Take away assignment
Probability i - the exercise book.

Mount Kenya University

UNIVERSITY EXAMINATION 2013/2014

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF NATURAL SCIENCES

BACHELOR OF EDUCATION SCIENCE
SCHOOL BASED

UNIT CODE: BMA 221 UNIT TITLE: LINEAR ALGEBRA I

DATE: DECEMBER 2013 MAIN EXAM TIME: 2 HOURS

ANSWER QUESTION ONE IN SECTION A AND ANY OTHER TWO QUESTIONS FROM SECTION B

SECTION A

1. A. Give the difference between a vector and a scalar. (2 Marks)

b. Given that $\underline{a} = 4i - j - k, b = 2i + 8j + 5k$ find the magnitude of $\underline{c} = \underline{a} + \underline{b}$. (3 Marks)

c. i. Define dot product of vectors $\underline{c and d}$. (1 Mark)

ii. Find the dot product of vectors $\underline{c} = 2i - k + j$ and $\underline{d} = i + j - k$ (2 Marks)

iii. Find the angle between vectors $\underline{b} = 3i + 2j - 4k$ and $\underline{c} = i - 6j + 5k$ (4 Marks)

d. i. Given $\underline{a} = (1, -2, 3)$, $\underline{b} = (-7, -6, -2)$, $\underline{c} = (3, 3, 3)$ and $\underline{d} = (3, -2, -4)$, find the linear combination $-3\underline{a} + \underline{b} - \frac{1}{3}\underline{c} - \underline{d}$. (3 Marks)

ii. Check whether the vectors $\underline{a}=\begin{pmatrix}2\\1\end{pmatrix}$, $\underline{b}=\begin{pmatrix}4\\2\end{pmatrix}$ and $\underline{c}=\begin{pmatrix}3\\3\end{pmatrix}$ are linearly dependent.

(3 Marks)

e. Briefly describe any two properties that a vector space must satisfy.

(2 Marks)

f. What do you understand by ~~Gauss~~ Cross product of two vectors $\underline{a}$ and $\underline{b}$?

(2 Marks)

g. Solve for p, q, r and s given that $\begin{pmatrix}3 & 2\\6 & 4\end{pmatrix}=\begin{pmatrix}r-s & p-q\\2r+s & p+q\end{pmatrix}$ (2 Marks)

h. Find the parametric equation of the line passing though A(1,-3,4) and parallel to the vector $\underline{n}=(3,5,-6)$.

(2 Marks)

i. Find the equation of the plane passing through m(2,2,1) and parallel to the vectors $\underline{q}=(3,2,5)$ and $\underline{b}=(1,-1,0)$

(3 Marks)

j. Define a basis of a vector space.

(1 Mark)

SECTION B

2. a. What do you understand by a linear transformation? (3 Marks)

b. Show that the mapping $f:R^3\to R^2$ defined by $f\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}a_1+a_2\\a_2-a_3\end{pmatrix}$ is a linear transformation.

(5 Marks)

c. Determine 2A+3B given that $A=\begin{pmatrix}-2 & 7\\3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}2 & 1\\3 & 4\end{pmatrix}$ (2 Marks)

d. Reduce the matrix $\begin{pmatrix}1 & 2 & 3 & 1\\2 & 1 & 2 & 0\\1 & -1 & 3 & 3\end{pmatrix}$ to echelon form and state its rank.

(5 Marks)

Question Three (20 Marks)

a) Use Cramer's rule to solve the following system of linear equations.

$3x + y - z = 14$

$x + 3y - z = 16$

$x + y - 3z = -10$ (10Marks)

b) Reduce $\begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{pmatrix}$ to row reduced echelon form (canonical form) (6 Marks)

c) Show that the following vectors are linearly dependent.

$V_1(2,1,1)$ $V_2(-1,3,1)$ $V_3(-5,1,-1)$ (4 Marks)

Question Four (20 Marks)

a) Let $\underset{\sim}{V}$ be a vector space over a field K. For any scalar $k \in K$ and $O \in \underset{\sim}{V}$, KO=O. Prove. (4 Marks)

b) $\left|\underset{\sim}{A} \times \underset{\sim}{B}\right|$ is the area of a parallelogram with sides $\underset{\sim}{A}$ and $\underset{\sim}{B}$. Prove. (4 Marks)

c) i) Define linear combination of a vector $\underset{\sim}{V}$ over a field K. (2 Marks)

ii) For what values of k will the vectors $\underset{\sim}{U} = (1, -2, k)$ in IR^3 be a linear combination of the vectors $\underset{\sim}{V} = (3,0,-2)$ and $\underset{\sim}{W} = (2,-1,-5)$? (3 Marks)

d) Find the coordinates of the intersection of the line

$\frac{x-2}{2} = \frac{y+3}{3} = \frac{z-3}{-6}$ and the plane $3x + 4y + 3z = 6$. (5 Marks)

e) Given that:

$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 3 \\ 4 & 7 \end{pmatrix}$

Determine $3A + 4B$. (2 Marks)

Question Five (20 Marks)

a) Find the point of intersection of the lines

$l_1 = \frac{x-3}{3} = \frac{y-1}{4} = \frac{7}{5}$ and

$l_2 = \frac{x-1}{4} = \frac{y-1}{7} = \frac{z+4}{-4}$ (6 Marks)

b) Determine whether or not the following form a basis for the vector space IR^3.
$(1,1,2), (1,2,5)$ *and* $(5,3,4)$ (4 Marks)

c) Write the vector $\underset{\sim}{V}(2,-5,3)$ in IR^3 as a linear combination of the vectors.
$\underset{\sim}{e_1} = (1,-3,2)$, $\underset{\sim}{e_2} = (2,-4,-1)$ and $\underset{\sim}{e_3} = (1,-5,7)$. (6 Marks)

d) Show that the vectors
$\underset{\sim}{A} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{B} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{C} = \underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ can form sides of a right angled triangle. (4 Marks)