KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

INSTITUTIONAL BASED PROGRAMME (IBP) AUGUST SESSION

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SMA 103: ANALYTIC GEOMETRY

DATE: Wednesday, 28th December 2011 **TIME:** 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS:

Answer question ONE and any other TWO questions.

1. (a) Find the angle between two lines whose equations are $x + 2y = 4$ and $4x - y = 2$ (4 marks)

(b) Find the distance of the line $3x + 4y - 3 = 0$ from the point (4,5) (2 marks)

(c) Find the equation of a circle with centre at (1, -2) and passes through (4, -3) (4 marks)

(d) Express the equation $x^2 + y^2 = 7 - 6y$ in polar coordinates. (4 marks)

(e) Show that the line segments joining (-3, 11), (2, -1) and (14, 4) form a right triangle. (4 marks)

(f) Find the vertex, focus and equation of a directrix of a parabola whose equation is $2x^2 - x - y = 1$ (4 marks)

(g) Find the equation of the hyperbola with a asymptotes $x - y = -1$ and $x + y = -3$ and vertex (3, -1). (4 marks)

(h) Find the equation of the ellipse with vertices at (1,5) and 1, -1), and foci at (1,4) and (1,0)

2. (a) Find the locus of a point P(x, y) which moves so that its distance from (2, 4) is twice its distance from (0, 0). Describe the locus. (7 marks)

(b) The points $A(x_1, y_1)$ and $B(x_2, y_2)$ are the ends of a diameter of a circle. Find the equation of the circle. (4 marks)

(c) Show that the circles $x^2 + y^2 - 2ax + C^2 = 0$, $x^2 + y^2 - 2by - c^2 = 0$ are orthogonal. (6 marks)

(d) Find the length of the tangent from the point (1, 1) to the circle $x^2 + y^2 - 4x - 6y + 12 = 0$. (3 marks)

3. (a) Find the equation of a parabola with vertex (-2, -4) and directrix $x = 3$ (6 marks)

(b) Given the equation of a parabola is $8y = 12 - 4x + x^2$, determine the coordinates of the vertex, focus and equation of the directrix. Hence sketch its graph. (8 marks)

(c) Find the rectangular equation of the "rose" $r = 4 \sin 2\theta$ (6 marks)

4. (a) By use of definition of the hyperbola, find the equation of the hyperbola with foci at (1, 2) and (11, 2) with a transverse axis of 8. (7 marks)

(b) Write the following equation in standard form $5x^2 - 4y^2 + 20x + 8y = 4$. Hence identify and sketch the curve. (7 marks)

(c) Analyse the following equation

$$9x^2 - 90x + 25y^2 - 150y + 225 = 0$$ (6 marks)

The tangent to the circle $x^2 + y^2 - 4x - 6y - 7 = 0$ at $(4,1)$ meets the x-axis at A and the y-axis at B. Find the area of the triangle OAB, where O is the origin.

[4 marks]

Question 4 (20 marks)

(a) (i) Define a parabola [2 marks]

(ii) With the help of a sketch derive the general equation of a parabola that opens to the left with focus $(-c, 0)$ and the vertex at the origin. [4 marks]

(iii) Find the equation of the parabola whose vertex is at $(4, 1)$ and the directrix is $x = 2$, hence sketch it. [4 marks]

Analyse the curve

$2x^2 + 5y - 3x + 4 = 0$ by finding

(i) Vertex

(ii) Focus

(iii) Directrix

(iv) Axis of symmetry

(v) Latus rectum

Sketch the curve and show the above features. [10 marks]

Question 5 (20 marks)

(a) Analyse the hyperbola

$4x^2 - 5y^2 - 16x + 10y + 31 = 0$.

Sketch its graph indicating clearly the centre, vertices, foci and asymptotes.

[8 marks]

(b) Determine the equation of the hyperbola with its centre at the point $(2,3)$, transverse axis on the line x=2, eccentricity $\frac{2\sqrt{3}}{3}$ and length of latus rectum 18.

[7 marks]

QUESTION 2 (20 MARKS)

a) Derive the equation of the Locus of a point P(x, y) which moves so that the product of the gradients of the straight lines joining P to the points (-2, 1) and (4, 5) is 3. By completing the squares in the variables involved, identify the curve as either a parabola, ellipse or hyperbola. (7 marks)

b) A parabola whose axis is vertical passes through the points (-19, 27) (17, 9) and $(-1, \frac{9}{2})$. If the vertex of the parabola lies on the line $3x - y - 15 = 0$, find its equation in standard form. (7 marks)

c) The lines $x + 3y + 11 = 0$ and $2x - y - 13 = 0$ are diameters of a circle. If the circle passes through (-4, 1), find its equation in standard form. (4 marks)

QUESTION 3 (20 MARKS)

a) An ellipse has equation $9x^2+25y^2+72x-100y+19=0$. Determine the standard form of the ellipse and hence find its centrer, foci, vertices, directrix, eccentricity and length of the latus rectum.

b) The end points of the major and minor axes of an ellipse are (7, 7), (10, -3), (7, 1) and (4, -3). Find the equation of the ellipse in standard form. (4 marks)

c) Find the shortest distance between the circles

$x^2+y^2-8x+6y=0$ *and* $x^2+y^2+16x-4y+59=0$. (6 marks)

QUESTION 4 (20 MARKS)

a) One focus of a hyperbola is at (1, -3) and the corresponding directrix is the line $y = 2$. If the eccentricity $e=\frac{3}{2}$, determine the equation in standard form.

(10 marks)

b) Determine the center, vertices, foci, eccentricity asymptotes, directrix and length of the latus rectum for the hyperbola. $16x^2-9y^2+32x+72y+16=0$.

(10 marks)

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The tangent to the circle $x^2 + y^2 - 4x - 6y - 7 = 0$ at $(4,1)$ meets the x-axis at A and the y-axis at B. Find the area of the triangle OAB, where O is the origin.

[4 marks]

Question 4 (20 marks)

(a) (i) Define a parabola [2 marks]

(ii) With the help of a sketch derive the general equation of a parabola that opens to the left with focus $(-c, 0)$ and the vertex at the origin. [4 marks]

(iii) Find the equation of the parabola whose vertex is at $(4, 1)$ and the directrix is $x = 2$, hence sketch it. [4 marks]

(b) Analyse the curve

$2x^2 + 5y - 3x + 4 = 0$ by finding

(i) Vertex

(ii) Focus

(iii) Directrix

(iv) Axis of symmetry

(v) Latus rectum

Sketch the curve and show the above features. [10 marks]

Question 5 (20 marks)

(a) Analyse the hyperbola

$4x^2 - 5y^2 - 16x + 10y + 31 = 0.$

Sketch its graph indicating clearly the centre, vertices, foci and asymptotes.

[8 marks]

(b) Determine the equation of the hyperbola with its centre at the point $(2,3)$, transverse axis on the line x=2, eccentricity $2\sqrt{3}$ and length of latus rectum 18.

[7 marks]