Instructions: Answer ALL questions.

1. The random variable $X$ has probability function

$$P(X = x) = \begin{cases} \frac{k}{x}, & x = 1, 2, 3, \\ \frac{1}{2}, & x = 4, 5 \end{cases}$$

Where $k$ is a constant.

a) Find the value of $k$. 

(2 marks)

b) Find the exact value of $E(X)$. 

(2 marks)

c) Show that $E(X) = 1.47$. 

(4 marks)

e) Find to 1 decimal place, $\text{Var}(4 - 3X)$. 

(2 marks)

2. The random variable $Y$ has probability generating function $M_Y(t)$ given by

$$M_Y(t) = \frac{1}{2} t^2 \left( \frac{1}{3} t^2 + 1 \right)$$

a) Find $E(Y)$, when $t = 1$. 

(5 marks)

The random variable $X$ has a binomial distribution with $n = 5$ and $p = \frac{1}{2}$.

b) Show that the probability generating function of the random variable $W = 5 - X$ is

$$G_W(t) = \left( \frac{2 + t}{3} \right)^5$$

(2 marks)

3. A continuous random variable $X$ has a probability density function $f(x)$ defined by

$$f(x) = \begin{cases} k(x - 2), & 2 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}, \text{ where } k \text{ is a constant.}$$

Find

a) The value of $k$. 

(3 marks)

b) $E(X)$. 

(2 marks)

c) The $\text{Var}(3X - 5)$. 

(4 marks)