Mount Kenya University

UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BAS/BEDA/BECS/BEDSC
REGULAR

UNIT CODE: BMA2102 **UNIT TITLE: PROBABILITY AND STATISTICS II**

DATE: AUGUST 2015 **MAIN EXAM** **TIME: 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE IN SECTION A AND ANY OTHER TWO QUESTIONS FROM SECTION B

1. a) Define random variable. (2 Marks)

b) A random variable X has the distribution function shown below.

X	0	1	2	3	4	5	6	7	8
F(x)	k	3k	5k	7k	9k	11k	13k	15k	17k

k is a positive constant, find;

i) The probability distribution of x (4 Marks)

ii) The value of k (3 Marks)

iii) $P(3 \leq x < 6)$ (3 Marks)

c) In a certain community the probability of a female birth is 0.6. If 10 individuals are randomly selected from the community, find;

i) The probability that exactly 4 of them are females. (2 Marks)

ii) The expected number of females in the sample. (2 Marks)

d) The probability density function of a random variable X is given by;

$$f(x)=\begin{cases} e^{-x} x \geq 0 \\ 0, otherwise \end{cases}$$

Find;

i) The moment generating function of x (3 Marks)

ii) E(X) (3 Marks)

e) The random variable X is normally distributed with mean μ and variance μ^2. Given that $P(x \leq 8)=0.95$, determine $P(4 \leq x \leq 11)$ (6 Marks)

f) A machine produces bolts which are 10 percent defective. Find the probability that in a random sample of 400 bolts produced by the machine, at most 30 of the bolts will be defective. (3 Marks)

2. a) According to the National office of vital statistics of the U.S department of health and human services, the average number of accidental drowning per year in the United States is 3 per 100000 people. In a certain city the population is 200000.
   i) Justify the use of poisson approximation for the distribution of the number of drownings per year in this city. (2 Marks)
   ii) Find the probability that in this city there will be not be not more than 4 accidental drownings. (5 Marks)

   b) A random variable X has probability density function given by;

$$f(x)=\begin{cases} 3x^{k}, 0 \leq x \leq 1 \\ 0, \text{elsewhere} \end{cases}$$

k is a positive constant, find;

i) The value of k (4 Marks)

ii) The mean of k (3 Marks)

iii) The mode of x (2 Marks)

iv) The median of x (4 Marks)

3. a) The joint density function of two random variables x and y is,

$$f(x,y)=\begin{cases}\frac{xy}{96}, 0\langle 2\langle, 1\langle y\langle 5\\ 0, elsewhere\end{cases}$$

Find;

i) P(1< x< 2, 2< y<3) (3 Marks)

ii) The marginal density functions of x and y (7 Marks)

b) Let x be a random variable with density function

$$f(x)=\begin{cases}\frac{\lambda^{x}e^{-\lambda}}{x!}, x=0,1,2,....(\lambda is a positive constant)\\ 0, otherwise\end{cases}$$

Find;

i) The moment generating function of x (4 Marks)

ii) Hence the mean and the standard deviation of x. (6 Marks)

4. a) The mean weight of 500 male students at a certain college is 151 lb and the standard deviation is 15 lb. Assuming that the weights of the students are normally distributed, find how many students weigh.

i) Between 120 lb and 155 lb (5 Marks)

ii) More than 185 lb (5 Marks)

b) The table below shows the probability distribution of random variable x.

X	0	1	2	3
F(x)	0.125	0.375	0.375	0.125

i) Find the distribution function of x (2 Marks)

ii) Find the expected value of x (3 Marks)

iii) Find the standard deviation of x (3 Marks)

iv) Graph the distribution function (2 Marks)

5. a) A random variable x has a probability function given by;

$$f(x)=\begin{cases}\dfrac{\chi^{\alpha-1}(1-\chi)^{\beta-1}}{B(\alpha,\beta)}, x=0\langle x\langle 1(\alpha,\beta\rangle 0) \\ 0, otherwise\end{cases}$$

B(α,β) is the Beta function. Show that mean and variance of the distribution is $\frac{\alpha}{\beta}$ and $\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)}$ respectively. (10 Marks)

b) The joint probability function of two discrete random variables x and y is given by

C is a positive constant

Find;

i)	The value of C	(3 Marks)
ii)	P(x=2),y=1)	(2 Marks)
iii)	P(x≥ 1, y≥ 2)	(2 Marks)
iv)	The marginal probability function of x	(3 Marks)