Mt Kenya University

UNIVERSITY EXAMINATIONS 2011

SCHOOL OF EDUCATION
DEPARTMENT OF CURRICULUM AND INSTRUCTION
BACHELOR OF EDUCATION (SCHOOL BASED)
END OF SEMESTER EXAMINATION
EMA: 222 PROBABILITY AND STATISTICS 11

TIME 2HOURS APRIL 2011 SERIES

Instructions: Answer questions ONE (COMPULSORY) and any other TWO questions

Question One (compulsory -30marks)

a) The random variable $X \sim B(150, 0.02)$. use a suitable approximation to estimate $P(X \geq 7)$ [4marks]

b) The random variable X is Uniformly distributed over the interval $[-1, 5]$
   i. Sketch the probability density function $f(x)$ of X [2 marks]
   ii. Find $E(X)$ [1 mark]
   iii. Find $Var(X)$ [2 marks]
   iv. $P(-0.3 < x < 3.3)$ [2 marks]

c) It is estimated that in a certain country 4% of people have green eyes. In a random sample of size n, the expected number of people with green eyes is 5.
   i. Calculate the value of n [2marks]

The expected number of people with green eyes in a second random sample is 3.

   ii. Find the standard deviation of the number of people with green eyes in this second sample [4 marks]

d) The random variable Y has probability generating function $M_y(t)$ given by

$$M_y(t) = \frac{1}{3^{10}}(2+t)^5(2t+1)^5$$

   i. Find $E(Y)$, when $t = 1$ [4 marks]

The random variable X has a binomial distribution with $n = 5$ and $p = \frac{1}{3}$

   ii. Show that the probability generating function of the random variable $w = 5 - x$ is

$$G_W(t) = \frac{(2t+1)^5}{3^5}$$ [5marks]

e) A random sample $x_1, x_2, \ldots\ldots\ldots\ldots, x_{10}$ is taken from a normal population with mean 100 and standard deviation 14.
   i. Write down the distribution of $\bar{x}$, the mean of this sample. [2marks]
   ii. Find $P(|\bar{x} - 100|) > 5$ [2marks]

$\mu = np$
$5 = n \times 0.04$
$5/0.04 = n$
$n = 125$

$\mu = 3$
$\sigma^2 = np(1-p)$

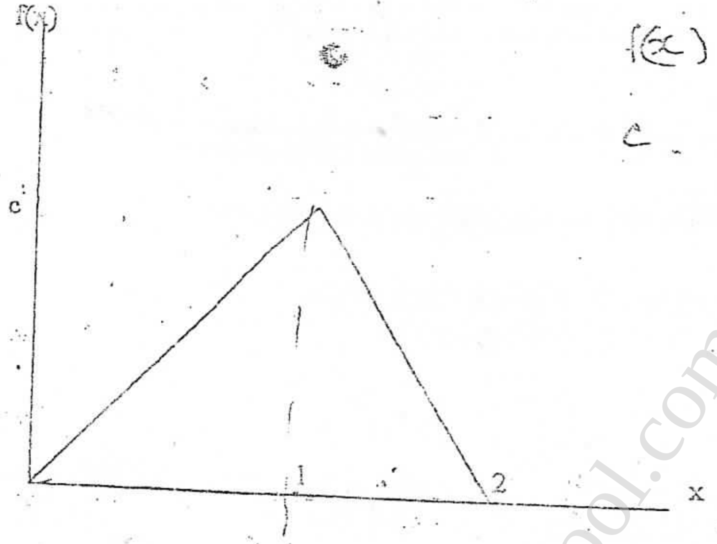
$E(x)$

$\int_a^b x f(x)$

Question two (20 marks)

The total number of radio taxi calls received at a control centre in a month is modeled by a random variable X (in tens of thousands of calls) having the probability density function

$$f(x) = \begin{cases} cx, & 0 < x < 1 \\ c(2-x), & 1 \le x < 2 \\ 0 & otherwise \end{cases}$$



$f(x)$

$c \cdot\ c\left[\frac{x^2}{2}\right]_0 = \frac{1}{2}c$

$c\left[2x - \frac{x^2}{2}\right]_1^2$

$(4-2) - (2 - 1/2)$

$2 - 1/2 = 1/2\, c$

$\frac{1}{2}c + 1/2\, c = 1$

$c = 1.$

a) Show that the value of c is 1

b) Write down the probability that $x \le 1$ [2marks]

c) Show that the cumulative distribution function of X is [1 mark]

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}x^2, & 0 \le x < 1 \\ 2x - \frac{1}{2}x^2 - 1 \;\; {-3/2} & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

d) Find the probability that the control centre receives between 8000 and 12000 calls in a month [5marks]

[3 marks]

A colleague criticizes the model on the grounds that the number of radio calls must be discrete, while the model used for X is continuous

e) State briefly whether your consider that it was reasonable to use this model for X [2 marks]

f) Give two reasons why the probability density function the diagram above must be unsuitable model.

g) Sketch the shape of a more suitable probability density function [2marks]

h) The random variable X~B(150, 0.02) use a suitable approximation to estimate P (X > 7) [1mark] [4matks]

$\mu = np$

$= 150 \times 0.02 = 3$

0	1	2	3	4	...

var $\sigma^2 = np(1-p)$ $P(x)$

$= 3 \times (1 - 0.02)$

$= 2.94$

$X = 7$

$Z = \frac{7-3}{1.7146}$

$= 2.333$

from tables

$\frac{12\,000}{10,000}$