

Mount Kenya



University

UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BEDA/BEDS/BSNE/BENH
SCHOOL BASED

UNIT CODE: BMA1102

UNIT TITLE: CALCULUS I

DATE: JUNE 2015

SUPP / SPECIAL EXAM

TIME: 2 HOURS

INSTRUCTIONS: Answer questions one and any other two questions.

1. a) Evaluate; $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$ (4 Marks)
- b) Find the values of a and b that make f(x) continuous on the entire real number line. $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \leq x \leq 2 \\ 4x & \text{if } x > 2 \end{cases}$ (5 Marks)
- c) A small spherical balloon is inflated at the rate of 2 cm³/s
- i) What is the rate of growth of the radius? (3 Marks)
- ii) Find the rate increase of the radius when the volume of the balloon is 50cm³ (4 Marks)
- d) Given $y = \frac{t^2 + 2}{t^3 - 4t}$ find $\frac{dy}{dx}$ (5 Marks)
- ii) Differentiate $y = \sin(xy)$ implicitly (4 Marks)

e) The parametric equations of a curve are $x=e^t$, $y=\sin t$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (5 Marks)

2. a) Find the equation of the tangent and the normal lines to the curve.

$x^2y^2 - 2x = 4 - 4y$ At the point (2, -2) (6 Marks)

b) Find the maximum and minimum value of the function. $y = 2\sin t + \cos 2t$ (6 Marks)

c) A body moves along a straight line according to the law.

$$s = t^3 - 6t^2 + 9t + 4$$

Find:

i) s and a when $v=0$ (5 Marks)

ii) s and v when $a=0$ (3 Marks)

3. a) A farmer has an adjustable electric fence that is 100m long. He uses this fence to enclose a rectangular grazing area on three sides, the fourth side being a fixed hedge. Find the minimum area he can enclose. (6 Marks)

b) Find the derivatives of;

i) $y = \tan h x$ (4 Marks)

ii) $y = \sec h x$ (3 Marks)

c) Find $\frac{dy}{dx}$ in the following;

i) $y = \cosh^{-1} \sqrt{x^2 + 1}$ (4 Marks)

ii) $y = \ln(\tanh^{-1} x)$ (3 Marks)

4. a) Differentiate the following using the rule indicated in brackets.

i) $y = \cos^4 \left(2\theta - \frac{\pi}{5} \right)$ (Chainrule) (5 Marks)

ii) $y = 12x^3 + 2x^2$ (first principles) (5 Marks)

iii) $y = (1 - 6x^3)(4x^2 - 6x + 2)$ (product rule) (5 Marks)

iv) $y = \frac{x^2 - 1}{x^2 + 1}$ (Quotient rule) (5 Marks)

5. a) Evaluate;

i) $\lim_{x \rightarrow \infty} \left(\frac{x^4 - x + 4}{3x^4 + x^3 + 3} \right)$ (3 Marks)

ii) $\lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{x^2 - 4} \right)$ (3 Marks)

b) Discuss the continuity of;

i) $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ \frac{x^2 - x - 2}{x^2 - 4} & \text{if } x = 2 \end{cases}$ (3 Marks)

ii) $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{for } x < 2 \\ \frac{x^2 - x - 2}{x^2 - 4} & \text{for } x > 2 \end{cases}$ (3 Marks)

c) Differentiate the following implicitly with respect to x.

$$x^2 y^3 - xy^2 = 10xy \quad (4 \text{ Marks})$$

d) Differentiate; $y = \log_7 \tan^3 x$ (4 Marks)