5.0 MATHEMATICS (121)

This Mathematics report is based on an analysis of performance of candidates who sat the year 2006 KCSE mathematics examination.

The KCSE Mathematics examination tested the candidate abilities in two papers; ***Paper 1 (121/1)*** and ***Paper 2 (121/2)*** under the revised curriculum. For the first time, candidates were allowed to use calculators. In terms of content coverage, the two papers supplement each other. Paper 1 (121/1) covered mainly form 1 and 2 work while paper 2 (121/2) covered form 3 and 4 work. It is hoped that this report will be helpful to teachers in the teaching/teaching process as well as in preparing candidates for future examinations.

5.1 GENERAL CANDIDATES' PERFORMANCE

The table below shows the overall performance of candidates in both papers for the last four years.

Table 8: Candidates Overall Performance in Mathematics for the Last Four Years

Year	Paper	Candidature	Maximum Mark	Mean Score	Standard Deviation
2003	1		100	17.17	16.31
	2		100	21.45	19.86
	Overall	**205, 232**	**200**	**38.62**	**26.17**
2004	1		100	14.57	15.42
	2		100	22.63	20.43
	Overall	**221, 295**	**200**	**37.20**	**35.85**
2005	1		100	14.87	15.73
	2		100	17.04	16.74
	Overall	**259,280**	**200**	**31.91**	**31.00**
2006	1		100	22.71	20.09
	2		100	15.36	15.97
	Overall	**238,684**	**200**	**38.08**	**35.00**

From the table above, it can be observed that the overall mean for the Mathematics examination improved from ***31.91*** in the year 2005 to ***38.08*** in the year 2006.

The discussion that follows is based on a random sample of candidates' scripts for the year 2006 Mathematics examination. Questions in which candidates performance was poor in the selected sample scripts were identified and have been analyzed in detail in the report.

5.2 PAPER 1 (121/1)

Question 2

All prime numbers less than ten are arranged in descending order to form a number.
(a) Write down the number formed.
(b) State the total value of the second digit in the number formed in (a) above.

This question tested the candidates' knowledge of prime numbers and total value of a digit in a given number.

Weaknesses

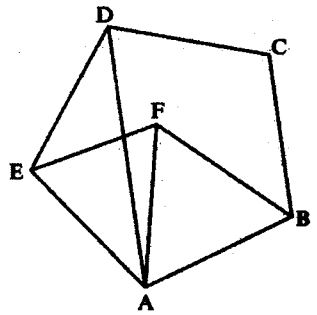
Majority of candidates included ***"1"*** as a prime number. This affected their response for the total value.

Expected Response

Candidates were expected to write the prime numbers less than ten in descending order as: 7 5 3 2 Then, the total value of the second digit is 500.

Question 4

In the figure below, ABCDE is a regular pentagon and ABF is an equilateral triangle.



Find the size of

(a) $\angle ADE$

(b) $\angle AEF$

(c) $\angle DAF$

This question tested the candidates' knowledge on angle properties of a regular pentagon and an equilateral triangle.

Weaknesses

Many candidates were unable to score even a mark in this question because they did not recognise that each of the interior angles of a regular pentagon is equal to 108° and that of an equatorial triangle is equal to 60°.

Expected Response

(a) $$\angle ADE = \frac{180° - 108°}{2} = 36°$$

(b) $$\angle AEF = \{180° - (108° - 60°)\} \div 2$$
$$= 66°$$

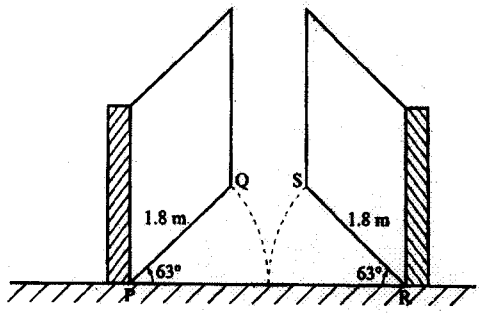
(c) $$\angle DAF = 108° - (60° + 36°)$$
$$= 12°$$

Calculations on the sum of interior angles of a regular plain figure should be emphasized during teaching.

Question 11

The diagram below represents a school gate with double shutters. The shutters are each opened through an angle of 63°.

The edges of the gate, PQ and RS are each 1. 8 m.



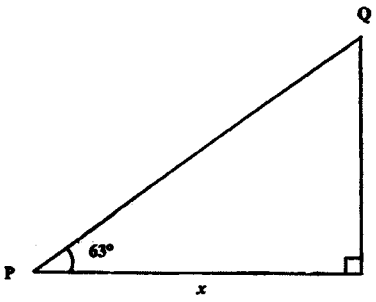
Calculate the shortest distance QS, correct to 4 significant figures.

This question tested candidates' ability to use trigonometric ratios to calculate lengths.

Weaknesses

Most candidates were unable to recognize that the distance PR = distance PQ + Distance RS.

Expected Response



Since PR = PQ + RS

$PR = 3.6\text{m}$

$x = 1.8\cos 63°$

$= 1.8 \times 0.454$

$= 0.8172$

$QS = 3.6 - 2 \times 0.8172$

$= 3.6 - 1.6344$

$= 1.9656$

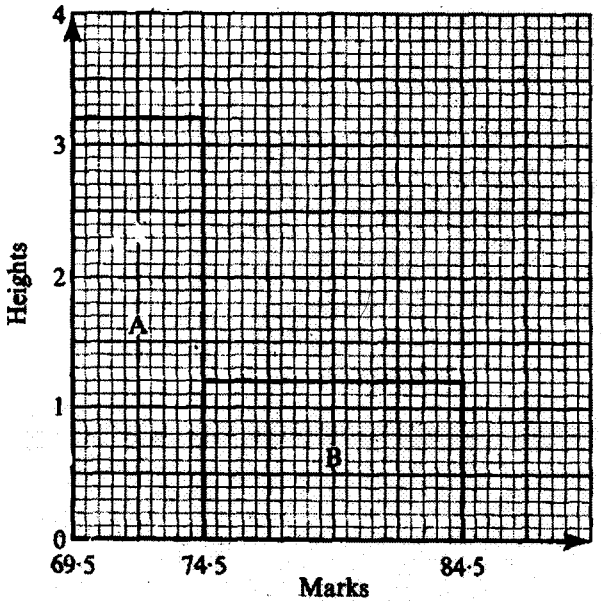
$= 1.966m$

Teachers are advised to give plenty of exercises on this concept to learners.

Question 15

The histogram below represents the distribution of marks obtained in a test.

The bar marked A has a height of 3. 2 units and a width of 5 units. The bar marked B has a height of 1. 2 units and a width of 10 units.



If the frequency of the class represented by bar B is 6, determine the frequency of the class presented by bar A.

This question tested candidates' knowledge on the histogram.

Weaknesses

Most candidates depicted lack of knowledge on the concept related to the histogram. Candidates were unable to conceptualize that frequency is proportional to areas of the corresponding bar in a histogram.

Expected Response

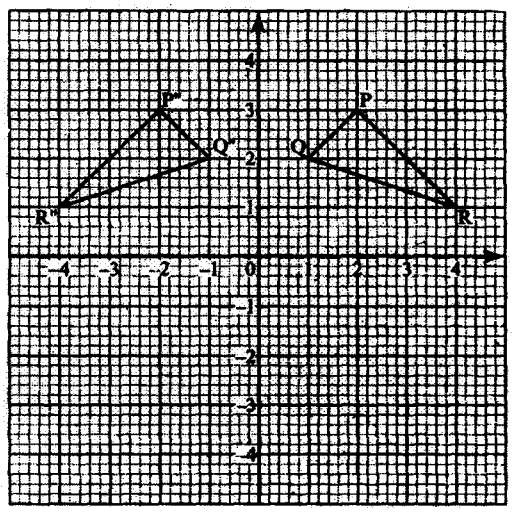
Area A = 5 x 3.2
B = 10 x 1.2

$\therefore$ 16: 12 = f: 6
12f = 96
f = 8

Teachers are advised to explain to the learners the difference between a bar graph and a histogram.

Question 18

On the cartesian plane below, triangle PQR has vertices P(2,3), Q(1,2) and R(4, 1) while triangle P"Q"R" has vertices P"(-2,3), Q"(-1 ,2) and R"(-4, 1).



(a) Describe fully a single transformation which maps triangle PQR onto triangle P"Q"R".

(b) On the same plane, draw triangle P'Q'R', the image of triangle PQR, under reflection in line $y = -x$.

(c) Describe fully a single transformation which maps triangle P'Q'R' onto triangle P"Q"R".

(d) Draw triangle P'"Q'"R'" such that it can be mapped onto triangle PQR by a positive quarter turn about (0,0).

(e) State all pairs of triangles that are oppositely congruent.

This question tested candidates' knowledge on two types of transformation; ***reflection*** and ***rotation.***

Weaknesses

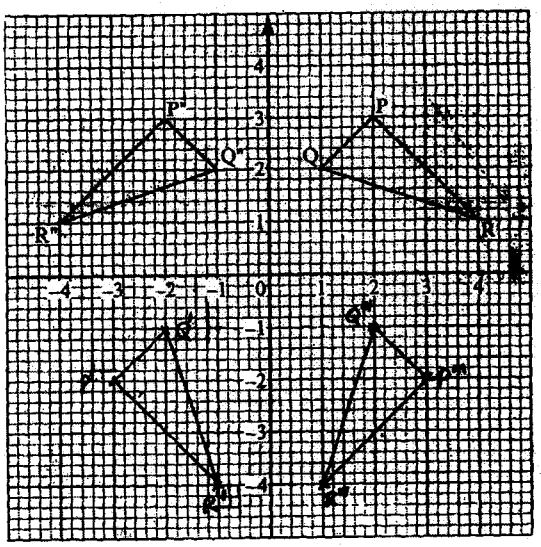
The question was the most poorly performed by candidates in the year 2006 KCSE mathematics paper one examination. Majority of the candidates displayed total lack of knowledge on the types of transformation and could not comprehend and interpret what ***congruency*** is.

Expected Response

In part (a) of the question, candidates were expected to recognize that the y-axis is a mirror line between triangles PQR onto triangle P"Q"R". Therefore, the single transformation that maps triangle PQR onto triangle P"Q"R" is the line axis x = 0(or the y-axis).

In part (b), candidates needed to reflect triangle PQR in the line y = - x to obtain triangle P'Q'R' whose coordinates are P' (-3, -2), Q' (-2, -1) and R' (-1, -4).

Part (c) of the question, like in part (a), required a description of a transformation that maps triangle P'Q'R' into P"Q"R", that is, negative quarter turn about (0,0). Candidates who were able to correctly work out part (a) to (c) were also able to draw the image P'"Q'"R'", which is as shown below.



For candidates to have earned the full marks for part (e), they had to state the following pairs of triangles which are oppositely congruent.

- PQR and P"Q"R"
- P'Q'R' and P'"Q'"R'"
- PQR and P'Q'R'
- P"Q"R" and P'"Q'"R'"

Question 20

A bus left Mombasa and travelled towards Nairobi at an average speed of 60 km/h. After 2½ hours, a car left Mombasa and travelled along the same road at an average speed of 100 km/h. If the distance between Mombasa and Nairobi is 500 km, determine

a) (i) the distance of the bus from Nairobi when the car took off.

(ii) the distance the car travelled to catch up with the bus.

(b) Immediately the car caught up with the bus, the car stopped for 25 minutes. Find the new average speed at which the car travelled in order to reach Nairobi at the same time as the bus.

This question tested the candidates' ability to solve problems on relative motion.

Weaknesses

Although the question was quite simple, many candidates were unable to comprehend the movements of the bus and car in terms of their coverage speeds and the times given.

Expected Responses

Candidates were expected to display their solutions as follows:

(a) (i) Distance of bus from Nairobi
500 - 2.5 x 60
= 350 Km.

(ii) Let distance required be x km
For bus x = 150 + 60t
For car x = 150 + 60 t
100t =150 + 60t
t = 3 ¾ h

Hence distance required = 100 x 3 ¾ = 375 km

(b) Distance yet to be covered
= 500 - 375 = 125km

Time taken by bus
$$= \frac{125}{60}$$
= 2 h 5 min (or 125 min)

New speed of car
$$= \frac{\frac{125}{(125-25)}}{60}$$
= 75Km/h

5.3 PAPER 2 (121/2)

Question 4

By correcting each number to one significant figure, approximate the value of 788 x 0. 006. Hence calculate the percentage error arising from this approximation.

This question tested the candidates' ability to express given numbers to one significant figure and then approximating the product of the given number. Candidates were also required to find the percentage error arising from that approximation.

Weaknesses

Most candidates were not able to correct 788 to one significant figure as well as realize that 0.006 correct to one significant figure is still 0.006.

Expected Response

Writing each number to one significant figure: $788 \approx 800$ and $0.006 = 0.006$

Thus, $800 \times 0.006 = 4.8$

$$\% \text{ error} = 4.8 - \frac{(788 \times 0.006)}{788 \times 0.006} \times 100\ \%$$

$$= \frac{0.072 \times 100\%}{4.728}$$

$$= 1.523\% \text{ or } (1.52284264\%)$$

Teachers need to concentrate more on guiding learners on writing numbers to specific significant figures or decimal places.

Question 5

The data below represents the ages in months at which 6 babies started walking:
9, 11, 12, 13, 11 and 10.
Without using a calculator, find the exact value of the variance of the data.

This question tested candidates' knowledge on calculating variance of discrete data.

Weaknesses

Surprisingly, majority of the candidates were not conversant with finding the variance of discrete data.

Expected Response

Finding the mean value of the data:

$$\bar{x} = \frac{9+11+12+13+11+10}{6} = 11$$

Find the square deviation from the mean,

$$\left(x - \bar{x} \right)^2 = 4, 0, 1, 4, 0, 1$$

$$S^2 = \frac{4+0+1+4+0+1}{6}$$

$$= \frac{10}{6}$$

$$= 1\frac{2}{3}$$

Teachers need to give more practice to learners on finding variance of discrete data.

Question 8

The table below is a part of tax table for monthly income for the year 2004.

Monthly taxable income in Ksh	Tax rate percentage (%) in each shilling
Under Ksh 9681	10%
From Ksh 9681 but under Ksh 18801	15%
From Ksh 18801 but under Ksh 27921	20%

In the year 2004, the tax on Kerubo's monthly income was Ksh 1916.
Calculate Kerubo's monthly income.

This question tested candidates' ability to calculate monthly income given income tax.

Weaknesses

Many candidates were unable to calculate the income that was subjected to 15% taxation.

Expected Response

Tax on the first sh 9680 $= \frac{10}{100} \times 9680 = 968$

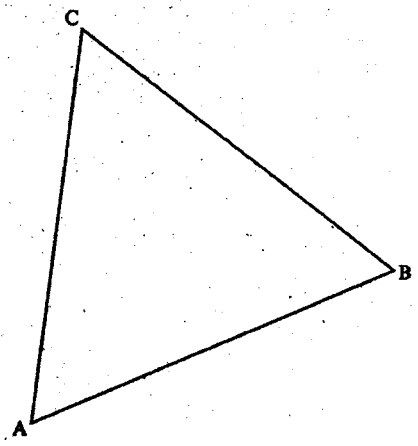
Monthly income $= \frac{(1916 - 968)}{15} \times 100 + 9680$

$= 6320 + 9680$
$=$ Shs. 16, 000

Teachers need to expose learners to not only calculating tax on monthly income but also to finding monthly income given the tax.

Question 13

The figure below is drawn to scale. It represents a field in the shape of an equilateral triangle of side 80 m.



The owner wants to plant some flowers in the field. The flowers must be at most, 60 m from A and nearer to B than to C. If no flower is to be more than 40 m from BC, show by shading, the exact region where the flowers may be planted.

This question tested candidates' ability to apply their knowledge of Loci on construction using a pair of compasses and ruler only.

Weaknesses

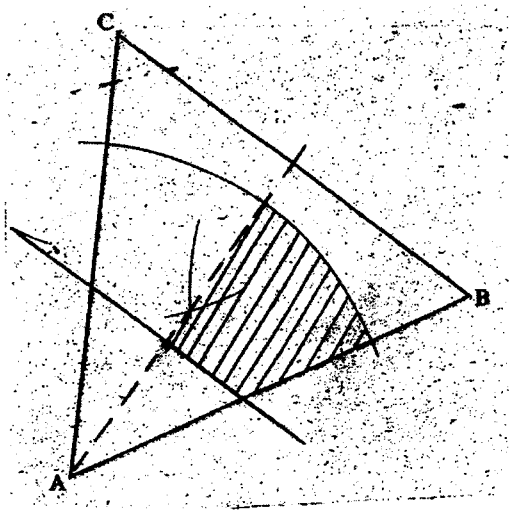
Most candidates could not draw or construct as demanded by the question since they could not scale down the given measurements. Those who could were unable to accurately follow the instructions.

Expected Responses

Candidates were expected to carry out the following tasks:

- Draw an arc centre A, radius 6 cm.
- Draw a perpendicular bisector of BC dotted.
- Draw a parallel line 4cm from BC.
- Shade the required region.

Candidates were to come up with the figure given below.



More exercises should be given to learners in order for them to perfect the art of construction.

Question 16

Two places P and Q are at (36°N, 125°W) and (36°N, 55°E) respectively.

Calculate the distance in nautical miles between P and Q measured along the great circle through the North Pole.

This question tested the candidates' knowledge on navigation and distance between two places measured along the great circle through the North Pole.

Weaknesses

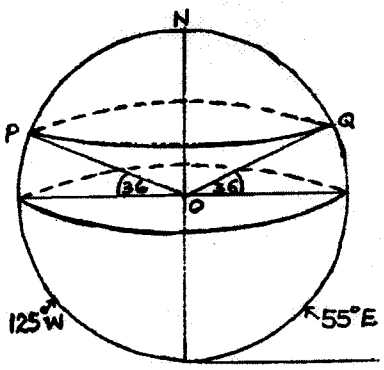
Candidates were not able to determine the angle between the two places P and Q from the equator.

Expected Response

Required angle $POQ = 180 - (36 \times 2) = 108°$

Distance $PQ = 108 \times 60$
= 6480 nautical miles

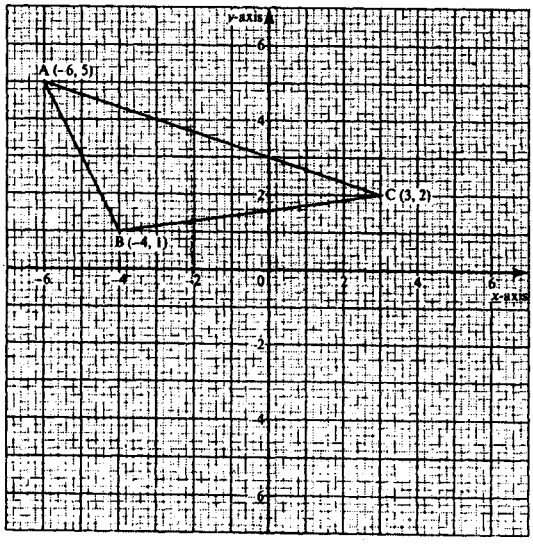
The figure below would assist to understand the above working.



Navigation is one of the topics that require creativity and imagination. Teachers are therefore advised to take more time and use teaching aids when teaching this topic.

Question 19

Triangle ABC is shown on the coordinate plane below.



(a) Given that A(-6, 5) is mapped onto A'(-6, -4) by a shear with y-axis invariant,

(i) draw triangle A'B' C', the image of triangle ABC, under the shear.

(ii) determine the matrix representing the shear.

(b) Triangle A' B' C' is mapped onto A" B" C" by a transformation defined by the

matrix $\begin{pmatrix} -1 & 0 \\ 1\frac{1}{2} & -1 \end{pmatrix}$

(i) Draw triangle A" B" C"

(ii) Describe fully a single transformation that maps ABC onto A" B" C".

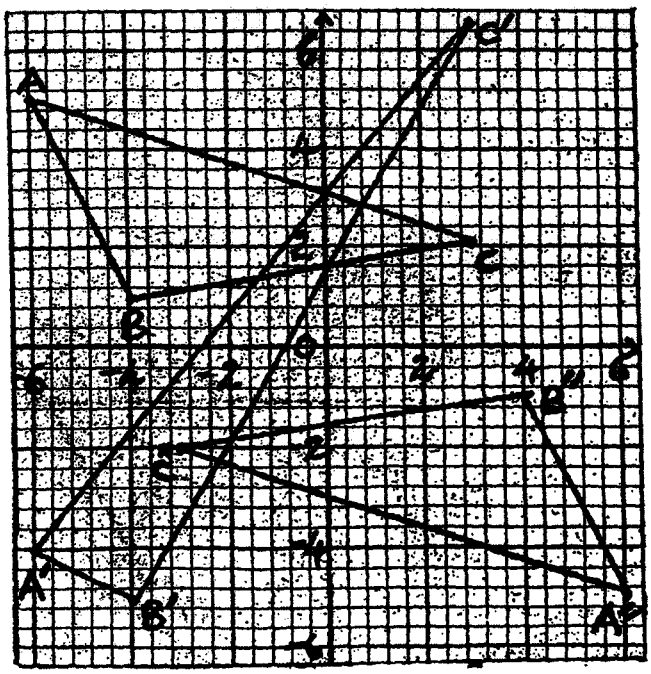
This question tested the shear and how to find the formation matrix. It is important to note that the shear was tested for the first time in the year 2006 KCSE Mathematics examination after so many years.

Weaknesses

Very few candidates chose this question and many of those who did could not even start off. Those who chose the question were not able to realize that since $y-axis$ is variant, then the shear should be parallel to this axis.

Expected Response

(a) Using the mapping of $A(-6,5)$ onto $A'(-6,-4)$ by a shear with $y-axis$ invariant.
Candidates were expected to draw triangle ***A' B' C'***, the image of ABC as shown below:



Since the shear maps $I(1,0) \longrightarrow I'(1,1\frac{1}{2})$, then the matrix representing the shear is $\begin{pmatrix} 1 & 0 \\ 1\frac{1}{2} & 1 \end{pmatrix}$

(a) To draw the image $A''B''C''$ candidates were expected to obtain the coordinates of $A''B''C'$.

$$\therefore \begin{pmatrix} -1 & 0 \\ 1\frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} A' & B' & C' \\ -6 & -4 & 3 \\ -4 & -5 & 6\frac{1}{2} \end{pmatrix} = \begin{pmatrix} A'' & B'' & C'' \\ 6 & 4 & -3 \\ -5 & -1 & -2 \end{pmatrix}$$

Points A" (6, -5) B" (4, -1) and C" (-3, -2) are then plotted and triangle A"B"C" be drawn as indicated above.

From the figure, it can be noted that a single transformation that maps ABC onto A"B"C" is half turn about (0,0) or an enlargement with scale factor -1 about the origin.

Teachers are advised to allow learners to practically draw transformations in order to acquire the skill.

Question 20

(a) Two integers, x and y, are selected at random from the integers 1 to 8. If the same integer may be selected twice, find the probability that:

(i) $|x-y| = 2$

(ii) $|x-y|$ *is 5 or more*

(iii) $x > y$

(b) A die is biased so that when tossed, the probability of a number r showing up, is given by P(r) = Kr where K is a constant and r = I, 2, 3, 4, 5, 6 (the numbers on the faces of the die).

(i) Find the value of K.

(ii) If the die is tossed twice, calculate the probability that the total score is 11.

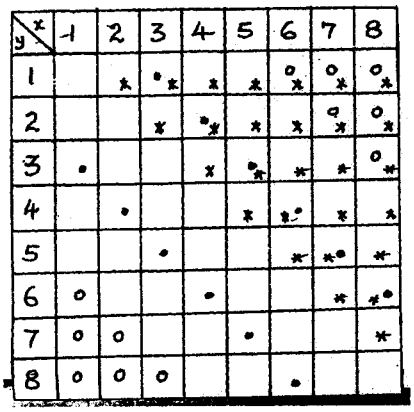
This question tested the candidates' knowledge on probability space.

Weaknesses

Very few candidates chose this question and those who did were unable to come up with the probability space. In fact, majority of the bright candidates avoided the question altogether.

Expected Responses

Candidates needed to come up with the following probability space to assist them in answering parts of the question.



(a) (i) $P(Ix - yI = 2) = \frac{12}{64} = \frac{3}{16}$

(ii) $P(Ix - yI \geq 5) = \frac{12}{64} = \frac{3}{16}$

(iii) $P(x \geq y) = \frac{28}{64} = \frac{7}{16}$

(b) (i)

$$k + 2k + 3k + 4k + 5k + 6k = 1$$

$$21k = 1$$

$$k = \frac{1}{21}$$

(ii) $$P(11) = \frac{5}{21} x \frac{6}{21} + \frac{6}{21} x \frac{5}{21}$$

$$= \frac{60}{441} = \frac{20}{147}$$

Teachers are again advised to give as many questions as possible to learners for practice in this area.

5.4 GENERAL COMMENTS

The analysis of performance in the two papers given above reveals a number of weaknesses in students' mathematics attainment.

5.4.1 Most glaring is the students' lack of knowledge of elementary techniques and their ignorance of simple algorithms and processes. Many students displayed inability to use both logarithm tables and calculators. The use of calculators should not be overdone. Simple basic calculations should do without the calculator unless otherwise stated.

5.4.2 The fact that paper 1 (121/1) tests mainly form 1 and 2 work should not be misused. The analysis of candidates work revealed that a number of schools concentrated in teaching form 1 and 2 work only. Learning should not only aim at passing examinations, but should also aim at enabling learners to acquire knowledge and skills that are useful in the world of work and life.