Mount Kenya University

UNIVERSITY EXAMINATION 2012/2013

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF NATURAL SCIENCES

UNIT CODE: ECS 211 UNIT TITLE: BASIC DISCRETE MATHEMATICS

DATE: AUGUST 2013 EXAMS (TAKE AWAY CAT)

INSTRUCTIONS: ANWER ALL QUESTIONS

1. If a universal set U is given by U={0,1, 2, 5, 7, 10} and the subsets of U are A, B, and C defined as A={1, 2, 5, 10}, B={0, 1, 7, 10} and C={0, 2, 5}
   (a) Draw t a venn diagram to represent the relationship between U, A, B and C. (4mks)
   (b) Using the venn diagram or otherwise find the members of the sets (8mks)
   (i) $A \cap B$
   (ii) $A \cap C$
   (iii) $B \cap C$
   (iv) $A \cap B \cap C$
   (v) AUB
   (vi) ~~AUB~~
   (vii) BUC
   (viii) AUBUC
2. If A and B are propositions, prove that $A\Lambda\bar{B} \rightarrow \overline{A \rightarrow B}$ is a tautology (7mks)
3. If p and q are propositions show that $(\bar{p}Vq)\Lambda(p\Lambda\bar{q})$ is a contradiction (7mks)
4. Eight-two individuals have complained to the customer protection Agency about a certain car make. The information contained in letters of complaint is summarized as below:
   25 complained about steering
   23 complained about comfort
   22 complained about visibility

11 complained about steering and comfort

7 complained about steering and visibility

5 complained about all the three

33 complained about none of the three.

(a) How many people complained about comfort and visibility but not steering? (8mks)

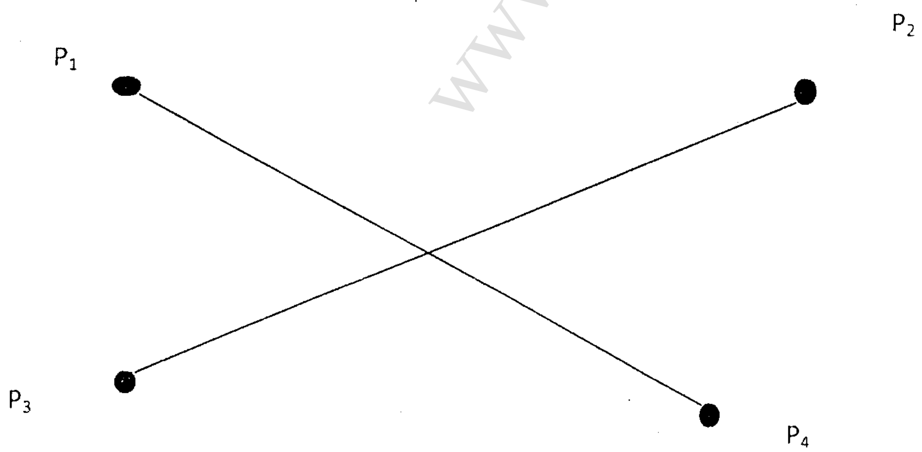
(b) How many complained about exactly one of the three items (2mks)

5. In how many ways can a team of eleven be picked from fifteen possible players? (3mks)
6. How many different arrangements can be many by taking five letters at a time from the word "special" (3mks)
7. Use mathematical induction to prove the formula

   $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots n(n+1) = \frac{n(n+1)(n+2)}{3}$
8. Draw the graph with the following adjacency matrix

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{matrix}$$

9. Represent the following graph using the adjacency matrix



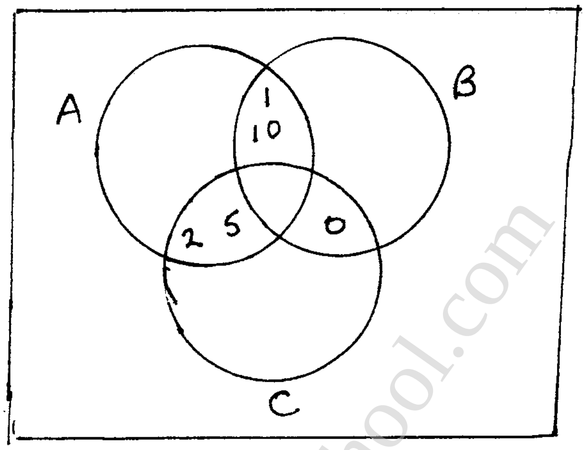
ECS 211: Discrete maths August session 2013.
TAKEAWAY CAT.
MARKING SCHEME:

1. (a) $U = \{0, 1, 2, 5, 7, 10\}$ → universal

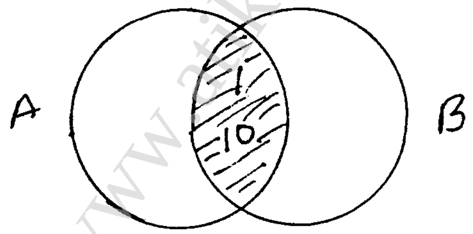
$A = \{1, 2, 5, 10\}$

$B = \{0, 1, 7, 10\}$

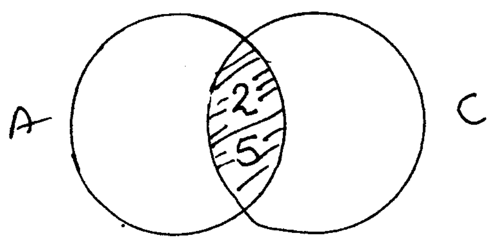
$C = \{0, 2, 5\}$



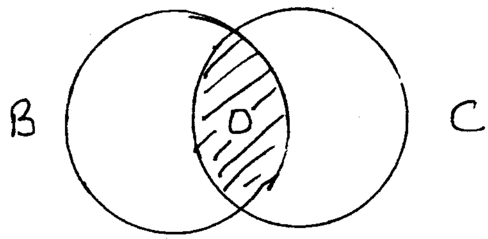
(b) (i) $A \cap B = \{1, 10\}$



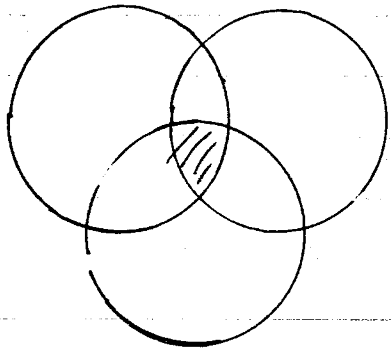
(ii) $A \cap C = \{2, 5\}$



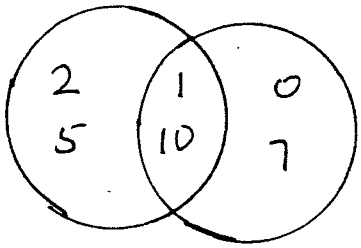
iii $B \cap C = \{0\}$



iv $A \cap B \cap C = \{ \} =$ empty set ϕ

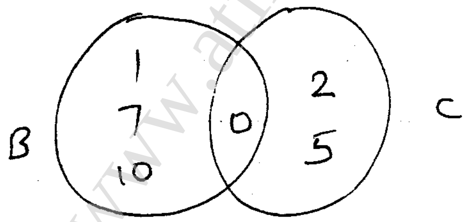


(v) $A \cup B$

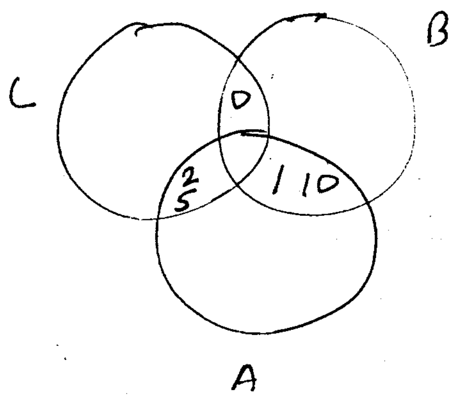


$= \{0, 1, 2, 5, 7, 10\}$

vi $B \cup C = \{0, 1, 2, 5, 7, 10\}$



$A \cup B \cup C = \{0, 1, 2, 5, 7, 10\}$



2.

A	B	$\bar{B}$	$A \wedge \bar{B}$	$A \to B$	$\overline{A \to B}$	$A \wedge \bar{B} \to \overline{A \to B}$
T	T	F	F	T	F	T
T	F	T	T	F	T	T
F	T	F	F	T	F	T
F	F	T	F	T	F	T

Tautology (all true)

3.

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \vee q$	$p \wedge \bar{q}$	$\bar{p} \vee q \wedge p \wedge \bar{q}$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

contradiction

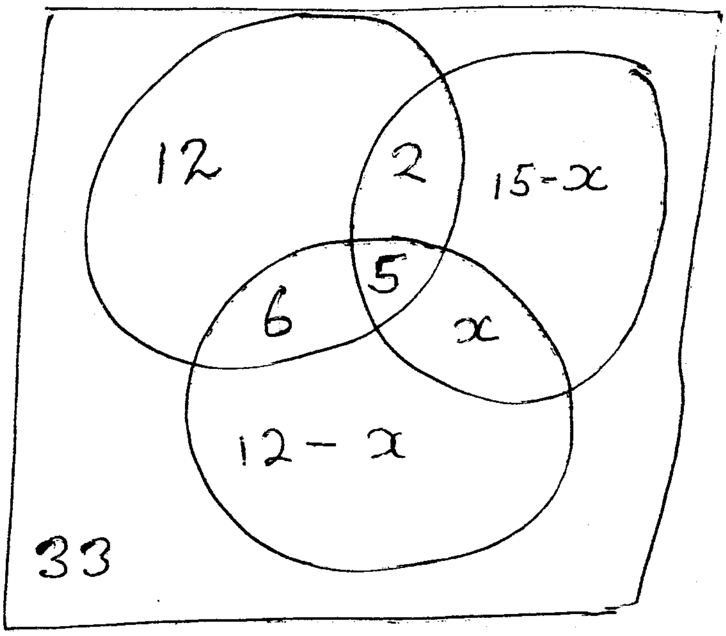
4. $n = 82$

$S = 25$
$C = 23$
$V = 22$

$S \cap C = 11$

$S \cap V = 7$

$S \cap C \cap V = 5$



$$12 + 2 + 5 + 6 + x + 12 - x + 15 - x = 82 - 33$$

$$52 - x = 49$$
$$-x = -3$$
$$x = 3$$

(b) $= 12 + 9 + 12$

$= 33.$

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5. $^nC_r = {}^{15}C_{11}$

$= \frac{15!}{11!(15-11)!}$ 3

$= 1365$

6. $^7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = 2520$

$1\cdot2 + 2\cdot3 + 3\cdot4 + \cdots\ n(n+1) = \frac{n(n+1)(n+2)}{3}$

for $n = 1$, $\frac{1(1+1)(1+2)}{3}$

$= \frac{2\cdot3}{3} = 2$ true

$n = K$

Hypothesis $1\cdot2 + 2\cdot3 + 3\cdot4 = K(K+1) = \frac{K(K+1)(K+2)}{3}$

conclusion for $n = K+1$

$1\cdot2 + 2\cdot3 + \cdots (K+1)(K+2) = \frac{(K+1)(K+2)(K+3)}{3}$

Add the $(K+1)^{th}$ term to hypothesis

$1\cdot2 + 2\cdot3 + \cdots K(K+1) + (K+1)(K+2) = \frac{K(K+1)(K+2)}{3} + (K+1)(K+2)$

$= \frac{K(K+1)(K+2) + 3(K+1)(K+2)}{3}$

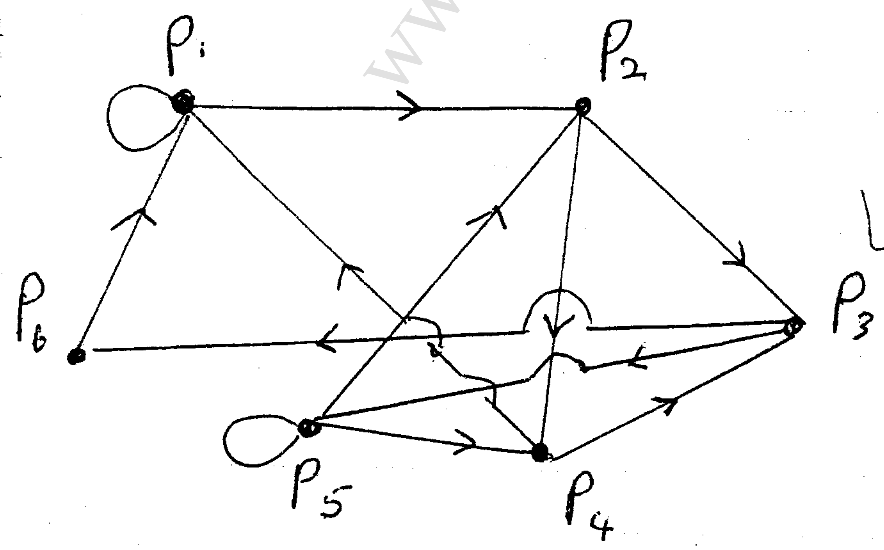
$$= \frac{K(K+1)(K+2) + 3(K+1)(K+2)}{3}$$

$$= \frac{(K+1)(K+2)(K+3)}{3}$$

$\therefore$ Hypothesis is equal to conclusion

8.

$$\begin{array}{c|cccccc} & P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \\ \hline P_1 & 1 & 1 & 0 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 1 & 1 & 0 & 1 \\ P_3 & 0 & 0 & 0 & 0 & 1 & 1 \\ P_4 & 1 & 0 & 1 & 0 & 0 & 0 \\ P_5 & 0 & 1 & 0 & 1 & 1 & 0 \\ P_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$



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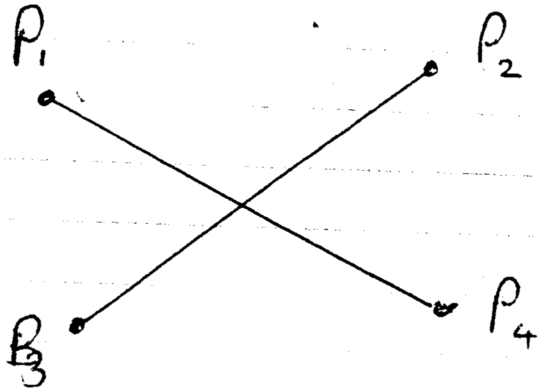
9.



$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

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