2 marks)

(d) Use the scale drawing to determine:

(i) the angle of depression of C from D; (1 mark)

(ii) the distance of C from P. (2 marks)

21 In the figure below, P, Q, R and S are points on the circumference of the circle centre O. TP and TR are tangents to the circle at P and R respectively. POQ is a diameter of the circle and angle PQR = 64°.



Giving reasons in each case, find the size of:

(a) ∠ROP; (2 marks)

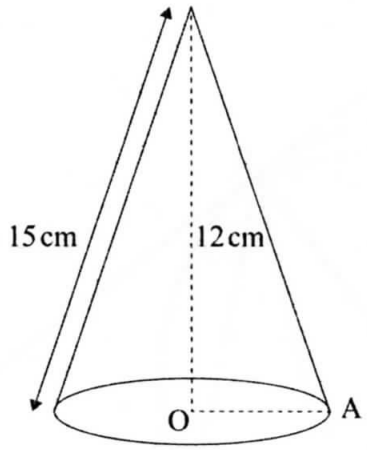
(b) ∠PSR; (2 marks)

(c) ∠ORP; (2 marks)

(d) ∠TRP; (2 marks)

(e) ∠RTP. (2 marks)

22 The figure below represents a cone whose vertical height is 12 cm and slant height is 15 cm.



(a) Calculate:

(i) the radius, OA, of the cone; (2 marks)

(ii) the volume of the cone. (2 marks)

(b) A smaller cone of radius 6 cm is cut off from the cone above to leave a frustum. Calculate:

(i) the height of the smaller cone; (2 marks)

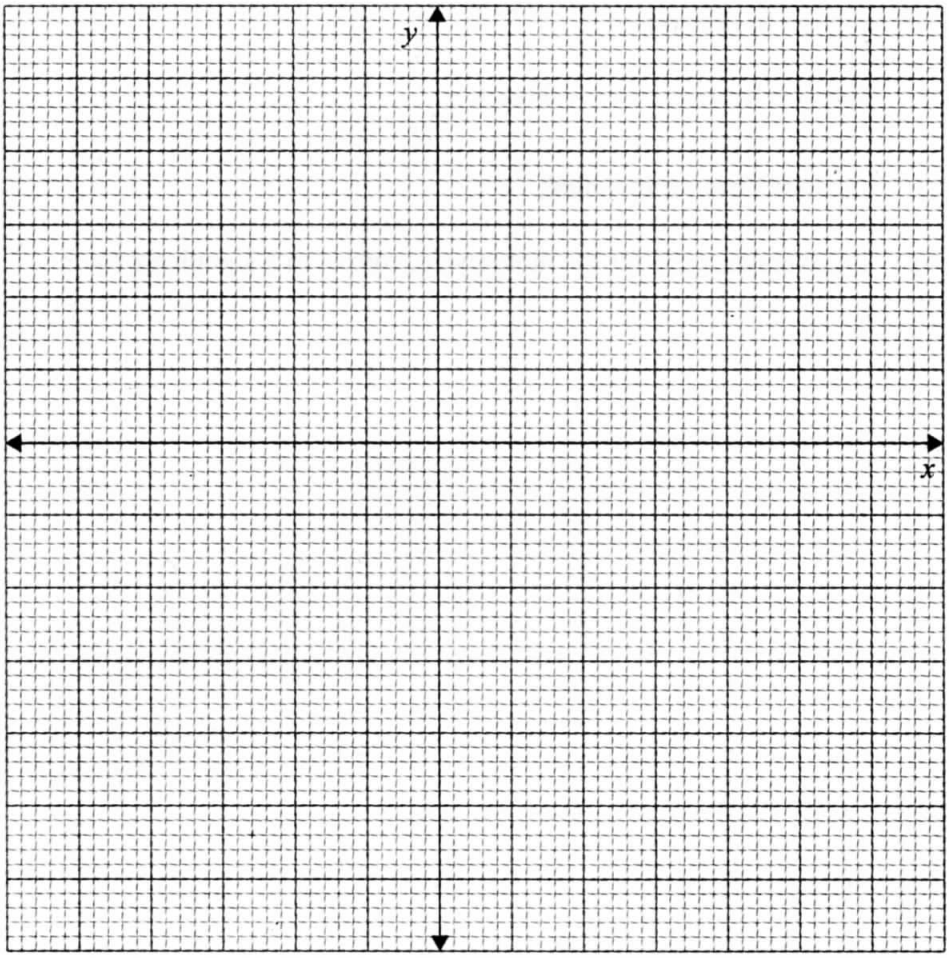
(ii) the volume of the smaller cone; (2 marks)

(iii) the volume of the frustum. (2 marks)

23 The vertices of a trapezium ABCD are A(2, 0), B(4, 0), C(6, 2) and D(2, 2).

(a) On the grid provided below, draw:

(i) the trapezium ABCD; (1 mark)



(ii) A′B′C′D′ the image of ABCD under a reflection in the line $y = -x$; (2 marks)

(iii) A″B″C″D″ the image of A′B′C′D′ under a rotation of $-90°$, centre (0, 0). (2 marks)

(b) Describe a transformation that maps A″B″C″D″ onto ABCD. (2 marks)

(c) State pairs of trapezia that are directly congruent and those that are oppositely congruent. (3 marks)

24 A racing motorcycle started from rest and moved with a constant acceleration of 1 m/s^2 for 15 seconds. It then accelerated at 3.5 m/s^2 for the next 10 seconds and maintained a constant speed for the next 10 seconds. It decelerated constantly and came to rest after 25 seconds.

(a) On the grid provided, draw the velocity–time graph for the motorcycle. (4 marks)

(b) Use the graph to determine:

(i) the deceleration of the motorcycle; (2 marks)

(ii) the total distance travelled; (2 marks)

(iii) the average speed for the motorcycle, correct to 3 significant figures. (2 marks)

4.2.2 Mathematics Alt. B Paper 2 (122/2)

SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1 Round off each of the numbers in the expression $\frac{4.957}{0.2628-0.0149}$ to 3 significant figures, hence evaluate the expression. (2 marks)

2 Given the matrices $\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$, find $\mathbf{AB} - 5\mathbf{B}$. (3 marks)

3 Three types of coffee, A, B and C are mixed such that A:B = 4:3 and B:C = 1:2. Determine the mass of type C in a mixture of 52 kg. (3 marks)

4 In a Geometric Progression (G.P), the 4th term is 24, and the 6th term is 96. Determine:

(a) the common ratio of the G.P; (2 marks)

(b) the first term of the G.P. (2 marks)

5 Two fair dice are rolled together and the sum of the numbers showing on the top faces noted.

(a) Represent all the possible outcomes in a probability space. (2 marks)

(b) Determine the probability that the sum is greater than 6 but less than 10. (1 mark)

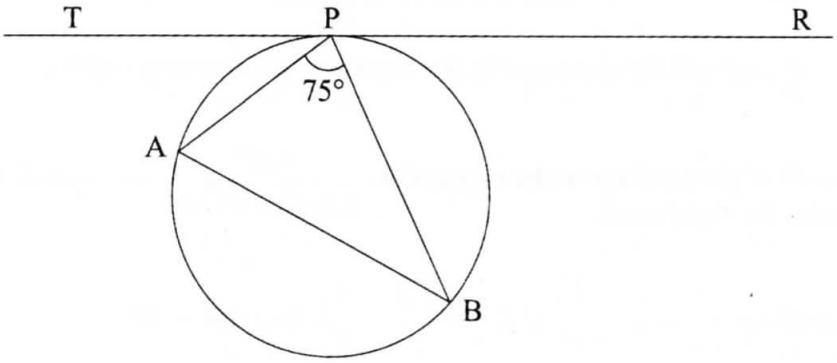
6 Two points A and B are such that $\mathbf{OA} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\mathbf{AB} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$. M is a point on **AB** such that **AM** : **MB** = 3:1.

Determine:

(a) **OB**; (2 marks)

(b) the coordinates of M. (2 marks)

7 In the figure below, TPR is a tangent to the circle at P. Angle APB = 75° and angle BPR is twice angle APT.



Determine the size of angle BAP. (2 marks)

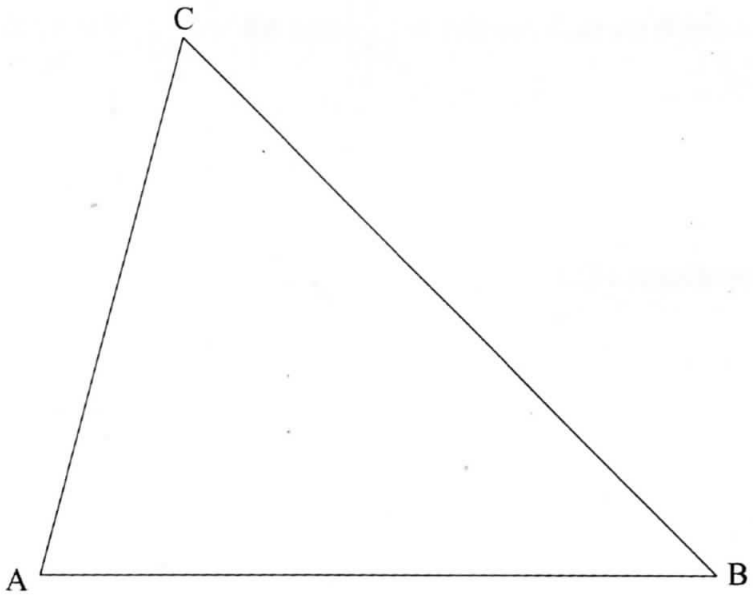
8 Given that $2\cos(x - 30)° = -0.9$, determine the value of x for $0° \leq x \leq 180°$ correct to 2 decimal places. (3 marks)

9 The vertices of a triangle RST are R(1, 3), S(1, 7) and T(−1, 4). Triangle RST is mapped onto triangle R′S′T′ by transformation matrices $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ followed by $\mathbf{Q} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

Find the coordinates of R′S′T′. (3 marks)

10 Using the method of completing the square, solve the equation $2x^2 + 8x = 15$, correct to one decimal place. (3 marks)

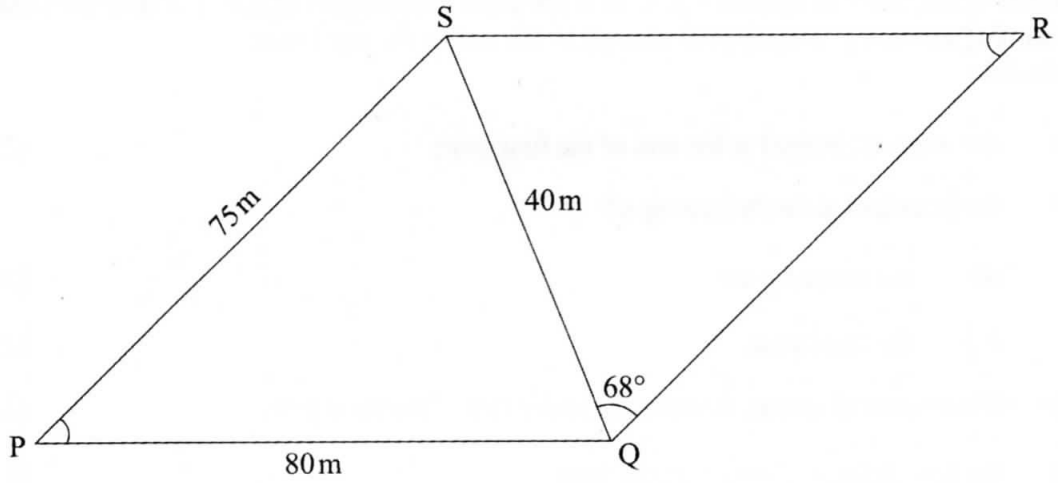
11 (a) Using a ruler and a pair of compasses only, construct an inscribed circle in triangle ABC given below. (2 marks)



(b) Measure the radius of the circle. (1 mark)

12 In a camp, there was enough food to feed 2000 people on equal rations for 90 days. After 20 days 500 more people joined the camp. Calculate the number of days that the remaining food would be used to feed the people. (4 marks)

13 Figure PQRS below represents a garden in which,
PQ = 80 m, PS = 75 m, SQ = 40 m, ∠SQR = 68° and ∠SPQ = ∠SRQ.



Calculate, to 2 significant figures, the length of SR. (3 marks)

14 The table below shows part of income tax rates in a certain year.

Monthly Income in Ksh	Tax Rate in each Shilling
Up to 10 164	10%
From 10 165 to 19 740	15%
From 19 741 to 29 316	20%

In a certain month of that year, Abdala's income was Ksh 21 820. He was entitled to a monthly personal tax relief of Ksh 1162. Calculate the income tax paid by Abdala that month. (4 marks)

15 Ali and Kinjo bought the same type of pencils and rubbers from the same shop. Ali bought 2 pencils and 3 rubbers for Ksh 66, Kinjo bought 7 pencils and 2 rubbers for Ksh 129. Find the cost of a pencil. (3 marks)

16 The table below shows marks scored by students in a mathematics test.

Marks	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99
Number of students	4	6	8	10	9	7	4	2

On the grid provided below, draw an ogive to represent the data. (3 marks)

SECTION II (50 marks)

*Answer only **five** questions from this section in the spaces provided.*

17 Kurao borrowed Ksh 300 000 from a financial institution. The institution charged compound interest at the rate of 18% per annum on the outstanding balance at the end of each year. At the end of the first and second years, he made equal repayments of Ksh 134 000. He repaid the remaining amount of money plus interest at the end of the third year.
Calculate:

(a) the interest charged at the end of the first year; (2 marks)

(b) the principal at the beginning of:

(i) the second year; (2 marks)

(ii) the third year. (2 marks)

(c) the amount of money Kurao paid at the end of the third year. (2 marks)

(d) the total interest charged on the loan. (2 marks)

18 (a) The nth term of a sequence is given by $U_n = n^2 - n + 3$

Determine:

(i) the 10th term of the sequence; (2 marks)

(ii) the difference between the 30th and the 20th terms of the sequence; (2 marks)

(iii) the value of n given that $U_n = 243$. (3 marks)

(b) In a research, it was found that the number of bacteria tripled every hour. Given that the number of bacteria at the start of a certain hour was 180:

(i) write an expression for the number of bacteria after *t* hours; (1 mark)

(ii) determine the number of bacteria, to the nearest million, after 12 hours. (2 marks)

19 The time in minutes each student in a group took to solve a certain mathematics question is shown in the table below.

Time in Minutes	0 – 1	1 – 2	2 – 3	3 – 4	4 – 5	5 – 6	6 – 7	7 – 8
Number of students	2	4	3	5	8	6	5	3

(a) State the modal class. (1 mark)

(b) The above data was represented in a pie-chart, determine the angle that represented the number of students who answered the question in 4 – 5 minutes. (2 marks)

(c) Calculate the mean time taken to solve the question. (4 marks)

(d) On the grid provided, draw a bar graph to represent the data. (3 marks)

20 A curve is represented by the equation $y = -2x^2 + 7x - 3$.

(a) Make a table of the values of x for: $-1 \leq x \leq 4$ and the corresponding values of y for the curve. (2 marks)

(b) On the grid provided, draw the graph of $y = -2x^2 + 7x - 3$ for $-1 \leq x \leq 4$. (3 marks)

(c) Use the graph to determine:

(i) the roots of the equation $-2x^2 + 7x - 3 = 0$; (2 marks)

(ii) the instantaneous rate of change of the curve at $x = 1$. (3 marks)

21 Four vectors are such that $\mathbf{OA} = -2\boldsymbol{i} + \boldsymbol{j}$, $\mathbf{OB} = 3\boldsymbol{i} + 5\boldsymbol{j}$, $\mathbf{OC} = -8\boldsymbol{i} - 12\boldsymbol{j}$ and $\mathbf{OD} = 2\boldsymbol{i} - 4\boldsymbol{j}$.

(a) Express in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$, the vectors:

(i) **AB**; (2 marks)

(ii) **CD**. (2 marks)

(b) Determine the co-ordinates of the mid-point of **AD**. (3 marks)

(c) Calculate to 3 significant figures, the magnitude of **BC**. (3 marks)

22 Two points P and Q lie on the equator. The position of P is (0°, 12° E) and that of Q is (0°, 60° W).
(Take the radius of the earth to be 6370 km and $\pi = \frac{22}{7}$).

(a) (i) Calculate the distance from P to Q in kilometres. (3 marks)

(ii) Determine the local time at Q when the time is 9.00 pm at P. (3 marks)

(b) A point T is due North of Q. An aeroplane flying from Q at 1001 km/h takes 2 h to reach T. Determine the position of T. (4 marks)

23 A relation connecting three variables R, C and T is such that R varies directly as the square of C and inversely as T. When R = 30, C = 6 and T = 2.4.

(a) Find:

(i) the constant of proportionality; (3 marks)

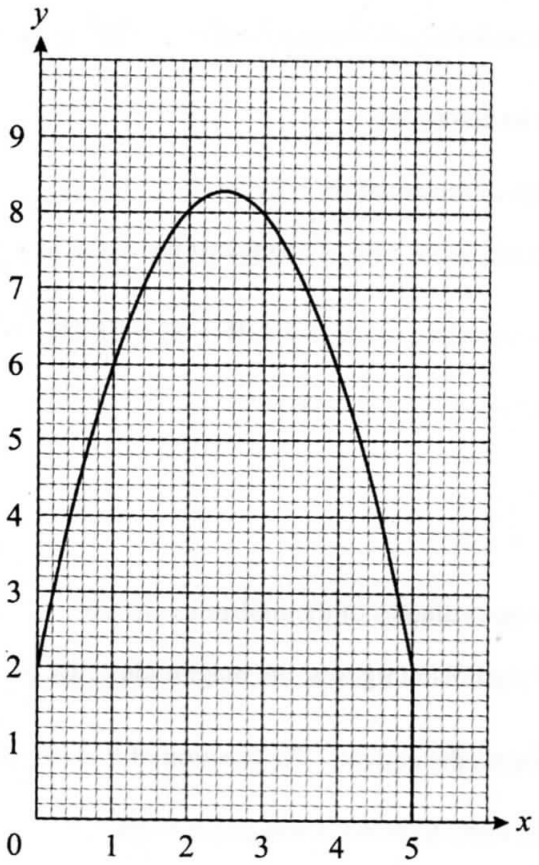
(ii) the equation connecting R, C and T. (1 mark)

(b) Given that R = 40 and C = 8, determine:

(i) the value of T; (2 marks)

(ii) the percentage change in R when C decreases by 10% and T increases by 8%. (4 marks)

24 In the figure below the area bounded by the curve, the y-axis, the x-axis and the line $x = 5$ represents a map of a piece of land.



(a) Estimate the area of the map in cm^2 by:

(i) the counting technique; (2 marks)

(ii) using the trapezium rule with 5 strips of equal width. (3 marks)

(b) Given that the actual area of the map is $30\frac{5}{6}$ cm^2, calculate:

(i) the percentage error, correct to 2 significant figures, when the trapezium rule is used to estimate the area of the map; (2 marks)

(ii) the actual area in hectares of the piece of land if the scale used was 1:12000. (3 marks)