Mount Kenya University

UNIVERSITY EXAMINATION 2013/2014

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BACHELOR OF EDUCATION ARTS AND BACHELOR OF EDUCATION SCIENCE SCHOOL BASED

UNIT CODE: BMA1202 **UNIT TITLE: BASIC DISCRETE MATHEMATICS**

DATE: AUGUST2014 **MAIN EXAM** **TIME: 2 HOURS**

Instructions: Answer question one and any other two

1 a) Define the following terms;

i) Theorem
ii) Proposition
iii) Tautology
iv) Contradiction (4 Marks)

b) State the two De Morgan's laws. (2 Marks)

c) Let p: '*I am a student*' and q: '*I am a lecturer*'

Express the statements $\neg(p \vee q)$ and its logical equivalence given by De Morgan's law in words. (4 Marks)

d) Let p and q be two statements show that $(p \vee q) V \left[(\neg p) \wedge (\neg q)\right]$ is a tautology. (5 Marks)

e) Prove that the square of an odd number is also odd. (3 Marks)

f) Use the principle of mathematical induction to prove that

$1^3+2^3+3^3+\ldots .n^3=\frac{1}{4}n^2(n+1)^2$ (5 Marks)

g) Define $f:\mathbb{R}\rightarrow\mathbb{R}$ by $f(x)=3x+2$ and $g:\mathbb{R}\rightarrow\mathbb{R}$ by $g:(x)=2x-3, x\geq 0$ verify that $(f^{\circ}g)^{-1}=g^{-1\circ}f^{-1}$ (7 Marks)

2 a) Show that the logical statements $p\rightarrow q$ and $q\rightarrow p$ are logically equivalent

a∧r

(4 Marks)

b) Let p and q be two statements. Construct the complete truth table for the logical statements $\neg(p\wedge q)\wedge(\neg r)$ and verify that is a tautology. (6 Marks)

c) State the converse and the contra positive of the statement 'If you get an A in logic, then I will buy you a new pair of shoes' (5 Marks)

d) A man was caught on the King's property and was brought before the King to be punished. The king told the man, "You must make a statement, if your statement is true, then you will be killed by the lion and if your statement is false then you will be killed by the bufallo' then the man said "I will be killed by the buffalo". Discuss the truth values of the truth values of the sentence "I will be killed by the buffalo" in this context hence state whether it is statement. (5 Marks)

3 a) Prove that if two integers have opposite parity then their sum is odd. (4 Marks)

b) Let a, b, c $\in\mathbb{Z}$ prove that if a\b and b\c then a\c (4 Marks)

c) Outline the procedure for the proof of a conditional statement of the form using contra positive hence use it to prove that if 7x+9 is even, then x is odd. (6 Marks)

d) Use the principle of mathematical induction to prove that

$1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)$ (6 Marks)

4 Derive the following terms;

i) Injective function
ii) Surjective function
iii) Bijective function (6 Marks)

b) Prove that the function $f:\mathbb{R}\rightarrow\mathbb{R}$ defined by f(x)=4x+3 is both injective and surjective hence find its inverse. (7 Marks)

c) i) Show that the function $f:\mathbb{R}\rightarrow\mathbb{R}$ defined by $(f\circ g)^{-1} = q^{-1}\circ f$ is bijective. (4 Marks)

ii) Show that the function $f:\mathbb{R}\rightarrow\mathbb{R}$ defined by $f(x)=x^2+5$ is not bijective. (3 Marks)

5 a) Define the following terms as used in sets theory.

i) One to one correspondence
ii) Equivalent sets (4 Marks)

b) Draw Venn diagrams such that sets A, B and C satisfies the following conditions;

$A \subset B, C \cap B = \phi, A \cap C = \phi, A \cap \overline{B}$ (4 Marks)

c) Use Venn diagram to solve the following counting problem. A survey in a town showed that 10,000 people were smokers and 4,000 were drunkard. There were 2000 people who smoked and drunk. Given the above information

i) How many people smoke and do not drink?
ii) How many people drunk but did not smoke?
iii) How many people either smoke or drink? (12 Marks)

MARKING SCHEME

1. a) (i) Theorem: is a statement that has been studied and determined to be true.

(ii) A statement that is either true or false but not both.

(iii) Tautology is a statement which is true irrespective of the variables.

(iv) contradiction is a statement which is false irrespective of the variable propositions

4

(b) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

2

(c) It is not the case, I am a student or a lecturer.

$\neg p \wedge \neg q \rightarrow$ I am not a student and I am not a lecturer.

(d)

p	q	$p \vee q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$(p \vee q) \vee [(-p) \vee (-q)]$
T	T	T	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	F	T	T	T	T

5

(e) Let x be an odd number

Then $x = (2a+1)$

$x^2 = (2a+1)^2$

$= (2a+1)(2a+1)$

$= 4a^2 + 2a + 2a + 1$

$= 4a^2 + 4a + 1$

$= 2(2a^2+2a) + 1$ Let n be $2a^2+$ ...

$= 2n+1 \rightarrow$ odd number

5

(f) $1^3 + 2^3 + 3^3 + \cdots n^2 = \frac{1}{4} n^2 (n+1)^2$

for $n = 1$ $\frac{1}{4} 1^2 (1+)^2$

$= \frac{4}{4} = 1$ true

for Let $n = k$.

for $1^3 + 2^3 + 3^3 + \cdots K^3 = \frac{1}{4} K^2 (\ldots+1)^2$ – hypothesis

for $k+1$

$1^3 + 2^3 + 3^3 + K^3 + (k+1)^3 = \frac{1}{4} (k+1)^2 (k+2)^2$ – conclusion

5

Add $(k+1)^{th}$ term to hypothesis

$\frac{1}{4} (K+1)^2 (K+2)^2 + (K+1)^3$

$= \frac{(K+1)^2 (K+2)^2 + 4(K+1)^3}{}$

$= \frac{(K+1)^2 [(K+2)^2 + 4(K+1)]}{4}$

$= \frac{1}{4} k^2(k+1)^2 + 1(k+1)^3$

$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$

$\frac{(k+1)^2[k^2 + 4(k+1)]}{4}$

$= \frac{(k+1)^2(k+2)^2}{4}$

6

(a) $f(x) = 3x+2$

$g(x) = 2x-3$

$f \circ g = f[g(x)] = 3[2x-3) + 2$

$= 6x - 9 + 2$

$y = 6x - 7$

$f(x) = 6x - 7$

$f(x) + 7 = 6x$

$x = \frac{f(x) + 7}{6}$

$f^{-1}(x) = \frac{x+7}{6}$

$f(x) = 3x+2$

$f(x) - 2 = 3x$

$\frac{f(x) - 2}{3} = x \Rightarrow f^{-1}(x) = \frac{x-2}{3}$

$g(x) = 2x - 3$

$g(x) + 3 = 2x$

$\frac{g(x) + 3}{2} = x \Rightarrow g^{-1}(x) = \frac{x+3}{2}$

$g^{-1} \circ f^{-1} = g^{-1}(f^{-1}(x))$

$= \frac{\frac{x-2}{3} + 3}{2}$

$= \frac{x - 2 + 9}{3} \div 2$

$\frac{x+7}{3} \times \frac{1}{2} = \frac{x+7}{6}$

$\therefore (f \circ g)^{-1} = g^{-1} \circ f^{-1}$

7

2 (a)

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

(b)

~~p~~	~~q~~	~~r~~
~~T~~	~~T~~	
~~T~~	~~F~~	
~~F~~	~~T~~	
~~F~~	~~F~~	

p	q	r	$p \wedge q$	$-(p \wedge q)$	$-r$	$-(p \wedge q) \wedge -r$
T	T	T	T	F	F	F
T	T	F	T	F	T	F
T	F	F	F	T	T	T
F	F	F	F	T	T	T
T	F	F	F	T	T	T
T	T	F	T	F	T	F
T	F	T	F	T	F	F
F	T	F	F	T	T	T

6

(c) if you get A in logic, then I will buy you a new pair of shoes

converse $\Rightarrow$ if I buy you a pair of shoes, then you will get A in logic

contra-positive

If I do not buy you a pair of shoes, then you will not get A in logic

5

3 (a) Let the integer be x and y

parity $x - y = n$, positive

$x + y = k$

$x = 2a$

$y = 2a + 1$

$x = n + y$

$x + y = 2a + 2a + 1$

$= 4a + 1$

$2(2a) + 1$

$y + n + y = k$

$2y = k - n$

$y = \frac{k-n}{2}$

Let $2a$ and $2a+1$ - odd

4

(b) $a/b = n \Rightarrow a = nb$. integer.

$\frac{b}{c} = 2$ - integer.

$b = 2c$

$\frac{b}{2} = c$.

then $a/c = \frac{nb}{} \div \frac{b}{2}$.

$= \frac{nb}{} \times \frac{2}{\cancel{b}}$

$= n2$

$\therefore a/c$ since n and 2 integers

(i) Start by assuming the true statement is false then show that if the statement is false then other
(ii) statement is also false.

Let x be even instead of odd, then

$x = 2n$

Thus $7x+9 = 7(2n)+9$

$= 14n+9$

$= \cancel{2(7n)+4}$

$= 2(7n+4)+1$

take $7n+4$ be integer d

$2d+1$

Thus $2d+1$ is odd.

$\overline{6}$

Thus by contrapositive if the x is odd then $7x+9$ is odd.

(d) $1^2+2^2+3^3+ \cdots n^2 = \frac{1}{6}n(n+1)(2n+1)$

Let $n=1$, $1^2=1$ true. $\frac{1}{6}1(1+1)(2+1)$

$\frac{2\times3}{6} = \frac{6}{6} = 1$

Let $n=k$.

$1^2+2^2+\cdots+k^2 = \frac{1}{6}k(k+1)(2k+1)$ - hypothesis

for $n=k+1$

$1^2+2^2+\cdots+k^2+(k+1)^2 = \frac{1}{6}(k+1)(k+2)(2(k+1)+1$

nad $(K+1)^{th}$ term to hyp.

$$1^2+2^2+\dots+k^2+(k+1)^2 = \frac{1}{6}k(k+1)(2k+1)+(k+1)^2$$

$$= \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$$

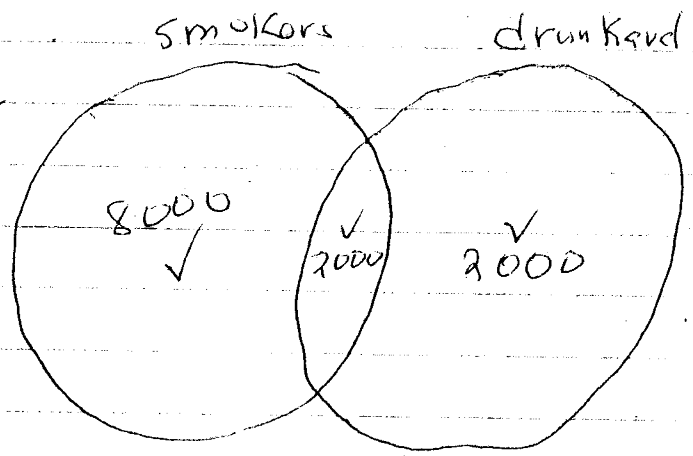
$$= \frac{1}{6}\left[(k+1)\left[k(2k+1)+6(k+1)\right]\right]$$

$$= \frac{1}{6}(k+1)\left[2k^2+k+6k+6\right].$$

$$= \frac{1}{6}(k+1)(2k^2+7k+6)$$

$$= \frac{1}{6}(k+1)(k+2)(2k+3)$$

6



(i) 10,000 − 2,000

= 8,000

(ii) 4000 − 2000

2000

(iii) 8 000 + 2000

= 10,000

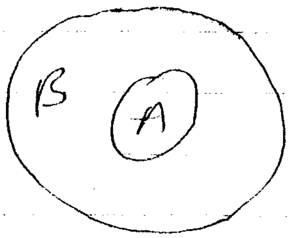
12

5 (a) (i) One to one correspondence mean two sets with equal number of elements

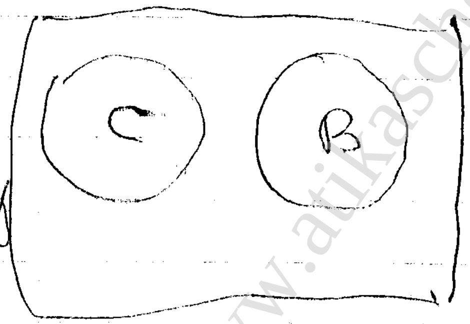
(ii) Equivalent sets are set with the same sets.

(b)

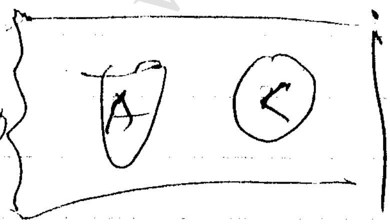
$A \subset B$

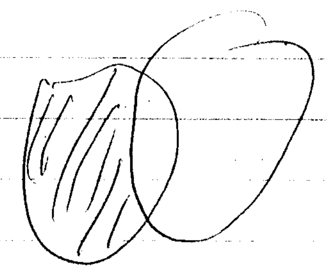


$C \cap B = \emptyset$



$A \cap C = \emptyset$





$\frac{1}{5}$

4. (i) Injective function is a function in the for $f(x)_1 = f(x)_2$

Surjective is the form $f(a) = b$, b is codomain of domain a

Bijective function → both injective and surjective

(b) $4x_1 + 3 = 4x_2 + 3$

$4x_1 = 4x_2$

$x_1 = x_2$

$f(x) = 4x + 3$

$f(x) - 3 = 4x$

$x = \frac{1}{4}(f(x) - 3)$

Thus $f(x)$ is surjective function

~~$g \circ f \circ g$~~ $(f \circ g)^{-1} = f(g(x))^{-1}$

$f_1 = f_2$

$g_1^{-1} = g \circ^{-1} f$

pro

$$x_1^2 + 5 = x_2^2 + 5$$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$

This is not true.