1521/203 1601/203
1522/203 1602/203
MATHEMATICS
June/July 2016
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN ELECTRICAL AND ELECTRONIC TECHNOLOGY (POWER OPTION) (TELECOMMUNICATION OPTION)

MODULE II

MATHEMATICS

3 hours

MANYAM FRANCHISE
Discover|Learn|Apply

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Mathematical tables/non-programmable scientific calculator;

Answer booklet.

Answer any ***FIVE*** *of the following* ***EIGHT*** *questions.*

All questions carry equal marks.

Maximum marks for each part of a question are as shown.

Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

© 2016 The Kenya National Examinations Council

1\. (a) Given the matrices $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -1 \\ 0 & -2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -2 & 3 \\ -3 & 2 & -1 \\ 1 & -4 & -2 \end{bmatrix}$, determine the matrix $D = C^2 + (AB)^T$. (8 marks)

(b) Three currents I_1, I_2 and I_3 in an electric circuit satisfy the simultaneous equations:

$$\begin{aligned} I_1 + I_2 + I_3 &= 4 \\ -I_1 + 2I_2 - I_3 &= 2 \\ 2I_1 - I_2 + I_3 &= 1 \end{aligned}$$

Use the inverse matrix method to determine the values of the currents. (12 marks)

2\. (a) Write down the term in x^5 in the binomial expansion of $(2x - 3y)^{11}$, and determine its value when $x = \frac{1}{3}$ and $y = \frac{1}{2}$. (7 marks)

(b) (i) Expand $(1 - 8x)^{-\frac{1}{2}}$ as far as the term in x^3.

(ii) Hence, by putting $x = \frac{1}{100}$ in the result in (i), determine the value of $\frac{1}{\sqrt{23}}$ correct to four decimal places. (6 marks)

(c) The magnetic field strength H due to a magnet of length 2ℓ and moment M at a point on its axis distance x from the centre is given by $H = \frac{M}{2\ell}\left\{\frac{1}{(x-\ell)^2} - \frac{1}{(x+\ell)^2}\right\}$.

Use the binomial theorem to show that $H = \frac{2M}{x^3}$, approximately. (7 marks)

3\. (a) Prove the identities:

(i) $1 - \frac{\sin x \tan x}{1 + \sec x} = \cos x$;

(ii) $\text{Cosec}2\theta \text{Cos}2\theta = \frac{1}{2}(\cot\theta - \tan\theta)$.

(8 marks)

(b) If A, B and C are angles of a triangle, prove that:

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C.$$ (8 marks)

(c) Solve the equation $\cos 2\theta + \cos\theta + 1 = 0$, for $0° \leq \theta \leq 180°$. (4 marks)

MANYAM FRANCHISE
Discover!Learn!Apply

4. (a) Given $3x^2 - kx + 12$ is positive for all values of x, determine, by completing the square, the possible values of k. (6 marks)

(b) Solve the equation: $3(3^{2x+2}) - 7(3^{x+1}) + 2 = 0$, correct to four decimal places. (6 marks)

(c) Three forces F_1, F_2 and F_3 necessary to keep a body in equilibrium satisfy the simultaneous equations:

$$\begin{aligned} 2F_1 + 3F_2 - F_3 &= 12 \\ -2F_1 + F_2 + 3F_3 &= 16 \\ F_1 - 2F_2 + F_3 &= 1 \end{aligned}$$

Use the substitution method to solve the equations. (8 marks)

5. (a) Determine the values of p and q such that $3\cosh x + 5\sinh x = pe^{x} + qe^{-x}$. (5 marks)

(b) Given the trigonometric identities

(i) $1 + \cot^2 x = \operatorname{cosec}^2 x$

(ii) $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$,

use Osborne's rule to derive the corresponding hyperbolic identities. (3 marks)

(c) Solve the equations:

(i) $3\cosh x + 4\sinh x = 3$;

(ii) $\cos 2\theta + \sin\theta = 0$, for $0^\circ \leq \theta \leq 360^\circ$. (12 marks)

6. (a) Determine a unit vector that is perpendicular to the vectors:

$\underset{\sim}{A} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{B} = -\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$. (5 marks)

(b) Given the scalar field $\phi = x^3y^2 + yz$, determine, at the point (1,-1,1):

(i) $\nabla\phi$;

(ii) the directional derivative of ϕ in the direction of the vector $\underset{\sim}{A} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$. (8 marks)

MANYAM FRANCHISE
Discover!Learn!Apply

(c) The magnetic field $\underset{\sim}{B} = xy\underset{\sim}{i} + x^2y\underset{\sim}{j} - z^2\underset{\sim}{k}$ exists in a region of space. Determine, at the point $(1, 0, 1)$:

(i) $\nabla \cdot \underset{\sim}{B}$;

(ii) $\nabla \times \underset{\sim}{B}$.

(7 marks)

7. (a) Given:

(i) $y = \frac{1}{x^2}$, find $\frac{dy}{dx}$ from first principles;

(ii) $y = a\cos 3x + b\sin 3x$, show that $\frac{d^2y}{dx^2} + 9y = 0$.

(10 marks)

(b) Determine the stationary points on the curve $y = 2x^3 + 3x^2 - 12x + 7$ and state their nature.
(10 marks)

8. (a) Given $u = \frac{x - y}{x + y}$, prove that $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$. (7 marks)

(b) Locate the stationary points of the function $z = 8 - 2x^2 - 3y^2 + 6xy - 8x$, and determine their nature.
(8 marks)

(c) Evaluate the integrals:

(i) $\int \sin 3x \sin x \, dx$;

(ii) $\int_0^1 \left(\frac{x^{-\frac{1}{2}} + x^{-\frac{3}{2}}}{x^{-\frac{3}{2}}} + 2x \right) dx$.

(5 marks)

MANYAM FRANCHISE
Discover!Learn!Apply

THIS IS THE LAST PRINTED PAGE.