

REAL ANALYSIS FOR SCHOOL BASED 2015 APRIL BMA 3107

Q1a(i). Show that if $s = \sqrt{n+1} - \sqrt{n-1}$ for any integer $n \geq 1$, then s is irrational. (4mks)

b) Let x and y be positive real numbers show that (6mks)

i) $x + y$ is also positive

$0 < x$ and $0 < y$

ii) $x < y$ if and only if $x^2 < y^2$

hence $0 < 0 < 0 < x + y$

$x + y > 0$ hence positive.

iii) $x < y$ implies $\frac{1}{y} < \frac{1}{x}$

c) Using the concept of neighbourhood of a point $x_0 \in A \subset \mathbb{R}$, Determine whether the set $A = \{x: x \in \mathbb{R}, -2 \leq x < 3\}$ is closed or not. (3mks)

d) i) When is a sequence of real numbers said to be Cauchy in \mathbb{R} ? (2mks)

ii) Proof that $\frac{1}{n}$ is Cauchy in \mathbb{R} (5mks)

Q2 a) Show from the first principles that the sequence $x_n = 3 + (-1)^n \frac{1}{n^3} \forall n \in \mathbb{N}$ Converges to 3. (10mks)