3.0 PART ONE: ANALYSIS OF DIFFICULT QUESTIONS

3.1 MATHEMATICS ALT A (121)

In the year 2019 Mathematics Alternative A was tested in two papers. **Paper 1 (121/1)** and **Paper 2 (121/2)**. Each paper consisted of two sections: Section 1 (50 marks) consisting of 16 compulsory short answer questions of not more than four marks each and Section II (50 marks), with eight questions of 10 marks each where candidates answer any five.

Paper 1 (121/1) tests mainly Forms 1 and 2 work while Paper 2 (121/2) tests mainly forms 3 and 4 work of the syllabus.

This report is based on an analysis of performance of candidates who sat the year 2019 KCSE Mathematics Alt A.

3.1.1 CANDIDATES' GENERAL PERFORMANCE

The table below shows the performance of both papers in the last five years.

Table 8 Candidates' Performance in Mathematics Alt A for the last five years, 2015 – 2018

Year	Paper	Candidature	Maximum Score	Mean Score	Standard Deviation
2015	**1**		100	25.53	20.39
	2	**520274**	100	28.23	22.81
	Overall		**200**	**53.76**	**40.87**
2016	**1**		100	23.74	21.24
	2	**570398**	100	17.84	21.09
	Overall		**200**	**41.56**	**41.20**
2017	**1**		100	24.49	22.03
	2	**609525**	100	26.47	22.43
	Overall		**200**	**50.95**	**43.46**
2018	**1**		100	24.07	21.16
	2	**658904**	100	28.82	20.85
	Overall		200	**52.88**	**41.1**
2019	**1**	**694445**	100	31	24.037
	2	**694347**	100	23	20.904
	Overall		**200**	**55.08**	**43.91**

From the table the following observations can be made:

i) There was an improvement of 6.96 marks in paper 121/1 compared to the previous year 2018. However, in paper 121/2 there was a 5.82 drop in the mean as compared to the previous year 2018.

ii) There was a 2.81 marks improvement in the overall mean.

3.1.2 INDIVIDUAL QUESTION ANALYSIS

The following is a discussion of some of the questions in which the candidates had major weakness in, as a result of which these questions were poorly performed. The discussion is based on comments from the chief examiners reports and from analysis the students' responses and scores from sampled scripts.

3.1.3 Mathematics Paper 1 (121/1)

In section 1 of 121/1, questions 1, 3, 4, 5 and 10 were performed well by most of the candidates. In Section II, question 21 was performed well by the candidates who choose it.
In section II, questions 18 and 21 were the most popular among the candidates.

Question 7
Three villages A, B and C are such that B is 53 km on a bearing of 295° from A and C is 75 km east of B.
(a) Using a scale of 1 cm to represent 10 km, draw a diagram to show the relative positions of villages A, B and C. (2 marks)
(b) Determine the distance, in km, of C from A. (2 marks)

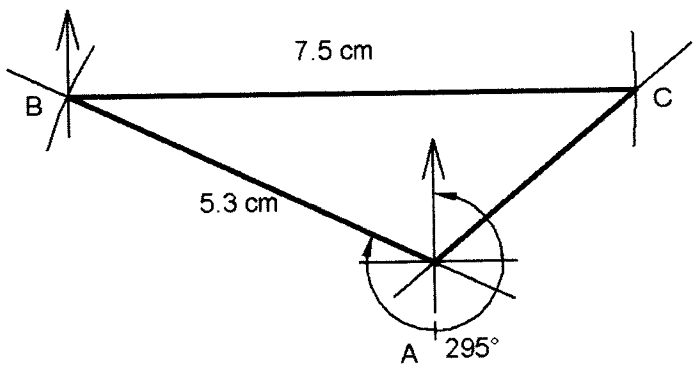
The question tested on solving of problems involving bearing.

Weaknesses

Inability to draw parallel northings.

Expected response

(a)



(b)

$$AC = (3.5 \pm 0.1)\text{cm}$$
$$AC = (35 \pm 1)\text{Km}$$

Advice to teachers

Teachers need to emphasize on parallel northings at various locations in the scale drawing.

Question 9

Given that $\sin 2x = \cos(3x - 10°)$, find $\tan x$, correct to 4 significant figures. (3 marks)

The question tested on use of the relationship of sine and cosine of complimentary angles.

Weaknesses

Most students did not know the relationship of sine and cosine of complementary angles.

Expected response

$$\sin 2x = \cos(3x - 10)$$

$$2x + (3x - 10) = 90$$

$$5x = 100$$

$$x = 20°$$

$$\tan 20° = 0.3640$$

Advice to teachers

In addition to giving sufficient practice on the relationship of sine and cosine of complementary angles, teachers need to emphasize to students on need to know the definitions of sine and cosine in a right angled triangle and how the relationship can be derived from first principles. This will reduce reliance on memory and rote learning.

Question 12

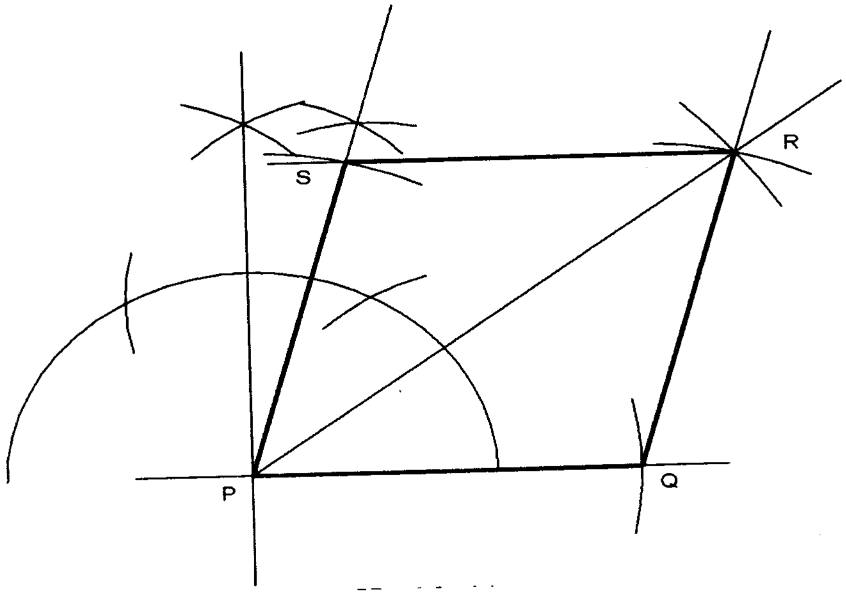
Using a ruler and a pair of compass only, construct a rhombus PQRS such that $PQ = 6$ cm and $\angle SPQ = 75°$. Measure the length of PR. (4 marks)

The question tested on construction of a rhombus using a ruler and a pair of compasses only.

Weaknesses

Most candidates were unable to accurately construct an angle of $75°$ at P using a ruler and a pair of compasses only. Some candidates also had difficulties locating point R either by transferring the angle of $75°$ or by using properties of a rhombus.

Expected response



PR = 9.5 ± 0.1 cm

AAdvice to teachers

More practice in the use of a ruler and a pair of compasses only in the accurately construction of angles whose values are multiples of 7.5°.

Question 14

Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 4 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} x & 1 \\ 2 & 3 \end{pmatrix}$ and that **AB** is a singular matrix, find the value of x.
(3 marks)

The question tested on matrix multiplication and the application of the definition of a singular matrix.

Weaknesses
A number of candidates had problems multiplying the given matrices and also in the identification of the unique property of a singular matrix. (Determinant of a singular Matrix = 0).

Expected response

$$\begin{pmatrix} 2 & 3 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} x & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 2x+6 & 11 \\ 4x+8 & 16 \end{pmatrix}$$

$$\begin{vmatrix} 2x+6 & 11 \\ 4x+8 & 16 \end{vmatrix} = 0$$

$$16(2x+6) - 11(4x+8) = 0$$

$$32x + 96 - 44x - 88 = 0$$

$$32x - 44x = 88 - 96$$

$$-12x = -8$$

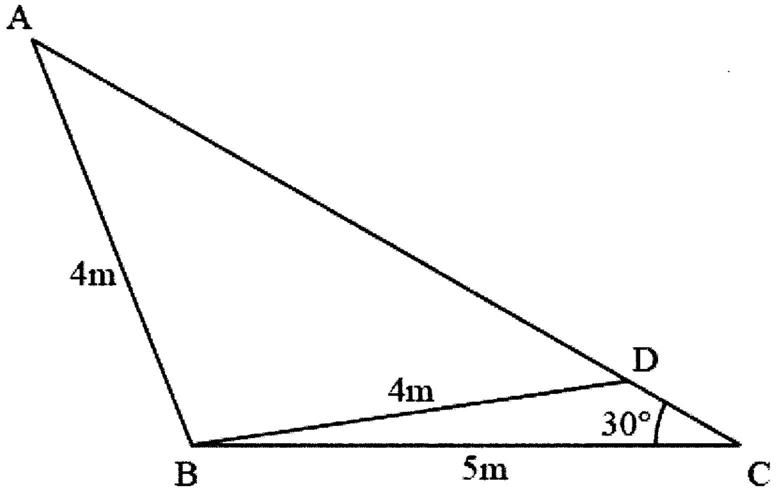
$$x = \frac{2}{3} = 0.\dot{6}$$

Advice to teachers

During introduction stage of subtopics in matrices, teachers need to emphasize on the definitions of the various types of matrices and possible applications in problem solving.

Question 22

The figure below represents a triangular flower garden ABC in which AB = 4 m, BC = 5 m and $\angle BCA = 30°$. Point D lies on AC such that BD = 4 m and $\angle BCD$ is obtuse.



Find, correct to 2 decimal places:

(a) $\angle BDC$; (3 marks)
(b) the length of AD; (3 marks)
(c) the length of DC; (2 marks)
(d) the area of the flower garden ABC. (2 marks)

The question tested on solution of triangles by use of the sine rule and cosine rules.

Weaknesses

A number of candidates assumed that $\angle ABD$ was a right angle. Other could not apply the sine or cosine rules.

Expected Response

(a) Let $\angle BDC = \theta$

$$\frac{\text{Sin } \theta}{5} = \frac{\text{Sin } 30°}{4}$$

$$\text{Sin } \theta = \frac{5 \times \text{Sin} 30}{4} = 0.625$$

Acute $\theta = 38.68°$

Obtuse $\theta = 141.32°$

(b) Length AD

$$\text{Angle ABD} = 180 - 38.68 \times 2$$

$$= 102.64$$

$$AD^2 = 4^2 + 4^2 - 2 \times 4 \times 4 \cos 102.64$$

$$= 39$$

$$AD = 6.24\text{m}$$

(c) Length of DC

$$\angle DBC = 180 - (30 + 141.32)$$

$$= 8.68°$$

Using sine rule

$$\frac{\text{Sin } 8.68}{DC} = \frac{\text{Sin} 30}{4}$$

$\Rightarrow$

$$DC = 8 \text{ Sin } 8.68$$

$$= 1.21\text{m}$$

(d) Area of ABC

$$= \frac{1}{2} \times 4 \times 5 \text{ Sin}(8.68 + 102.64)$$

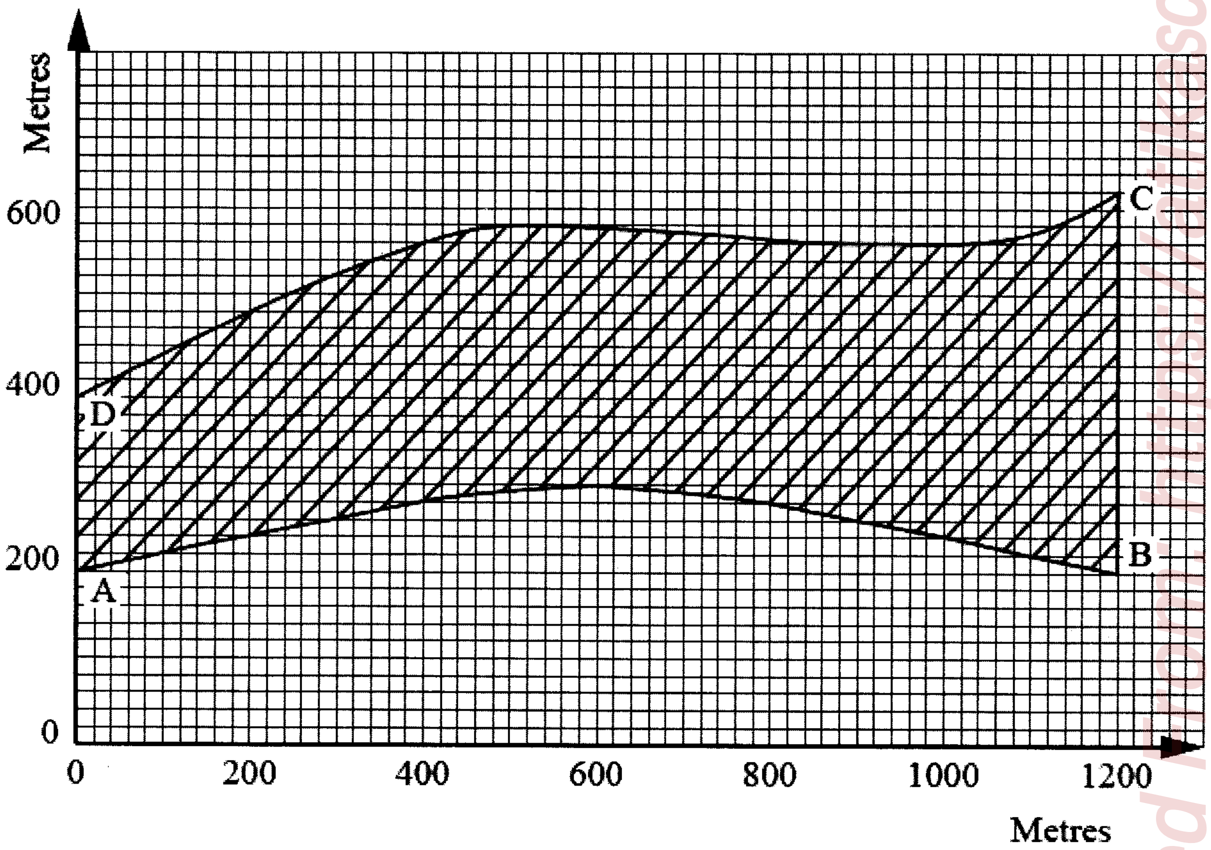
$$= 9.32\, m^2$$

Advice to teachers

Teachers need to advise candidates to read the introduction part before a diagram as it describes the diagram in details and not to make assumptions based on wrong eye judgments. The need to correctly state and apply accurately both the sine and the cosine rules needs to be emphasized to the learners when teaching. Candidates also need to be reminded of the unrestricted access to the formulas at the back of the KNEC Mathematical tables in an exam setting.

Question 23

The shaded region on the graph below shows a piece of land ABCD earmarked for building a sub-county hospital.



(a) Write down the ordinates of curves AB and DC for $x = 0, 200, 400, 600, 800, 1000$ and 1200. (2 marks)

(b) Use trapezium rule, with 6 strips to estimate the area of the piece of land ABCD, in hectares. (4 marks)

(c) Use mid-ordinate rule with 3 strips to estimate the area of the piece of land, in hectares. (4 marks)

The question tested on application of the trapezium and the mid ordinate rules to estimate the areas under curves.

Weaknesses

- Wrong interpretation of the vertical scale which caused the candidates to come up with wrong ordinates.
- Using the wrong strips leading to wrong ordinates.
- Inability to apply the trapezium rule and the mid ordinate rule.

Expected Response

x	0	200	400	600	800	1000	1200
Ordinates along AB	200	240	280	300	280	240	200
Ordinates along CD	400	500	580	600	580*	580	640

For ordinates along CD: At $x = 800$, accept $580 \leq y \leq 590$

(b) Area of piece of land ABCD using trapezium rule

Area under curve AB

$$= \frac{1}{2} \times 200\{(200 + 200) + 2(240 + 280 + 300 + 280 + 240)\}$$

$$= 100(400 + 2680)$$

$$= 308\,000 \text{ m}^2$$

Area under curve CD

$$= \frac{1}{2} \times 200\{(400 + 640) + 2(500 + 580 + 600 + 580 + 580)\}$$

$$= 100\ (1040 + 5680)$$

$$= 672\,000$$

Area of land ABCD

$$= 672000 - 308000$$

$$= 364000 \text{ m}^2$$

$$= \frac{364000}{10000} \text{ ha}$$

$$= 36.4 \text{ ha}$$

(c) Area using mid ordinate Rule:

$= 400\{(500+600+580)-(240+300+240)\}$

$= 400\times 900$

$= 360000\,\text{m}^2$

$= \dfrac{360000}{10000}$

$= 36\,\text{ha}$

Advice to teachers

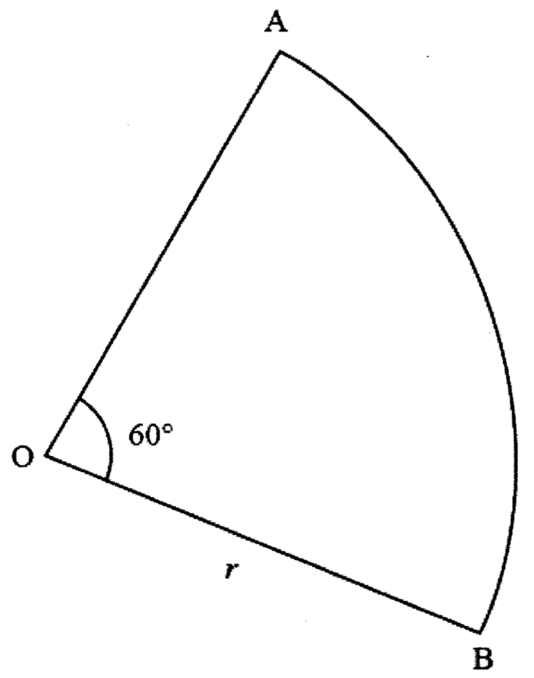
Teachers need to revisit what a small square represents in diverse scales and on how to read an ordinate accurately. More practice should be done in the correct application of trapezium and mid ordinate rule especially when a graph is given,

3.1.3 Mathematics Paper 2 (121/2)

In section 1 of 121/2, questions 2, 3, 10 and 12 were performed well by most candidates while in section II, questions 20 and 24 were performed well.
Questions 18 and 23 were popular among the candidates in Section II.

Question 8

OAB is a sector of a circle of radius r cm. Angle AOB = 60°. Find, in its simplest form, an expression in terms of r and π for the perimeter of the sector. (2 marks)



The question tested on finding the perimeter of a sector of a circle.

Weaknesses

- A number of students failed to include $2r$ in the perimeter.
- A number of students left the perimeter in terms of diameter d.

Expected response

$$\text{Perimeter of sector} = \frac{60}{360} \times 2\pi r + 2r$$

$$= 2r + \frac{1}{3}\pi r$$

Advice to teachers

Teachers need to spend more time on definition of perimeter to avoid cases where students capture incomplete perimeter.

Question 9

In a mathematics test, the scores obtained by 30 students were recorded as shown in the table below.

Score (x)	59	61	65	K	71	72	73	75
No. of students	2	3	5	6	7	4	2	1

The score K with a frequency of 6 is not given. Given that $\frac{\sum fd}{\sum f} = -1.2$

where $d = x - 69$, and using an assumed mean of 69, determine score K. (4 marks)

The question tested on calculation of mean using assumed mean method. The candidate was required to find an unknown score K.

Weaknesses

Most candidates could not connect the steps that could lead to the determination of the value of K.

Expected response

Score x	No. of students	$d = x - 69$	fd
59	2	-10	-20
61	3	-8	-24
65	5	-4	-20
k	6	k -69	6(k-69)
71	7	2	14
72	4	3	12
73	2	4	8
75	1	6	6
	$\Sigma f = 30$		

$$\bar{x} = A + \frac{\sum f(x - A)}{N}$$

$$= 69 + -1.2 = 67.8$$

Also,

$$\bar{x} = \frac{1632 + 6k}{30}$$

Therefore,

$$\frac{1632 + 6k}{30} = 67.8$$

$$k = 67$$

$$\frac{\Sigma fd}{\Sigma f} = \frac{6k - 438}{30} = -1.2$$

$$6k = 402$$

$$k = 67$$

Advice to teachers

Teachers need to emphasize on the use of the assumed mean method to determine the mean of given data.

Question 13

The position vectors of points A, B and C are $\mathbf{OA} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $\mathbf{OB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{OC} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$.

Show that A, B and C are collinear. (3 marks)

The question tested on use of vector method to show collinearity.

Weaknesses

When one vector can be expressed as a scalar multiple of another vector, then the two vectors are parallel. Most candidates failed to make that conclusion.

Expected response

$$\left.\begin{aligned}\mathbf{AB} &= \begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}-3\\4\end{pmatrix} = \begin{pmatrix}4\\-2\end{pmatrix}\\ \mathbf{AC} &= \begin{pmatrix}7\\-1\end{pmatrix} - \begin{pmatrix}-3\\4\end{pmatrix} = \begin{pmatrix}10\\-5\end{pmatrix}\end{aligned}\right\}$$

$$\begin{pmatrix}4\\-2\end{pmatrix} = k\begin{pmatrix}10\\-5\end{pmatrix}$$

$k = 0.4$

Thus

$\mathbf{AB} \mathbin{//} \mathbf{AC}$ *and A is a common point.*

$\therefore$ *Points A, B and C are collinear.*

Advice to teachers

Teachers need to emphasize that

(i) The scalar got should be confirmed in both the two ways.

(ii) Reinforce that a vector being expressed as a scalar multiple of another vector is proof of parallelism.

(iii)The sharing of common point is the proof of collinearity.

Question 14

The vertices of a triangle PQR are $P(-3,2)$, $Q(0,-1)$ and $R(2,-1)$. A transformation matrix M, maps triangle PQR onto triangle $P'Q'R'$ whose vertices are $P'(-7,2)$, $Q'(2,-1)$ and $R'(4,-1)$. Find M^{-1}, the transformation that maps $P'Q'R'$ onto PQR. (4 marks)

The question tested on the determination of the inverse of a transformation.

Weaknesses

- A number of candidates obtained matrix M and failed to get M^{-1}
- Other candidates were unable to extract the correct simultaneous equations.

Expected response

Let $M^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -7 & 2 & 4 \\ 2 & -1 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 0 & 2 \\ 2 & -1 & -1 \end{pmatrix}$$

$$\left.\begin{array}{l} -7a + 2b = -3 \\ 2a - b = 0 \text{ or } b = 2a \end{array}\right\}$$

$$-7a + 2 \times 2a = -3$$

$$-3a = -3$$

$$a = 1,\ b = 2$$

$$\left.\begin{array}{l} -7c + 2d = 2 \\ 2c - d = -1 \text{ or } d = 2c + 1 \end{array}\right\}$$

$$-7c + 2(2c + 1) = 2$$

$$-3c = 0$$

$$c = 0,\ d = 1$$

Therefore

$$M^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

Advice to teachers

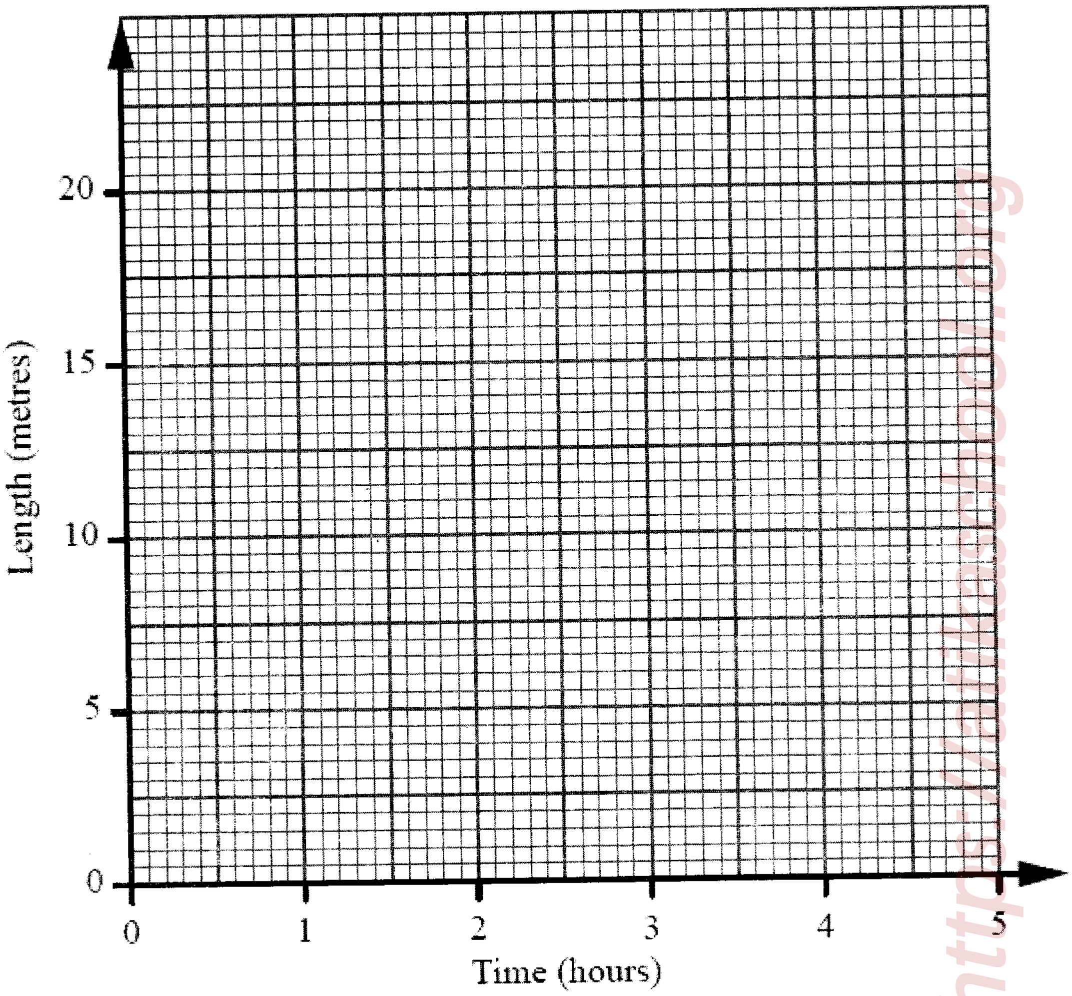
Teachers should expose students to the different methods of solving such questions and give sufficient practice on the same.

Question 16

The length of a shadow of a mast was measured at intervals of 1 hour and recorded as shown in the table below.

Time (h)	0	1	2	3	4	5
Length (m)	18.7	8.7	5.0	2.9	1.3	0

(a) On the grid provided, draw the graph of length against time. (2 marks)



(b) Determine the rate of change of the shadow length at t = 2. (2 marks)

The question tested on drawing of a curve given the table of values and also finding the rate of change of length at time t.

Weaknesses

- Most candidates could not plot the points accurately.
- Quite a number of students could not draw the correct tangent to the curve at the given point.

Expected response

Acceptable tangent drawn at $t = 2$; Tangent passes through points $(2, 5)$ and $(2.5, 3.5)$

$$\frac{\Delta L}{\Delta t} = \frac{3.5 - 5}{2.5 - 2} = \frac{-1.5}{0.5}$$

$$= -3.0 \text{ m/s}$$

Advice to teachers

Teachers need to give candidates sufficient practice on drawing of tangent to a curve at a point which should be marked.

Question 17

The first term of an Arithmetic Progression (AP) is equal to the first term of a Geometric Progression (GP). The second term of the AP is equal to the fourth term of the GP while the tenth term of the AP is equal to the seventh term of the GP.

(a) Given that a is the first term and d is the common difference of the AP while r is the common ratio of the GP, write the two equations connecting the AP and the GP. (2 marks)

(b) Find the value of r that satisfies the progressions. (4 marks)

(c) Given that the tenth term of the GP is 5120, find the values of a and d. (2 marks)

(d) Calculate the sum of the first 20 terms of the AP. (2 marks)

The question tested on two sequences; An AP and a GP. Some terms in the two sequences were related.

Weaknesses
A number of Candidates managed to form the equations in (a) but could not solve them

Expected response

(a)

$$ar^3 = a + d$$

$$ar^6 = a + 9d$$

(b)

From (a) above

$$d = ar^3 - a$$

$$a + 9(ar^3 - a) = ar^6$$

$$a + 9ar^3 - 9a = ar^6$$

$$ar^6 - 9ar^3 + 8a = 0$$

$$r^6 - 9r^3 + 8 = 0$$

$$(r^3 - 1)(r^3 - 8) = 0$$

$$\mathrm{r} = 1 \text{ or } \mathrm{r} = 2$$

$$\mathrm{r} = 2$$

(c)

$$ar^9 = 5120$$

$$a = \frac{5120}{2^9} = 10$$

$$a + d = 10 \times 2^3 = 80$$

$$\therefore d = 80 - 10 = 70$$

(d)

$$S_{20} = \frac{20}{2}\{20 + 19 \times 70\}$$

$$= 13500$$

Advice to teachers

Teachers are advised to expose students in more activities involving APs and GPs with terms that are related.

Conclusion

Major weaknesses have been observed in some areas of the syllabus for Mathematics Alt A. These areas include **Geometry (Bearings and scale drawing and Construction), Area below a curve, Graphical Methods, Sequences and Seris, Statistics 2,** and **Transformations.**

Application of learned concepts to real life situations was observed to be a challenge to many candidates. To help learners understand the concepts, it is necessary to have more applications relating to real life situations in the course of the teaching and learning.