ALGEBRAIC STRUCTURES ASSIGNME
2016 April.

MOUNT KENYA UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/16

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

BACHELOR OF EDUCATION ARTS & BACHELOR OF EDUCATION SCIENCE AND BSNE

SCHOOL BASED

BMA 1105: ALGEBRAIC STRUCTURES

AUGUST 2015

SUPPLEMENTARY/ SPECIAL EXAM DRAFT

TIME:2 HOURS

① c, g ② all, ③ a, b, c ④ h ⑤ b(ii)

INSTRUCTIONS: ANSWER QUESTION ONE IN SECTION A AND ANY OTHER TWO QUESTIONS FROM SECTION B

SECTION A

QUESTION ONE (30 MARKS)

(a) Given the meaning of the following symbols as used in algebraic structures(5 mks)

(i) $a \in A$

(ii) $\mathbb{R}$

(iii) $f: X \to Y$

(iv) $A \cup B$

(v) Z_3

(b) Give the meaning of a set and list the elements of the following sets (2mks)

(i) $A = \{a \text{ is even: } -3 \leq a \leq 4\}$ (2mks)

(ii) B = { prime numbers less than 11} (1mk)

✓(c) Given that $A = \{1,2,3,4,5,6,7\}$, $B = \{1,5,9\}$, and the universal set $E = \{x : 1 \leq x \leq 10\}$ find

(i) $A \cap B$ (2mks)

(ii) A-B (2mks)

(iii) the complement of the set B (2mks)

(d) State four examples of fields (4mks)

(e) Which between z_3 and z_4 is a field? Give reasons (4mks)

(f) Let G= (z, +) be the group of all integers under addition. If $H = \{3a: a \in z\}$, find all the disjoint left cosets of H in G (4mks)

✓(g) Let $\div$ be defined on z, the set of integers by $a \div b = a + b + 3$ for all $a,b \in z$

(i) show that $\ast$ satisfies the closure property, associativity and commutativity(4mks)
(ii) find the identity element (1mk)
(iii) find the inverse of $a \in z$ (1mk)

SECTION B

2. (a) Let G= (z_5, x_5) be the set of non-zero integers modulo 5 under multiplication modulo 5. Show that

(i) G is closed under multiplication modulo 5.(2mks)

(ii) Multiplication modulo 5 is associative in z_5.(2mks)

(b) (i) state the identity element in (a) above (2mks)

(ii) state the inverse of every element in z_5(2mks)

(c) show that G is abelian (2mks)

(d) let $f: R \longrightarrow R$ be defined by $f(x) = x/(x+1)$

(i) find f(-5), the domain and co domain of f (4mks)

(ii) find $f^{-1}(x)$ and $f^{-1}(1)$ (3mks)

(iii) show that f is 1-1 (3mks)

3. the following are cayley tables of a ring R

+	a	b	c	d	E	f	g
a	e	a	d	g	F	c	b
b	a	b	c	d	E	f	g
c	d	c	a	e	G	b	f
d	g	d	e	f	B	a	c
e	f	e	g	b	C	d	a
f	c	f	b	a	D	g	e
g	b	g	f	c	A	e	d

x	a	b	c	d	E	f	g
a	f	b	d	a	G	e	c
b	b	b	b	b	B	b	b
c	d	b	g	c	F	a	e
d	a	b	c	d	E	f	g
e	g	b	f	e	D	c	a
f	e	b	a	f	C	g	d
g	c	b	e	g	A	d	f

(a) state the identity element of R under + and x (2mks)

(g) give the inverse of each element under + (3mks)

(h) give the inverse of each element under x (3mks)

(i) solve for x and y if

(i) $(x+c)a = g^2$ (4mks)

(ii) $(exy) + d = e$ (3mks)

(j) complete the table below (5mks)

x	a	b	c	D	e	f	g
X^2							
X^3							

4. given that A={a, b, c, d, e, f},

B={1, 2, 3, 4, 5, 6, 7}

C= {p, q, r, s, t, u, v, w }

And that h and g are functions defined by

h: a→2, b→3, c→5, d→6, e→7, f→4,

g; 1→p, 2→q, 3→r, 4→s, 5→u, 6→v, 7→w,

(f) define the composition map gh (2mks)

(g) using a diagram, show how each element in A is mapped by the composite map gh (5mks)

(h) Explain whether or not the inverse functions f^{-1}: B→A: g^{-1}: C→B and $(gf)^{-1}$: C→A exist. (6mks)

(i) Find $g^{-1}(E)$ given that E= {p,r,u,v,w } (5mks)

(j) Find $f^{-1}(D)$ given that D={4,5,6,7} (4mks)

5. (a) let G ={A_1, A_2, A_3, A_4} be the multiplicative group of four matrices:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, A_4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Complete the cayley table below, hence show that G is a group. (5mks)

X	A_1	A_2	A_3	A_4
A_1				
A_2				
A_3				
A_4				

(b) State the zero divisors and units in z_6 ={0,1,2,3,4,5} the, ring of integers modulo 6 (5mks)

© The following is an operation table for a group G={e,a,b,c}

	e	a	b	C
E	e	a	b	C

A	a	b	c	e
B	b	c	e	A
C	c	e	a	B

(i) let H = {e, b}. show that H is a sub group of G (4mks)

(ii) find all the disjoint right cosets of H in G (4mks)

(iii) Show that the inverse of ac is $c^{-1}a^{-1}$ (2mks)