Name____________________________ Index No.____________________

3301/211
3302/211
3303/211
MATHEMATICS
Oct/Nov 2013
Time: 3 hours

Candidate's Signature______________

Date ____________________

THE KENYA NATIONAL EXAMINATIONS COUNCIL

HIGHER DIPLOMA IN LAND SURVEYING
HIGHER DIPLOMA IN PHOTOGRAMMETRY AND REMOTE SENSING
HIGHER DIPLOMA IN CARTOGRAPHY

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

Write your name and index number in the spaces provided above.
Sign and write the date of the examination in the spaces provided above.
You should have mathematical tables and a scientific calculator for this examination.
This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the spaces provided in this question paper.*
All *questions carry equal marks.*
Maximum marks to each part of a question are as shown.
Do ***NOT*** *remove any pages from this booklet.*
Candidates should answer the questions in English.

For Examiner's Use Only

Question	1	2	3	4	5	6	7	8	TOTAL SCORE
Candidate's Score									

This paper consists of 20 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Find the eigen values of the matrix.

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -1 \end{bmatrix}$$ (4 marks)

(b) Given that $[1 \quad 2 \quad 3]^T$ is an eigen vector of the matrix

$$A = \begin{bmatrix} \frac{7}{2} & \frac{-1}{2} & \frac{-1}{2} \\ 4 & -1 & 0 \\ \frac{-3}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix},$$

find:

(i) the corresponding eigen value;

(ii) the other two eigen values and their corresponding vectors;

(iii) a matrix B that diagonalizes A. (16 marks)

2. (a) Sketch the graph of the function

$$f(x) = \begin{cases} x^2 + x & -\pi < x < \pi \\ f(x + 2\pi) \end{cases}$$

in the range $-\pi < x < 3\pi$, and find its Fourier series representation. (16 marks)

(b) By setting $x = \pi$ in the expansion in (a) above, find the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$ (4 marks)

3. (a) Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \int_0^{\sin y} y^2 \cos x dx dy$$ (6 marks)

(b) Sketch the domain of integration and show that

$$\int_0^1 \int_{\frac{1}{2}y}^{y} \frac{xy^2}{\sqrt{x^3 + y^3}} dx dy + \int_1^2 \int_{\frac{1}{2}y}^{1} \frac{xy^2}{\sqrt{x^3 + y^3}} dx dy = \frac{4}{21}(3 - \sqrt{2})$$ (8 marks)

(c) Use a double integral to find the volume of the solid bounded by the surface $z = 4 - x^2 - y^2$ and the xy - plane. (6 marks)

4. (a) From first principles, find the Laplace transform of $f(t) = t^2 \cos t$ (9 marks)

(b) Use Laplace transforms to solve the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = 120\cos 3t$$

given that when $t = 0, x = 0$ and $\frac{dx}{dy} = 0$. (11 marks)

5. (a) Find the value of a, given that the three vectors $\underset{\sim}{A} = 4\underset{\sim}{i} + 5\underset{\sim}{j} - 6\underset{\sim}{k}$, $\underset{\sim}{B} = -2\underset{\sim}{i} + \underset{\sim}{j} + 8\underset{\sim}{k}$ and $\underset{\sim}{C} = 2\underset{\sim}{i} + 6\underset{\sim}{j} + a\underset{\sim}{k}$ are coplanar. (5 marks)

(b) Given that $\underset{\sim}{A} = xy^2\underset{\sim}{i} - xz\underset{\sim}{j} + x^2\underset{\sim}{k}$, find, at the point (-1, 2, 1):

(i) curl $\underset{\sim}{A}$;

(ii) div $\underset{\sim}{A}$. (7 marks)

(c) Given the scalar field $\phi = 4x^2y + z^3y^2$, find:

(i) grad ϕ at the point (-1, -2, -1);

(ii) the directional derivative of ϕ at the point (-1, 0, 1) in the direction of the vector $\underset{\sim}{A} = 2\underset{\sim}{i} - 4\underset{\sim}{j} + \underset{\sim}{k}$. (8 marks)

6. (a) Solve the differential equation

$$xy^2\frac{dy}{dx} + \frac{1+y^3}{1+x^2} = 0$$ (9 marks)

(b) Use the method of undetermined coefficients to solve the differential equation:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{-2x},$$

given that when $x = 0, y = 1$ and $\frac{dy}{dx} = -2$. (11 marks)

7. (a) Sketch the region bounded by the parabola $y = 4 - (x+3)^2$ and the straight line $x - y - 5 = 0$, and use integration to find the area of the region. (9 marks)

(b) A function is defined by the data in Table 1.

Table 1

x	0	1	2	3	4
f(x)	1	5	31	121	341

Using the Newton-Gregory difference interpolation formulas, determine:

(i) f (0.75);

(ii) f (4.9). (11 marks)

8. (a) A plant produces steel sheets whose weights are known to be normally distributed with a standard deviation of 2.4 kg. A random sample of 36 sheets had a mean weight of 31.4 kg. Find the 99% confidence limits for the population mean. (2 marks)

(b) Two makes of bulbs, Phillips and HMT were installed in private residential estate. A test was carried out to ascertain the average life of the bulbs from the two makes. The results are shown in Table 2.

Table 2

	Phillips	HMT
Number of bulbs on test	50	50
Average life	1500 hours	1512 hours
Standard deviation	60 hours	80 hours

Test the claim that there is no significant difference in the mean life of the two makes of bulb at 1% significance level. (8 marks)

(c) The heat output of wood is known to vary with the percentage of moisture content. Table 3 shows, in suitable units, the data obtained from an experiment carried out to assess the variation.

Table 3

Moisture content (X%)	50	8	34	2	45	15	74	82	60	30
Heat output Y	5.5	7.4	6.2	6.8	5.5	7.1	4.4	3.9	4.9	6.3

(i) Obtain the equation of the regression line for heat output on percentage moisture content, giving the values of the coefficients correct to three decimal places;

(ii) Hence estimate the heat output of wood with 40% moisture content. (10 marks)