Mount Kenya University

UNIVERSITY EXAMINATION 2015/2016

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BED (SCIENCE), BED (ARTS) AND BSNE
SCHOOL BASED

UNIT CODE: BMA2103 UNIT TITLE: LINEAR ALGEBRA I

DATE: DECEMBER 2015 MAIN EXAM TIME: 2 HOURS

Instructions: Answer question one and any other two

Question one (30 marks)

a) Use cramers rule to solve the following simultaneous equations (6 Marks)
   i. 2x+y=7
   ii. 3x-4y=5

b) Find the inverse of the following matrix $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & -1 \\ 2 & 0 & 2 \end{bmatrix}$ (6 Marks)

c) Use Gauss Jordan method to solve the following system of equations (6 Marks)

   X+2y=4
   y-z=0
   x+2z=4

d) Let w be a subspace of IR^{+} generated by the vectors (1,-2,5,-3), (2,3,1,-4) and (3,8,-3,-5)
   Find a basis and dimension of w. (6 Marks)

e) Find the vector equation of the line through the points A(1,2,3) and B(4,4,4,) and find the coordinates where the line meets the plane z=0 (6 Marks)

Question two (20 marks)

a) Solve the following system of linear equations; (10 Marks)

$$7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 = 8$$
$$-3x_1 - 3x_2 + 2x_4 + x_5 = -1$$
$$4x_1 - x_2 - 8x_3 + 20x_5 = 1$$

b) Compute the determinant of the matrix $M = \begin{pmatrix} 2 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}$

(5 Marks)

c) Show that the map $f: IR^2 \rightarrow IR$ defined by f(x,y)=x+y+1 is not linear

(5 Marks)

Question three (20 marks)

a) In the real vector space IR4 determine the intersection UnV of the subspaces.

(10 Marks)

U={w,x,y,z)∈(IR^{4}ix+2y=0w+2x-z}
V={ w,x,y,z)∈(IR^4;w+x+2y=0}

b) Consider the vector space V=IR^3 over IR show that the sequence S: (1,0,1), (1,1,1), (1,2,1), (1,-1,2) is a spanning sequence of V and find a subsequence which is a basis.

Question four (20 marks)

a) Let $f: v \rightarrow w$ be a linear transformation prove that;

i) Kerf is a subspace of V (5 Marks)

ii) Imf is a subspace of W (5 Marks)

b) Find row and column ranks of the matrix $A=\begin{bmatrix}1 & 2\\ 0 & 3\\ 5 & 1\end{bmatrix}$

(5 Marks)

c) Evaluate the following determinant. $\begin{bmatrix}4 & 5 & 1 & 2\\ 0 & 3 & 2 & 1\\ 3 & 0 & 1 & 3\\ 2 & 7 & 0 & 3\end{bmatrix}$ (5 Marks)

Question five (20 marks)

a) Given the matrices $A=\begin{pmatrix}1 & 0 & 1\\ 2 & -1 & 1\\ 3 & 2 & -2\end{pmatrix} B=\begin{pmatrix}3 & 4 & 6\\ 2 & 0 & 5\\ 1 & 1 & -2\end{pmatrix}$ determine; (10 Marks)

b)

i) $A^T + B^T$

ii) $A^{-1}B^{-1}$

c) Find the matrix representation of the linear mapping $T: IR^3 \to R^2$ where T is defined by T(v)=AV where v is a column matrix and $A=\begin{pmatrix}2 & 5 & -3\\ 1 & -4 & 7\end{pmatrix}$ using basis{(1,1,1)(1,1,0) and (1,0,0)} of IR^3 and basis (1,3), (2,5) of IR^2

(5 Marks)

d) Express w=(-7,7,11) as linear combination of vectors V_1=(1,2,1) V_2=(-4,-1,2) and V_3=(-3,1,3) (5 Marks)