TAKE AWAY HOME. SESSION 5.

REAL ANALYSIS

Mount Kenya University

UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BEDSC, BEDA, BAS

REGULAR

UNIT CODE: BMA3107 UNIT TITLE: REAL ANALYSIS

DATE: AUGUST 2015 MAIN EXAM TIME: 2 HOURS

Instructions: Answer Question ONE and ANY other TWO

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Show that if $x \neq 0$, then $x^{-1} \neq 0$ and x^{-1} is unique. (3mks)

b) For every $x \neq 0$, show that $x^2 > 0$, hence show that $1 > 0$. (3mks)

c) Let (S, <) be an ordered set and E a subset of S, if the least upper bound of E (lub. E) and the greatest lower bound of E (glb. E) exist. Show that lub.E and glb.E are unique. (6mks)

d) Show that $\sqrt{2}$ is an irrational number. (4mks)

e) State the completeness axiom for IR (2mks)

f) Let A be a nonvoid subset of IR which is bounded above. Define a set B by $B = \{-x; x \in A\}$, show that B is bounded below and -sup.A=inf.B. (4mks)

g) If a and b are given real numbers such that for every real number $\varepsilon > 0$, $a \le b + \varepsilon$, show that $a \le b$ (5mks)

h) What is an inductive set? (2mks)

QUESTION TWO (20 MARKS)

a) For any subset E of a metric space (X, ρ), prove that E^0 is an open set. (6mks)

b) Consider the metric space (IR, d) and let $f : IR \to IR$ be defined by $f(x) = |x|$. Show that f is uniformly continuous. (6mks)

c) Show that the limit of a convergent sequence is unique in a metric space (8mks)

QUESTION THREE (20 MARKS)

a) Let A and B be nonvoid subsets of IR and define the set $A + B = \{x + y; x \in A, y \in B\}$, show that

i. If A and B are bounded above, then so is A+B and sup(A+B)=sup.A+sup.B (5mks)

ii. If A and B are bounded below, then so is A+B and inf.(A+B)=inf.A+inf. B (5mks)

b) For every real numbers x and a, a>0, show that $|x| \le a$ iff $x \in [-a, a]$ (4mks)

c) Let A, B, C be nonvoid sets and $f : A \to B$ and $g : B \to C$ be bijections. Then the composition $g_0 f$ and $(g_0 f)^{-1} = f^{-1}{}_0 g^{-1}$. (6mks)

QUESTION FOUR(20 MARKS)

a) Show that every infinite set E contains a countable subset A. (7mks)

b) Differentiate between an algebraic and a transcendental number giving examples in each case (3mks)

c) Does the equation $x^2 + 1 = 0$ have a solution in IR? Show your working. (4mks)

d) Define the following terms;

i. A metric space (4mks)

ii. An interior point of E (2mks)

QUESTION FIVE (20 MARKS)

a) State and provide a proof of Cauchy –Schwarz inequality. (10mks)

b) Suppose that an open interval $(0,1)$ is equivalent to IR . Show that IR is uncountable (10mks)