Exam draft

Supplementary exams (Dec 2013) school based

ECS 211: Basic Discrete Mathematics

Question one (30 marks)

(a) Draw the truth table for the Boolean expressions $Z = A \cdot B + \bar{A} \cdot \bar{B}$ (4 marks)

(b) Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction using truth table (6 marks)

(c) Given that A, B and C are subsets of universal set U, draw and shade the Venn diagram to illustrate the following sets

(i) $A \cup B$

(ii) $\neg(A \cup B \cup C)$

(iii) $A \cap C$

(iv) $A \cap B \cap C$ (4 marks)

(d) Draw the graph represented by the adjacency matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (3 marks)

(e) Use mathematical induction to prove the formula $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n + 1) = \frac{n(n+1)(n+2)}{3}$ (8marks)

(f) How many ways can a football team of 11 players be selected from 18 players (3 marks)

(g) How many different orders can 10 mathematics books be selected if each set consists of 3 books (2 marks)

Questions two (20 marks)

(a) Find the value f:

(i) $20C_5$ (2 marks)

(ii) $10P_3$ (2marks)

(b) Given that $U = \{x: 1 \le x \le 10, x \text{ is an integer}\}$ $A = \text{set of odd numbers}$, $B = \text{set of factors of } 24$ and $C = \{3, 10\}$

(i) Draw a Venn diagram to show the relationship between sets U, A, B and C

(ii) Using the Venn diagram or otherwise find

$A \cup B$

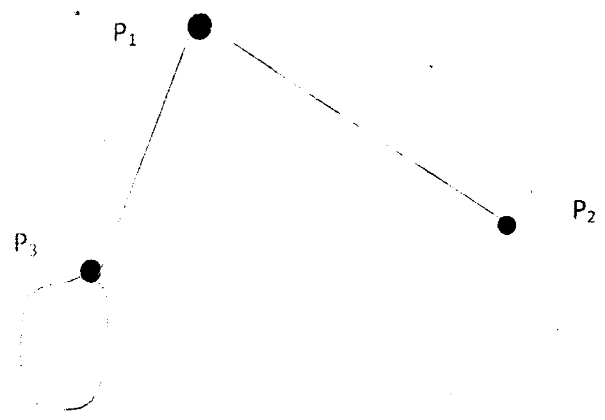
$A \cup C$

$A \cap B \cap C$

(c) Given the compound propositions $P = \overline{p \vee q}$ and $Q = \neg p \wedge \neg q$. Use the truth table to show that $P \equiv Q$

(d) (i) what is a loop of a graph (2 marks)

(iii) Write the matrix representing the graph below: (4marks)



Question three (20 marks)

(a) The local pet store surveyed 50 people about pets. Eleven of these people owned dogs, 13 own cats and 6 owned fish. One person owned all the three types of pets, 2 people owned only fish and dogs, 3 people owned only fish and cats and 5 people owned only cats and dogs. How many people owned none of these pets? (6 marks)

(b) Prove by mathematical induction the formula

$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$. (4marks)

(c) Show that the statements $(P \wedge q) \vee (\neg q)$ *and* $\neg((\neg p) \wedge q)$ are logically equivalent using the truth table. (6 marks)

(d) Three boys A, B, and C are throwing a ball among themselves such that A always throw the ball to B, but B and C are just as likely to throw the ball to a as they are to each other. Illustrate this information in form of a graph and write the matrix that represents the graph. (4 marks)

Question four (20 marks)

(a) Three Boolean variables x_1, x_2, and x_3 are such that $A = x_1 + x_2$, $B = \neg x_1 \cdot x_3$ and $C = (x_1 + x_2) \cdot x_3$ fill in the table below (5 marks)

X1	X2	X3	A	B	C	A.B	A+B+C
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

(b) In how many ways can we select a committee of 4 people from a group of 11 distinct people (3 marks)

(c) In how many different ways can an escort of four soldiers be chosen from nine soldiers (3 marks)

(d) What is a tautology (2marks)

(e) Show that $n \vee (n \wedge m)$is a tautology using truth table (4 marks)

(f) Let B be a set defined recursively by:

(i) $2 \in B$

(ii) If $n \in b$ $then$ $n^2 \in B$

Describe this set by listing method (3 marks)

Question five (20 marks)

(a) Eight-two individuals have complained to the customer protection Agency about a certain car make. The information contained in letters of complaint is summarized as below:

25 complained about steering
23 complained about comfort
22 complained about visibility
11 complained about steering and comfort
7 complained about steering and visibility
5 complained about all the three

33 complained about none of the three.

(i) How many people complained about comfort and visibility but not steering? (8mks)

(ii) How many complained about exactly one of the three items (2mks)

(b) If x and y are Boolean algebra complete the following table for multiplication and addition (4 marks)

x	y	X+y	x.y
1	1		
1	0		
0	1		
0	0		

1. Prove by mathematical induction that the sum of arithmetic progression $a + (a + d) + (a + 2d) + \cdots (a + (n - 1)d) = \frac{n(2a+(n-1)d)}{2}$ if n is any natural number (6 marks)

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Question one (30 mks).

(a) Draw the truth table for the following Boolean expression

$$Z = A \cdot B + \bar{A} \cdot B$$ (4mks)

(b) Verify that the proposition $(P \wedge Q) \wedge \neg(P \vee Q)$ is a contradiction.

(6mks)

(c) Given that A, B and C are ~~sets~~ of subsets of universal set U. Draw and shade the Venn diagram to illustrate the following sets

(i) $A \cup B$

(ii) $\neg(A \cup B \cup C)$

(iii) $A \cap C$

(iv) $A \cap B \cap C$ (4mks)

(d) Draw a graph represent by the adjacency matrix

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3 mks)

(e) use mathematical induction to prove the formula

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

(8 mks)

(f) How many ways can a football team of 11 player be selected of 18 players

3 (~~8~~ mks).

(g) How may different orders can 10 mathematics books be shelved if each set consis of 3 books 2 mks)

Question two (20 mks).

(a)

(i) Find the value of :-

(i) $20C_5$ (2 mks)

(ii) $10P_3$. (2 mks)

(b) Given $U = \{x : 1 \leq x \leq 10, x \text{ is an integer}\}$

A = Set of odd numbers.

B = Set of factors of 24

$C = \{3, 10\}$

(a) Draw a venn diagram to show the relationship of the sets. A B and C (3 mks)

(b) using the venn diagram or otherwise, find,

(i) $A \cup B$

(ii) $A \cup C)$

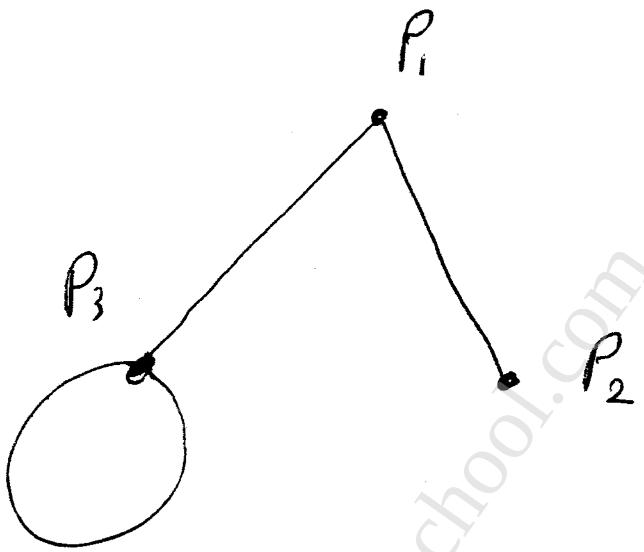
(iii) $A \cap B \cap C$ (3 mks)

(c) Given the compound propositions $P = p \vee q$ and $Q = \neg p \wedge q$. use Truth table to show that

$P \equiv Q$. (4 mks)

(d) (i) What is a loop of a graph. (2mks)

(ii) ~~How~~ Write the matrix represents the following graph



(4mks)

Quest 3. (20mks).

(a). The local pets store surveyed 50 people about pets. Eleven of these people owned dogs, 13 own cats and 6 owns fish. One person owns all the three pets, 2 people owned only fish and dogs, 3 people owned only fish and cats and five owned only cats and dogs. How many people owned none of theses (6mks).

(b) prove by mathematical induction the formula

$$1^3+2^3+3^3+4^3+\cdots n^3 = \frac{n^2(n+1)^2}{4}.$$

(4mks).

(C) ~~What is Boolean algebra~~

Show that the statements $(p \wedge q) \vee (\neg q)$ and $\neg((\neg p) \wedge q)$ are logically equivalent using the truth table. (6mks).

(d) Three boys A, B and C are throwing a ball among themselves such that A always throws the ball to B but B and C are just as likely to through the ball to A as they are to ~~each~~ other illustrate this information in form of ~~(4mks)~~ a graph and write the matrix of the graph (4mks)

Question 4 (20 mks).

(a) Three ~~variables x_1~~ Boolean algebra variable x_1, x_2 and x_3 are such that $A = x_1 + x_2$ $B = \neg x_1 \cdot x_3$

$C = (x_1 + x_2) \cdot x_3$ ~~construct the table of~~ Fill in the table below.

~~A + B~~

x_1	x_2	x_3	A	B	C	$A \cdot B$	~~C+A+B~~
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

(5 mks)

(b) In how many ways can we select a ~~group~~ committee of 4 people from a group of 11 distinct people.

(3 mks).

(c) In how many ~~ways~~ different ways can a escort of four soldiers be chosen from nine soldiers.

(3 mks)

(d) (i) What is tautology (2mks)

(ii) Show that $n \vee (n \wedge m)$ is a tautology if n and m are propositions using truth table.

(4mks)

(e) Let B be the set defined recursively as follows

(i) $2 \in B$

(ii) If $n \in B$, then $n^2 \in B$.

Describe this set by listing method.

Question 5 (20mks).

(a). ~~Eight~~ two individuals have complainted to customer protection Agency about a certain car make. The information is summarised as below:

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23 complained about comfort
22 complained about visibility
11 complain about steering and comfort.
7 complaind about steering and visibility
5 complained about all the three
33 complained about none of the three.

(i) How many people complaind about comfort and visibility but not steering. (5mks)

(ii) How many complained about exactly one of the three (2mks).

(b) If x and y are boolean algebra complete the table for the multiplication and addition below

x	y	x+y	x·y
1	1		
1	0		
0	1		
0	0		

(4mks)

(c) prove by mathematical induction that the sum of arithmetic progression

$$a+(a+d)+(a+2d)+\cdots(a+(n-1)d) = \frac{n(2a+(n-1)d)}{2}$$ if n is any natural number. (6mks)