Mount Kenya University

UNIVERSITY EXAMINATION 2015/2016

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BED (SCIENCE), BED (ARTS) AND BSNE AND BAS
SCHOOL BASED

UNIT CODE: BMA3107 UNIT TITLE: REAL ANALYSIS

DATE: DECEMBER 2015 MAIN EXAM TIME: 2 HOURS

Instructions; Answer ALL questions in Section A and any other TWO questions from Section B

QUESTION 1 (30MARKS)

SECTION A

a). List the elements of the following sets:

i. A = {prime numbers less than 20} (1 Mark)
ii. B={even numbers less than 16} (1 Mark)
iii. C={Multiplies of 4 less than 16} (1 Mark)

Find;

iv. The intersection of the sets B and C. (1 Mark)
v. The union of the sets A and B. (2 Mark)
vi. The set difference B-A. (2 Mark)
vii. If the universal set E=AUBUC find the complement of the set B (2 Marks)

b). i. Define an Irrational number. Show that $\sqrt{n+1}$ -$\sqrt{n-1}$ is irrational for all n∈ R^+ (5 Marks) c). Define the concept of a countable set. Show that the set Z of intergers is countable.

(5 Marks)

d).Determine the supremum and infimum of the following sets and state the one that belongs to the set.

i. $\left[\frac{1}{2}, \frac{3}{2}, \frac{1}{3}, \frac{4}{3} \frac{1}{4} \frac{5}{4}, \ldots\right]$ ii $\left[-2, \frac{1}{2}, \frac{-3}{2}, \frac{1}{3}, \frac{-4}{3}, \frac{1}{4}, \ldots\right]$

(4 Marks)

e) i).Define the concept of lim sup (x_n) and lim inf (x_n) . (2 Marks)

ii).By computing lim sup (x_n) and lim inf (x_n) determine whether each of the following sequences diverges or convergences in R.

1. $x_n = (-1)^2(1-\frac{1}{2})$ for all n $\in J^+$ (2 Marks)

2. $x_{n=\frac{n}{5}}$ - $\left[\frac{n}{5}\right]$ for all n$\in J^+$

Where [x] denotes the largest interger $\leq x$ (2 Marks)

QUESTION 2 (20 MARKS)

a) a). explain what is meant by the following:
   - (i) the set of natural numbers (2 Marks)
   - (ii) The set of integer (2 Marks)
   - (iii) A rational number (2 Marks)
   - (iv) The set of real numbers (2 Marks)

b) Give the difference between an even and an odd number (2 Marks)

c) Let m be an integer. Show that m is odd if and only if m^2 is odd (5 Marks)

d) Let (S, <) be an ordered set and E a subset of S, if the least upper bound of E (lub. E) and the greatest lower bound of E (glb. E) exist. Show that lub.E is unique.

(5 Marks)

QUESTION 3 (20 MQARKS)

a) Let (X, d) be a metric space and A be a subset of X(Take X to be R) By use of examples define the following terms.

i) An open set A (2 Marks)

ii) A closed set A (2 Marks)

iii) Neighborhood of a point $p \in X$ (2 Marks)

b) Prove that the set Q^c of irrational numbers is uncountable (4 Marks)

ii. Show that if t is irrational then $s = \frac{t}{t-1}$ is also irrational. (4 Marks)

c) Show from first principles that the sequences $(x_n) = 1 + (-1)^n \frac{1}{n^2}$ converges to 1.

(6 Marks)

QUESTION 4 (20 MARKS)

a) State the completeness axiom. Prove that if S is a bounded subset of real numbers then Sup S and *Inf*S both belong to *S Iff S* is closed (10 Marks)

b) Give the definition of the derivative of a function *g (x)* . Prove that if *g (x)* is differentiable at *C* then *g(x)* is continuous at *C* (10 Marks)

QUESTION 5 (20 MARKS)

a) For all real numbers *x, y* show that:

i. $| x+y | \leq |x| + |y|$ (4 Marks)

ii. $|x - y| \geq ||x| - |y||$ (4 Marks)

b) Consider the sequence (y_n) defined by $y_{1=\sqrt{2}}$, $y_{n+1} = \sqrt{2y_n}$ for all $n \geq 2$. Use induction to show that the sequence is monotonic increasing and that $y_n \leq 2$ for all $n \in J^+$. State with reasons whether y_n diverges or converges in R. (7 Marks)

c) Show that for a bounded set *S* of real numbers there exists a number G >0 such that $|x| \leq G \; \forall x \in s$

(5 Marks)