KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2009/2010
OPEN, DISTANCE AND E-LEARNING EXAMINATION FOR THE DEGREE
OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SMA 202: LINEAR ALGEBRA I

DATE: Wednesday 21st July, 2010 
TIME: 2.00 p.m – 4.00 p.m

INSTRUCTIONS:
Answer question ONE and any other TWO questions.

Question One- Compulsory

a) Given \( A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \)

Show that \((AB)^T = B^TA^T\)  
[3 marks]

b) Evaluate

i) \( \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{vmatrix} \)  
[2 marks]

ii) Hence find \( \begin{vmatrix} 18 & 21 & 6 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{vmatrix} \)  
[1 mark]

c) i) Let \( A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{pmatrix} \)

Find \( A^{-1} \)  
[6 marks]

ii) Hence solve the given system
\( x + 2y + z = 4 \)
\( 3x - 4y - 2z = 2 \)
\( 5x + 3y + 5z = -1 \)  
[4 marks]
d) Solve using Cramer’s rule
\[\begin{align*}
2x - 5y + 2z &= 7 \\
x + 2y - 4z &= 3 \\
3x - 4y - 6z &= 5
\end{align*}\] [4 marks]

e) Determine k so that the vectors \(u = (2, 3k, -4, 1, 5)\) and \(v = (6, -1, 3, 7, 2k)\) are orthogonal. [3 marks]

f) Write the vector \(v = (1, -2, 3)\) as a linear combination of the vectors \(e_1 = (1, 1, 1)\), \(e_2 = (1, 2, 3)\) and \(e_3 = (2, -1, 1)\). [6 marks]

h) Show that the vectors \((1, 1, 1, 1), (1, 1, 1, 1), (0, 1, 1, 1)\) and \((0, 0, 0, 1)\) form a basis for \(\mathbb{R}^4\). [2 marks]

**Question Two** (20 marks)

a) Find the value of \(\lambda\) if the matrix \(A\) below is singular
\[
A = \begin{bmatrix}
\lambda & \lambda \\
3 & \lambda - 2
\end{bmatrix}
\] [2 marks]

b) Reduce matrix \(A = \begin{bmatrix} 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}\) to row-reduced echelon form. [6 marks]

c) Use Cramer’s rule to solve the system of equations
\[
\begin{align*}
x_1 + 3x_2 + 2x_3 &= 3 \\
2x_1 + 4x_2 + 2x_3 &= 8 \\
x_1 + 2x_2 - x_3 &= 10
\end{align*}\] [6 marks]

d) Using Gauss – Jordon method, solve for \(x_1, x_2\) and \(x_3\)
\[
\begin{align*}
2x_1 - 4x_2 + 6x_3 &= 20 \\
3x_1 - 6x_2 + x_3 &= 22 \\
-2x_1 + 5x_2 - 2x_3 &= -18
\end{align*}\] [6 marks]