UNIVERSITY EXAMINATIONS 2011

SCHOOL OF EDUCATION
DEPARTMENT OF CURRICULUM AND INSTRUCTION
BACHELOR OF EDUCATION (SCHOOL-BASED)
END OF SEMESTER EXAMINATION
EMA: 222 PROBABILITY AND STATISTICS 1

APRIL 2011 SERIES

TIME 2 HOURS

Instructions: Answer questions ONE (COMPULSORY) and any other TWO questions.

Question One (compulsory - 30 marks)

(a) The random variable X \sim \text{B}(150, 0.22), use a suitable approximation to estimate \( P(X \geq 7) \).

\( \text{[2 marks]} \)

(b) The random variable X is uniformly distributed over the interval \([-1, 3]\).

i. Sketch the probability density function \( f(x) \) of X.

\( \text{[1 mark]} \)

ii. Find \( E(X) \).

\( \text{[2 marks]} \)

iii. Find \( \text{Var}(X) \).

\( \text{[2 marks]} \)

iv. \( P(-0.3 < x < 3.3) \)

\( \text{[2 marks]} \)

(c) It is estimated that in a certain country 4% of people have green eyes. In a random sample of size \( n \), the expected number of people with green eyes is 5.

i. Calculate the value of \( n \).

\( \text{[2 marks]} \)

The expected number of people with green eyes in a second random sample is \( \frac{3}{2} \).

ii. Find the standard deviation of the number of people with green eyes in this second sample.

\( \text{[4 marks]} \)

d) The random variable \( Y \) has probability generating function \( M_Y(t) \) given by

\[ M_Y(t) = \frac{1}{3} t^x \left( \frac{2}{3} + t \right) \left( \frac{1}{3} + t \right) \]

i. Find \( E(Y) \), when \( t = 1 \).

\( \text{[4 marks]} \)

The random variable \( X \) has a binomial distribution with \( n = 5 \) and \( p = \frac{1}{3} \).

ii. Show that the probability generating function of the random variable \( W = 5 - X \) is

\( \text{[3 marks]} \)

e) A random sample \( x_1, x_2, \ldots, x_{10} \) is taken from a normal population with mean 100 and standard deviation 14.

i. Write down the distribution of \( \bar{x} \), the mean of this sample.

\( \text{[2 marks]} \)

ii. Find \( P \left( \frac{X - 100}{14} > 5 \right) \).

\( \text{[2 marks]} \)
Question two (20 marks)

The total number of radio taxi calls received at a control centre in a month is modeled by a random variable \( X \) (in tens of thousands of calls) having the probability density function

\[
f(x) = \begin{cases} 
2x, & 0 < x < 1 \\
2(2-x), & 1 \leq x < 2 \\
0, & \text{otherwise}
\end{cases}
\]

a) Show that the value of \( c \) is 1

b) Write down the probability that \( x \leq 1 \).

c) Show that the cumulative distribution function of \( X \) is

\[
F(x) = \begin{cases} 
0, & x < 0 \\
\frac{1}{2}x^2, & 0 \leq x < 1 \\
2x - \frac{1}{2}x^2 - \frac{3}{2}, & 1 \leq x < 2 \\
1, & x \geq 2
\end{cases}
\]

d) Find the probability that the control centre receives between 8000 and 12000 calls in a month.

A colleague criticizes the model on the grounds that the number of radio calls must be discrete, while the model used for \( X \) is continuous.

e) State briefly whether you consider that it was reasonable to use this model for \( X \).

f) Give two reasons why the probability density function the diagram above must be unsuitable model.

g) Sketch the shape of a more suitable probability density function.

h) The random variable \( X \sim N(150, 10^2) \), use a suitable approximation to estimate \( P(X > 7) \).

\[
P(X > 7) = n(1-p)^{\frac{x-1}{p}}
\]

\[
= 160 \times 0.98^2 \approx 1.2 \times 10^2
\]

\[
\text{Var} = 3 \times (1 - 0.10^2)
\]

\[
= 2.98
\]