SECTION A

1. a. Define a discrete random variable. (2 Marks)
   b. Probability distribution function. (2 Marks)

A random variable \( Z \) has a probability function

<table>
<thead>
<tr>
<th>( Z )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(Z) )</td>
<td>( k )</td>
<td>3( k )</td>
<td>5( k )</td>
<td>7( k )</td>
<td>9( k )</td>
<td>11( k )</td>
<td>13( k )</td>
<td>15( k )</td>
<td>17( k )</td>
</tr>
</tbody>
</table>

c. Determine the value of \( k \) (3 Marks)

\[ \sum k = 1 \quad \Rightarrow \quad k = \frac{1}{8} \]

d. Find \( P(3 \leq Z < 6) \) (2 Marks)

\[ P(3 \leq Z < 6) = P(Z = 3) + P(Z = 4) = \left( \frac{7}{8} \right) + \left( \frac{9}{8} \right) = \frac{16}{8} = 2 \]

The continuous random variable \( X \) has probability density function given by:

\[ f(x) = \begin{cases} 
2x & \text{if } 0 \leq x \leq 1 \\
0, \text{otherwise} 
\end{cases} \]
(a) Show that the moment generating function of \( X \) is given by:
\[
M_X(t) = \frac{2[t^2(t-1)+1]}{t^2}
\]
(4 Marks)

By Expanding \( M_X(t) \) as a power series in \( t \), find \( E(X) \) and \( E(X^2) \). (5 Marks)

g. Find the variance of \( X \). (2 Marks)

Along a stretch of Thika super highways, breakdowns requiring the summoning of the breakdown services occur with a frequency of 2.5 per day, on average.

h. Find the probability that there will be exactly 2 breakdowns on a given day. (3 Marks)

i. Find the smallest integer \( c \) such that the probability of more \( c \) breakdowns in a day is less than 0.03. (4 Marks)

j. Find the probability that \( 0 < X < 3 \). (3 Marks)

### SECTION B

2. The probability density function of a random variable \( X \) is:
\[
f(x) = \begin{cases} 
  x, & \text{for } 0 \leq x \leq 1 \\
  2-x, & \text{for } 1 \leq x < 2 \\
  0, & \text{otherwise}
\end{cases}
\]

i. Find cumulative distribution function
(5 Marks)

ii. Find \( F(3/2) \)
(2 Marks)

iii. Find the median of the distribution
(3 Marks)

Under what circumstance would it be sensible to use?

a. The normal distribution as an approximation of the binomial distribution? (2 Marks)

b. The Poisson distribution as an approximation of the Binomial distribution? (2 Marks)