Mount Kenya University

UNIVERSITY EXAMINATION 2013/2014

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS STATISTICS AND ACTURIAL SCIENCE

BAS, BED (SCIENCE), BED (ARTS), AND BSNE
SCHOOL BASED

UNIT CODE: BMA2103 UNIT TITLE: LINEAR ALGEBRA

DATE: AUGUST 2014 MAIN EXAM TIME: 2 HOURS

Instructions; Answer ALL questions in Section A and any other TWO questions from Section B.

SECTION A

Question One (30 Marks)

a) Show that vector addition is associative under addition. (4 Marks)

b) Let A and B be the matrices

$$A = \begin{pmatrix} 2 & 1 & -4 \\ 6 & 3 & -9 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 12 \\ 4 & 0 \\ 2 & 6 \end{pmatrix}$$

Show that:

i) $A \cdot B$ is not a null matrix

ii) $AB \neq B \cdot A$ (5 Marks)

c) Given that

$\bar{a} = 3\bar{i} - 2\bar{j} - 4\bar{k}$

$\bar{b} = 2\bar{i} + 4\bar{j} - 3\bar{k}$

$\bar{c} = \bar{i} + 2\bar{j} - \bar{k}$ Find

i) $3\bar{a} - 2\bar{b} + 4\bar{c}$

ii) $|3\bar{a} - 2\bar{b} + 4\bar{c}|$

iii) A unit vector parallel to $(3\bar{a} - 2\bar{b} + 4\bar{c})$ (6 Marks)

d) Find the angle between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ (6 Marks)

e) Find the unit vector which is parallel to the line.
$\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-7}{12}$ (3 Marks)

f) Find the rank of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$$ (3 Marks)

g) Show that the set of vectors
$U_1 = (2,1,1)$; $U_2 = (-1,3,1)$ and $U_3 = (1,-2,-3)$ are linearly independent. (3 Marks)

SECTION B

Question Two (20 Marks)

a) Find the equation of a line through points P (3, 4, 1) and Q (2, 7, 6). (5 Marks)

b) Solve the following equations by Gauss – Jordan elimination method.

$$2x_1 - 2x_2 + x_3 = 3$$
$$3x_1 + x_2 - x_3 = 7$$
$$x_1 - 3x_2 + 2x_3 = 0$$ (5 Marks)

c) Given the matrix

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{bmatrix}$$, determine

i) A matrix C which contains all cofactors of A.

ii) Adjoint of cofactors of A

iii) Determinant of A

iv) A^{-1} (inverse of A) (10 Marks)

c) Given that $p=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix}$ $q=\begin{pmatrix}3 & 1 & 2\\ 4 & 5 & 6\\ -1 & 7 & 0\end{pmatrix}$

i. State the order of the matrix p. (1 Mark)

ii. Determine $3p-2q$ (2 Marks)

iii. Find P.Q (3 Marks)

d) Determine the rank of the matrix;

$$\begin{pmatrix}1 & -1 & 3 & 3\\ 1 & 2 & 3 & 1\\ 2 & 1 & 2 & 0\end{pmatrix}$$

(5 Marks)

e) Find the determinant of the matrix; $A=\begin{pmatrix}1 & -1 & 1\\ 1 & 3 & 2\\ 2 & 3 & 0\end{pmatrix}$ (3 Marks)

4. a) i. Find the inverse of the matrix;

$$A=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 2 & 1 & -1\end{pmatrix}$$

(5 Marks)

ii. Using a(i) above, solve the system of equations;

$$\begin{aligned} x+y+z&=7\\ x-y+2z&=9\\ 2x+y-z&=1\end{aligned}$$

(3 Marks)

b) Solve the system of equations by Gauss-Jordan elimination method.

$$\begin{aligned} 2a+b+3c&=4\\ a-2b+c&=1\\ a+4b-c&=2\end{aligned}$$

(5 Marks)

c) Solve the system of equations by Cramer's rule;

$$f - 8g \quad -h \quad = -1$$
$$-f + 4g \quad +h \quad = 3$$
$$3f - 2g \quad +6h \quad = 5$$

(7 Marks)

5. a) Let P(1, -1, -3) be a point on the line e which is parallel to the vector $\underline{a} = (2, -3, 4)$ find the;

   i. Vector equation of the line. (1 Mark)
   ii. Parametric equation of the line (2 Marks)
   iii. Symmetric equation of the line (2 Marks)
   iv. Coordinates of the point where this line meets the plane z=0. (3 Marks)

b) Given a plane $2x + 2y - z + 3 = 0$ find the distance of P(1, 1,1) from the plane. (5 Marks)

c) Find the point of intersection of the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{2}$ on the plane $2x + y + z - 6 = 0$ (4 Marks)

d) Find the equation of a plane passing through 0(0,0,0) $M_1(1, 2, 3)$ and $M_2(4, 5, 6)$. (3 Marks)

Question Three (20 Marks)

a) Use Cramer's rule to solve the following system of linear equations.

$3x + y - z = 14$
$x + 3y - z = 16$
$x + y - 3z = -10$

$-15 + 6 - 1$
-15

10 (~~6~~ Marks)

b) Reduce $\begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{pmatrix}$ to row reduced echelon form (canonical form) (6 Marks)

c) Show that the following vectors are linearly dependent.
$V_1(2,1,1)$ $V_2(-1,3,1)$ $V_3(-5,1,-1)$ (4 Marks)

Question Four (20 Marks)

a) Let $\underset{\sim}{V}$ be a vector space over a field K. For any scalar $k \in K$ and $O \in \underset{\sim}{V}$, KO=O. Prove. (4 Marks)

b) $\left\| \underset{\sim}{A} \times \underset{\sim}{B} \right\|$ is the area of a parallelogram with sides $\underset{\sim}{A}$ and $\underset{\sim}{B}$. Prove. (4 Marks)

c) i) Define linear combination of a vector $\underset{\sim}{V}$ over a field K. (2 Marks)

ii) For what values of k will the vectors $\underset{\sim}{U} = (1, -2, k)$ in IR^3 be a linear combination of the vectors $\underset{\sim}{V} = (3,0,-2)$ and $\underset{\sim}{W}(2,-1,-5)$? (3 Marks)

d) Find the coordinates of the intersection of the line
$\frac{x-2}{2} = \frac{y+3}{-3} = \frac{z-3}{-6}$ and the plane $3x + 4y + 3z = 6$. (5 Marks)

e) Given that:
$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 3 \\ 4 & 7 \end{pmatrix}$
Determine $3A + 4B$. (2 Marks)

Question Five (20 Marks)

a) Find the point of intersection of the lines
$l_1 = \frac{x-3}{3} = \frac{y-1}{4} = \frac{7}{5}$ and
$l_2 = \frac{x-1}{4} = \frac{y-1}{7} = \frac{z+4}{-4}$ (6 Marks)

$-11 + 3$

b) Determine whether or not the following form a basis for the vector space IR^3.
$(1,1,2), (1,2,5)$ *and* $(5,3,4)$ (4 Marks)

c) Write the vector $\underset{\sim}{V}(2,-5,3)$ in IR^3 as a linear combination of the vectors.
$\underset{\sim}{e_1} = (1,-3,2)$, $\underset{\sim}{e_2} = (2,-4,-1)$ and $\underset{\sim}{e_3} = (1,-5,7)$. (6 Marks)

d) Show that the vectors
$\underset{\sim}{A} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{B} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{C} = \underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ can form sides of a right angled triangle. (4 Marks)