Name……... **Index No** ……...

Candidate's Signature ……

Date: ……..

121/1

MATHEMATICS

Paper 1

2019

Time: $2^1/_2$ Hours

Revision Kit 2019

Kenya Certificate of Secondary Education (K.C.S.E)

MATHEMATICS

Paper 1

INSTRUCTIONS TO THE CANDIDATES

- *Write **your name** and **index number** in the spaces provided above*
- *This paper contains two sections; **Section** 1 and **Section 11**.*
- *Answer all the questions in **section 1** and only **five** questions from **Section 11***
- *All workings and answers must be written on the question paper in the spaces provided below each question.*
- *Marks may be given for correct working **even if** the answer is wrong.*
- *Calculations and KNEC Mathematical tables may be used **EXCEPT** where stated otherwise.*
- *Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.*

FOR EXAMINERS'S USE ONLY

Section 1

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Marks																	

Section 1I

Question	17	18	19	20	21	22	13	24	**Total**
Marks									

GRAND TOTAL

This paper consists of 16 printed pages. Candidates should check carefully to ascertain that all the pages are printed as indicated and no questions are missing.

SECTION I (50 MARKS)

Answer all questions in this section in the spaces provided.

1. Without using a calculator ,evaluate

$$\frac{3/4 + 1\,5/7 \div 4/7 \text{ of } 2\,1/3}{(1\,3/7 - 5/8) \times 2/3}$$ Giving your answer as mixed fraction (3mks)

2. Two boys and a girl shared some money. The younger boy got $5/18$ of it; the elder boy got $7/12$ of the remainder and the girl got the rest. Find the percentage share of the younger boy to the girl's share. (4mks)

3. Three numbers,1400, 1960 and **n** have a G.C.D and L.C.M of 70 and $2^2 \times 5^2 \times 7^2 \times 11$ respectively. Find the least possible value of **n** (3mks)

4. A bus starts off from Kitale at 9. a.m and travels towards Kakamega at a speed of 60km/hr. At 9.50 a.m, a matatu leaves Kakamega and travels towards Kitale at a speed of 60Km/h. How far from Kitale will the two vehicles meet. (3mks)

5. Find the equation of a straight line which is equidistant from the points **A**(2,3) and **B** (6,1) (3mks)

6. Simplify the expression completely (3mks)

$$\frac{12x^2 - 16x}{20 - 11x - 3x^2}$$

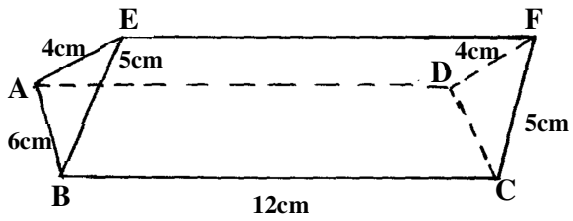
7. Given that $\sin\theta = \frac{2}{3}$ and θ is an a cute angle, find without using tables $tan^2\theta + Cos^2\theta$. Give your answer as a mixed fraction. (3mks)

8. Solve for ***y*** in the equation below. (4mks)

$8(2^2)^y = 6(2^y) - 1$

9. Using a ruler, a pair of compasses only and (proportional) a set square, construct on the upper side division of line **BC**, a line **BD** such that $\angle$**DBC** = 37.5^{o}. Use the line **BD** to divide **BC** into 4 equal portions. (3mks)

10. Sketch the net of the solid below. (2mks)

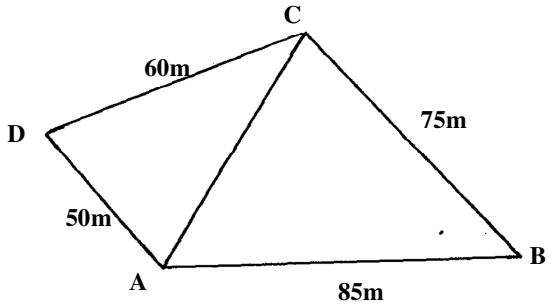


11. In a regular polygon, each interior angle is x^{o} and each exterior angle is $\left(\frac{x-36}{3}\right)^{o}$

(i) Find angle X^{o} (1mk)

(ii) Find the number of sides of the polygon (2mks)

12. The figure below represents a plot of land **ABCD** such that **AB**= 85m, **BC** 75m **CD**= 60m **DA** = 50m and angle **ACB** = 90^{o}. (not drawn to scale)



Determine the area of the plot, in hectares correct to two decimal places. (4mks)

13. An open rectangular box measures externally 32cm long , 27cm wide and 15cm deep. The box is made up of metal 1cm thick. If it has a mass of 1.5kg, what is the density of the box to 4 significant figures? (3mks)

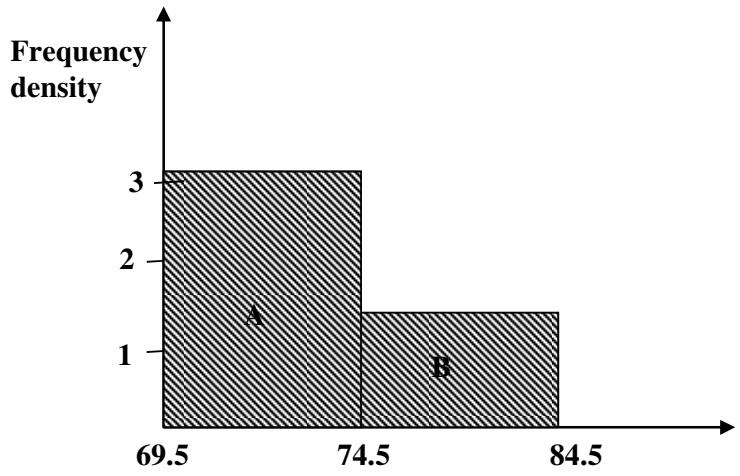
14. Find the integral values of x which satisfy the following inequalities;
$2x + 3 > 5x - 3 > -8$ (3mks)

15. A Kenyan bank buys and sells foreign currency as shown below.

	Buying Ksh	Selling Ksh
1 US dollar ($)	63.00	63.20
1 UK pound (£)	125.00	125.95

A tourist arrived in Kenya with £ 9600 which he converted into Kshs at a commission of 5%. He later used ¾ of the money before changing the balance of dollars at no commission calculate ; to the nearest dollar, the amount he received. (3mks)

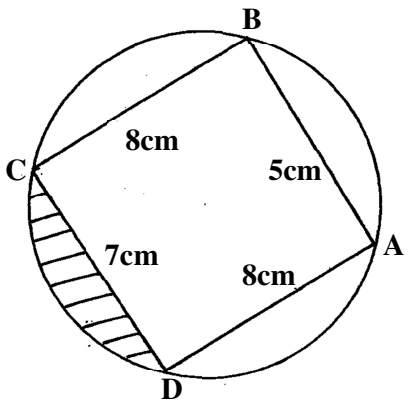
16. The histogram shown below represents the distribution of marks obtained in attest. The bar marked A has a height of 3.2 units while **B** has a height 1.2 units. If the frequency of the class represented by **B** is 6, find the frequency of the bar represented by **A**. (3mks)



SECTION II (50 MARKS)

Answer any five questions in this sections in the spaces provided.

17. The figure below (not drawn to scale) shows a quadrilateral **ABCD** inscribed in a circle. **AB** = 5cm, **BC** = 8cm,**CD** = 7cm and **AD** = 8cm. **AC** is one of the diagonals of length 10cm.



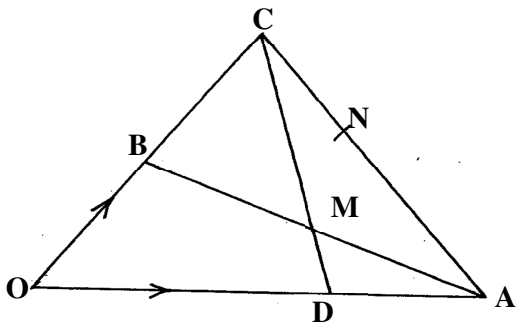
(a) Find the size of angle **ABC**. (3mks)

(b) Find the radius of the circle. (2mks)

(c) Hence, calculate the area of the shaded region. (5mks)

18. In the figure below $\overrightarrow{OB} = \underset{\sim}{b}$, $\overrightarrow{OC} = 3\overrightarrow{OB}$ and $\overrightarrow{OA} = \underset{\sim}{a}$

(a) Given that $\overrightarrow{OD} = 1/3\ \overrightarrow{OA}$ and $\overrightarrow{AN} = ½\ \overrightarrow{AC}$, $\overrightarrow{CD}$ and $\overrightarrow{AB}$ meet at **M**. Determine in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$



(i) $\overrightarrow{AB}$ (1mk)

(ii) $\overrightarrow{CD}$ (1mk)

(b) Given that $\overrightarrow{CM} = k\ \overrightarrow{CD}$ and $\overrightarrow{AM} = h\ \overrightarrow{AB}$ determine the values of the scalars **k** and **h** (5mks)

(c) Show that **O**, **M** and **N** are collinear. (3mks)

19. The table below shows the analysis of examination marks scored by 160 candidates.

Marks %	1-10	11-20	21- 30	31 – 40	41- 50	51 – 60	61 – 70	71 – 80	81- 90	91-100
No. of candidates	2	6	15	22	36	34	20	15	6	4

(a) Using an assumed mean of 45.5 , calculate

(i) The mean (3mks)

(ii) The standard deviation (4mks)

(b) Calculate the minimum mark for grade **A** if 40 students got grade **A-** (3mks)

20. **ABCD is** a quadrilateral with vertices as follows: **A**(3,1), **B** (2,4) **C** (4,3) and **D** (5,1)

(a) (i) On the grid provided draw the quadritoral **ABCD** and the image **A'B'C'D'** under a transformation

with matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Find the co-ordinates of **A'B'C'D'** (3mks)

Describe the transformation that maps **ABCD** onto **A'B'C'D'** fully (1mk)

(b) A transformation represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ maps **A'B'C'D'** onto **A''B''C''D''** find the co-ordinates of **A''B''C''D''.** Plot **A''B''C''D''** on the same grid. (3mks)

(c) Determine a single transformation that maps **A''B''C''D''** onto **ABCD**. Describe this transformation fully. (3mks)

21. Four towns **P,Q, R** and S are such that **Q** is 1500 km due east of town **P**. Town R is 1080 km due North of town **Q**. Town S is on a bearing of 330^{o} from P and on a bearing of 300^{o} from **R**.

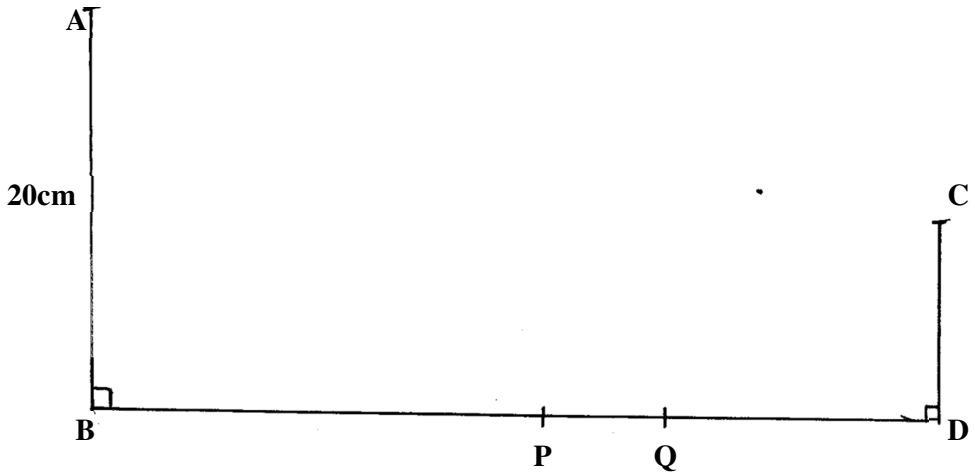
(a) Use a ruler and a pair of compasses and show the positions of town **P, Q,R** and **S.** (take a scale of 1cm = 3000km) (5mks)

(b) (i) Determine the distance of **PS** in Km (2mks)

(ii) Determine the distance **RS** in km. (2mks)

(iii) Determine the bearing of town **S** from **Q** (1mk)

22. The diagram below represents two vertical watch – towers **AB** and **CD** on a level ground. **P** and **Q** are two points on a straight road **BD**. The height of the tower **AB** is 20m and road **BD** is 200m



(a) A car moves from **B** towards **D**. At point **P,** the angle of depression of the car from points **A** is 11.3^{o} Calculate the distance **BP** to 4 significant figures. (2mks)

(b) If the car takes 5 second to move from **P** to **Q** at an average speed of 36km/hr, calculate the angle of depression of **Q** from **A** to 2 decimal places. (3mks)

(c) Given that **QC** = 50.9 cm, calculate
(i) The height of **CD** in meters to 2 decimal places; (2mks)

(ii) The angle of elevation of **A** from **C** to the nearest degree. (3mks)

23. The parents of a certain mixed secondary school decided to buy a school van worth Ksh. 900,000. Each student was to contribute the same amount of money. 50 students were transferred from the school; as a results each f the remaining students had to pay Ksh. 600 more.

(a) Find the original number of the students in the school. (5mks)

(b) Find the percentage change in contributions per student. (3mks)

(c) If the ratio of boys to girl in the school was 11: 7 find the amount money contributed by boys alone.

(2mks)

24. The distance **S** metres from a fixed point ,covered by a particle after **t** seconds is given y the equation;
$S = t^3 - 6t^2 + 9t + 5$

(a) Calculate the gradient tot the curve at **t** = 0.5 seconds. (3mks)

(b) Determine the values of **S** at the maximum turning points of the curve. (4mks)

(c) On the space provided, sketch the curve of $\mathbf{S = t^3 - 6t^2 + 9t + 5}$ (3mks)

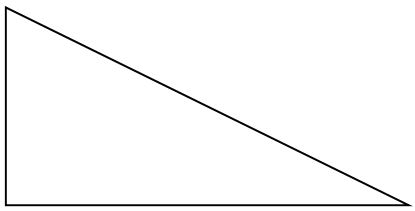
MARKING SCHEME

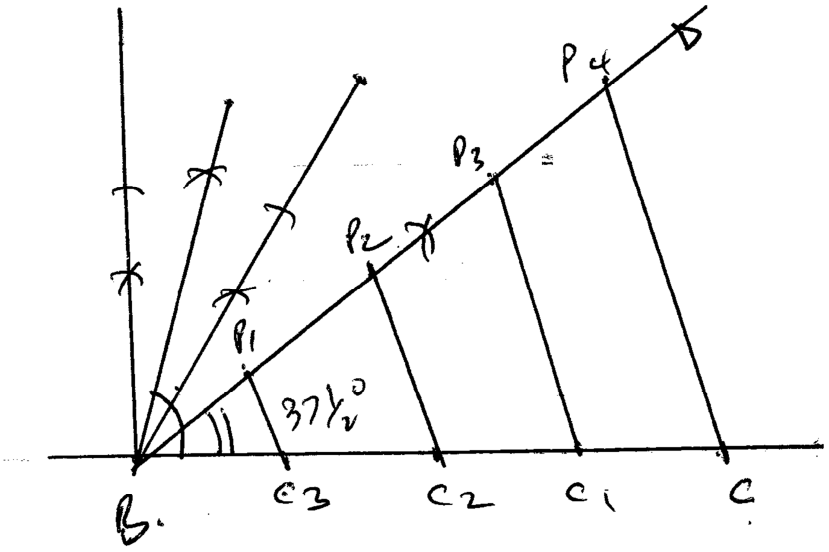
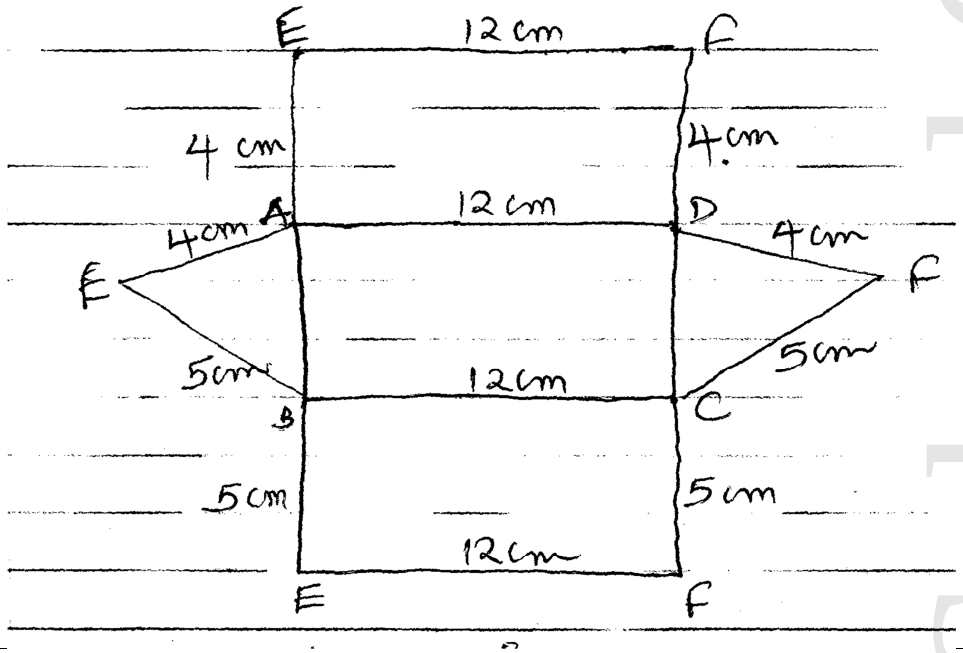
No.	SECTION 1		
1.	$\frac{3}{4}+1\frac{7}{7}\div\frac{4}{7}\times\frac{7}{3}$ $=\frac{3}{4}+\frac{9}{7}=\frac{21+36}{28}=\frac{57}{28}$ $\left(\frac{80-35}{56}\right)\times\frac{2}{3}$ $=45/56 \ X \ 2/7$ $=15/28$ $=\frac{57}{28}\times\frac{28}{15}=\frac{19}{5}$ $=3\,4/5$	M1 M1 A1	
2.	$\frac{7}{12}\times\frac{13}{18}=\frac{91}{216}$ $=\frac{91}{216}\times\frac{5}{18}=\frac{91+60}{216}=\frac{151}{216}$ $\left(\frac{5}{18}\div\frac{65}{216}\right)100$ $=\frac{5}{18}\times\frac{216}{65}\times100=\frac{1200}{13}$ $=92\frac{4}{13}\%$	M1 M1 M1 A1	
3	**L.C. M** $2^3\times5^2\times11$ **N=** $2^3\times5\times7^2$	M1	

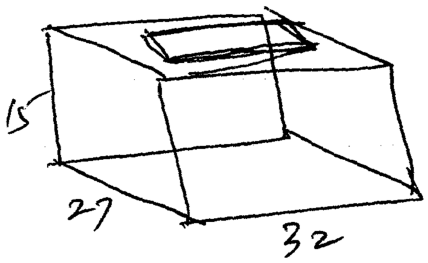
	1400
2	**700**
2	**350**
2	**175**
5	**35**
5	**7**
7	**1**

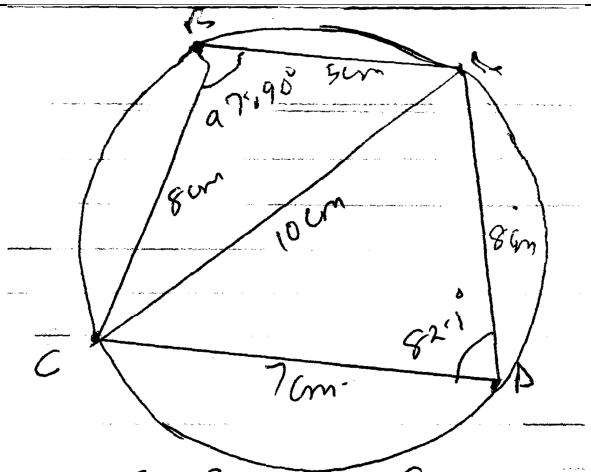
	1960
2	**980**
2	**490**
2	**245**
5	**47**
7	**7**
7	**1**

	<table><tr><td></td><td>70</td></tr><tr><td>2 5</td><td>35 7</td></tr></table> **L.C.M** $2^3 \times 5^2 \times 7^2 \times 11$ $n = 2 \times 5 \times 11 \times 7$ $= 770$	M1 A1	
4	At 9.50am , the bus has travelled $\left(\frac{20}{60} \times 60\right) = 20km$ The distance between the two vehicles at 9.50am $(65 - 45)$ $= 20km$ Rel . speed = 140 km/ h. It will take them $\frac{45}{140} hrs$ since Leaving Kitale. Distance covered at the time Matatu Met the bus $= \frac{45}{140} \times 80 = 25.71km$	B1 B1 A1	
5	$M\left(\frac{6+2}{2}, \frac{3+1}{2}\right)(4,2)$ Gradient = $\frac{1-3}{6-2} = \frac{-1}{2}$ $g \times \frac{-1}{2} = -1$ $g = 2$ $\frac{y-2}{\propto -4} = 2$ $y - 2 = 2x - 8$ $y = 2x - 6$	M1 M1 B1	

6.	$\frac{4x(3x-4)}{20-15x+4x-3x^2} = \frac{4x(3x-4)}{5(4-3x)+x(4-3x)}$ $= \frac{4x(3x-4)}{(5+x)(4-3x)}$ $= \frac{-4x(4-3x)}{(5+x)(4-3x)}$ $= \frac{-4x}{5+x}$	M1 M1 A1	
7.	 $y = \sqrt{9-4}$ $= \sqrt{5}$ $tan^2\theta + cos^2\theta$ $\left(\frac{}{\sqrt{5}}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2$ $= \frac{4}{5} + \frac{5}{9} = \frac{61}{45} = 1\frac{16}{45}$	B1 M1 A1	
8.	$8x(2^2)^Y = 6(2^Y) - 1$ $= 8x(2^Y)^2 = 6x(2^Y) - 1$ Let $2^Y = x$ $\therefore 8x^2 = 6x - 1$ $\Rightarrow 8x^2 - 6x + 1 = 0$ $\Rightarrow 8x^2 - 4x - 2x + 1 = 0$ $4x(2x-1) - 1(2x-1) = 0$ $\Rightarrow (4x-1)(2x-1) = 0$ $\Rightarrow \therefore x = 1/2 \; or \; 1/4$ $2^Y = \frac{1}{2} = 2^{-1} or 2^Y = 1/4 = 2^{-2}$ $\therefore y = -1$ *or* $y = -2$	M1 M1 M1 A1	
9.		B1 B1	√angle 37.5°

		B1	√ Division of BD √ Division of BC
10.		B1 B1 2	
11.	$x + \frac{x-36}{3} = 180^0$ $\Rightarrow 3x + x - 36 = 540$ $4x = 576$ $\Rightarrow x = 144^0$ Exterior angle $= \frac{144-36}{3} = \frac{108}{3} = 36^0$ $\therefore$ No of sides of the polygon $n = \frac{360}{36} = 10 sides$	M1 B1 B1 3	
12.			

	$AC^2 = 85^2 - 75^2$ $AC = \sqrt{85^2 - 75^2} = 40m$ Area of quadrilateral ABCD $\frac{1}{2} \times 40 \times 75 + \sqrt{75(75-60)(75-50)(75-40)}$ $= 1500 + \sqrt{984375}$ $= 2492m^2$ In hectares $= \frac{2492}{10000} = 0.2492 hectares$ $\approx 0.25 hectares.$	M 1 M 1 A1 4	
13.	 External volume $= (32 \times 27 \times 15) = 12960 cm^3$ Internal volume $= (30 \times 25 \times 14) = 10500 cm^3$ Volume of material $= (12960 - 10500)$ Metre used $= 2460 cm^3$ Density $= \frac{mass}{volume} = \frac{1500}{2460}$ $0.6098 g / cm^3$	M 1 M 1 A1 3	
14.	$2x > 5x - 3 > -8$ $2x + 3 > 5x - 3$ $-3x > -6$ $x < 2$ $5x - 3 > -8$ $5x > -5$ $x > -1$ $-1 < x < 2$	M 1 M 1 A1	

	The integral values are (0,1)	3	
15.	Remaining amount = $\frac{9600}{1E}\times\frac{95}{100}\times 125$ $= ksh1{,}140{,}000$ Amount speed = ksh $\frac{1140{,}000\times 3}{4}$ $= ksh855{,}000$ The Balance = ksh 1,140,000- 855,000 =ksh285,000 Amount in dollars = $\frac{285{,}000}{63.20}$ $= 4509.49 USdollars$	M1 M1 A1	
16	$1.2\times 10\times x = 6$ $\times = \frac{6}{12}$ $= 0.5$ $3.2\times 15\times 0.5 = 24$	M1 A1 A1	
17	 $10^2 = 8^2 + 5^2 - 2\times 8\times 5\,\text{Cos B}$ $\text{Cos B} = \frac{89-100}{80} = \frac{-11}{80}$ $\text{B} = \text{Cos}^{-1}\left(\frac{-11}{80}\right) = 97.90^0$ (b) $2\text{R} = \frac{10}{\sin 97.90}$ $R = \frac{5}{\sin 97.90} = 5.0479cm$	M1 M1 A1 M1 A1 M1	

	□□□(c) $\frac{10}{\sin 82.1^0} = \frac{7}{\sin A}$ $\text{Sin A} = \frac{7}{10}\sin 82.1 = 0.6984$ $\text{A} = \sin^{-1}(0.6934) = 43.90^0$ $\angle COD = 2 \times 43.90^0 = 87.80^0$ $\text{Area} = \frac{87.80}{360} \times \frac{22}{7} \times 5.0479 - \frac{1}{2} \times 5.04329 \sin 87.80$ =19.5316- 12.7313 $=6.800\ \text{cm}^2$	1 M 1 A1	
18	(a) (i) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ (ii) $\overrightarrow{CD} = -3\underset{\sim}{b} + -\frac{1}{3}\underset{\sim}{a}$ (b) $\text{CM} = \frac{1}{3}Ka - 3kb$ $\Rightarrow \text{AM} = -h\,\underset{\sim}{a} + h\,\underset{\sim}{b}$ $\text{CM} = \text{CA} + \text{AM}$ $= \frac{1}{3}\,\text{K}\,\underset{\sim}{a} - 3\,\underset{\sim}{b}$ $= a\,(1-h) + b(h-3)$ K = 3 -3h and $\frac{1}{3}\text{K} = 1 - h$ h= $\frac{3}{4}$ k= $\frac{3}{4}$ (c) $\overrightarrow{OM} = \frac{1}{4}\,\underset{\sim}{a} + \frac{3}{4}\,\underset{\sim}{b}$ $\overrightarrow{ON} = \frac{1}{2}\,\underset{\sim}{a} + \frac{3}{4}\,\underset{\sim}{b}$	B1 B1 M 1 M 1 M 1 B1 B1 B1	

	$\overset{\sim}{\rightarrow}$ $\overset{\sim}{\rightarrow}$ OM = $\frac{1}{2}$ ON Hence parallel, and point O is common $\Rightarrow$ O, M and N are collinear	B1 B1	
19			

class	x	f	d=x-a	df	df^2	cf
1-10	5.5	2	-40	-8	3200	2
11-20	15.5	6	-30	-180	5400	8
21-30	25.5	15	-20	-300	6000	23
31-40	35.5	22	-10	-220	2200	45
41-50	45.5	36	0	0	0	81
51-60	55.5	34	10	340	3400	115
61-70	65.5	20	20	400	8000	135
71-80	75.5	15	30	450	13500	150
81-90	85.5	6	40	240	9600	156
91-100	95.5	4	50	200	10000	160
		160		850	61300	

B1 For d
B1 For df
B1 For df^2
For cf

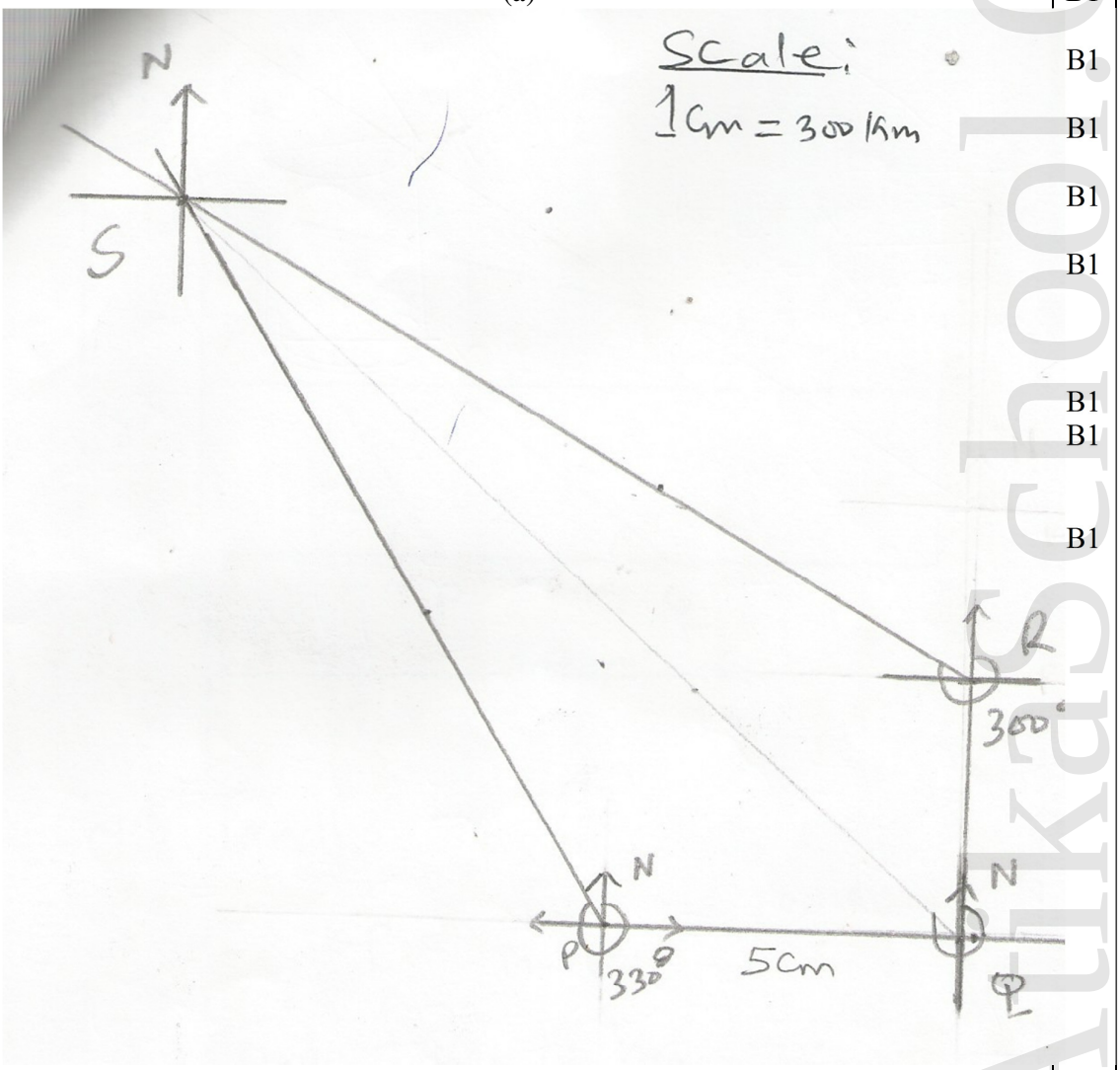
Mean = $45.5 + \frac{850}{160} = 50.8125$ M1 A1

Standard deviation = $\sqrt{\frac{6130}{160} - \left(\frac{850}{160}\right)^2}$ M1 A1
=18.84

) 121^{st} student M1 A1
$60.5 + \frac{121-115}{20} \times 10$

=60.5+ 3
=63.5

20	$(a)\begin{pmatrix} m & \\ 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} A & B & C & D \\ 3 & 2 & 4 & 5 \\ 1 & 4 & 3 & 1 \end{pmatrix}=\begin{pmatrix} A1 & B1 & C1 & D1 \\ -1 & -4 & -3 & -1 \\ 3 & 2 & 4 & 5 \end{pmatrix}$ $\Rightarrow A^1(-1,\ 3), B^1(-4,2)\ C^1(-3,\ 4) D^1(-1,5)$.	B1	
	(i)This is $\frac{1}{4}$ turn about the origin	B1	

	(b) $\begin{pmatrix} N & \\ 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} A1 & B1 & C1 & D1 \\ -1 & -4 & -3 & -1 \end{pmatrix} = \begin{pmatrix} A^{11} & B^{11} & C^{11} & D^{11} \\ -1 & -4 & -3 & -1 \\ -3 & -2 & -4 & -5 \end{pmatrix}$ $A^{11}(-1,-3), B^{11}(-4,-2), C^{11}(-3,-4), D^{11}(-1,-5).$	M1 A1	
	(c) $\begin{pmatrix} N & \\ 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} M & \\ 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ Det = 0-(+1) = -1 Inverse Matrix = $\frac{1}{-1}\begin{pmatrix} 01 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 01 \end{pmatrix}$ This is reflection on the line y = -x	B1 B1 B1	
21	(a)  	B1 B1 B1 B1 B1 B1 B1 B1 B1	Diagram positions

(b)(i) The distance PS= 11.9cm $(3570km.)$

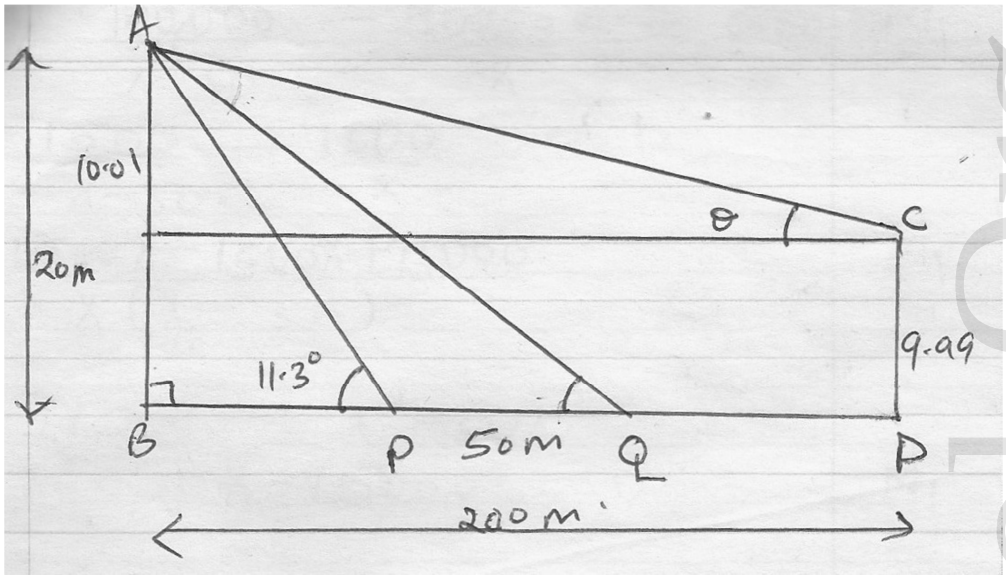
(ii) The distance RS= 12.8cm $(3840km)$

(iii) The bearing of Town S from Q

$= (270 + 42)$

$= 312^0$ or N 48^0 W

22



M 1

A1

M 1

M 1

A1

M 1

(a) Tan $11.3^0 = \frac{20}{BP}$

$\therefore$ BP = $\frac{20}{Tan11.3^0} = 100.09m$

A1

M 1

(b) Time Taken in hrs = $\frac{5}{3600}$

M 1

Distance between P and Q

$= \left(\frac{36}{1} \times \frac{5}{3600}\right)$

$= 0.05km$

A1

$50km$

B1

$\Rightarrow \tan \propto = \frac{20}{150.09}$

$\propto = tan^{-1}\left(\frac{20}{150.09}\right) = 7.59^0$

(c) (i) $CD^2 = 50.9^2 - 49.91^2 = 99.8019$

	CD= $\sqrt{99.8019} = 99.9901$ $= 9.99m$ (ii) $\tan \propto = \frac{10.01}{200} = 0.05005$ $\propto = \tan^{-1}(0.05005)$ $= 2.865$ $= 3.00$		
23.	$\frac{1500}{x-50} - \frac{1500}{x} = 1$ $\frac{1500x - 1500x + 75000}{x(x-50)}$ (a) $\frac{900,000}{x-50} - \frac{900,000}{x} = 600$ $75000 = x^2 - 50x$ $x^2 - 50x - 75000 = 0$ $x^2 - 300x + 250x - 75000 = 0$ $x(x-300) + 250(x-300) = 0$ $(x-300)(x+250) = 0$ $x = 300 or -250$ $\Rightarrow x = 300$ (b) $\frac{900,000}{250} - \frac{900,000}{300} \times 100\%$ $\frac{900,000}{300} = \left(\frac{3600-3000}{300}\right) \times 100\%$ $= 20\%$ (c) B:G 11:7 $\frac{11}{18} \times 900,000$ *Kshs*.550,000	M1 M1 M1 M1 A1 M1 M1 A1 M1 A1 10	

24.	(a) $S = t^3 - 6t^2 + 9t + 5$ $\frac{ds}{st} = 3t^2 - 12t + 9$ At t =0.5seconds. Gradient = $3(0.5)^2 - 12(0.5) + 9$ =3.75m (b) when $\frac{ds}{dt} = 0, 3t^2 - 12t + 9 = 0$ $t^2 - 4t + 3 = 0$ (t-1) (t-3) = 0 t= 1 or 3 seconds when t=1b, S=1-6+9+5 =9m When t = 3s, S= $3^3 - 6 \times 3^3 + 9 \times 3 + 5$ =5m. (c) $\frac{d^2s}{dt^2} = 6t - 12$ At t = 1b, $\frac{d^2t}{dt^2} = 6 \times 1 - 12 = -6$ $\therefore at(1,9)$ max turning point At t = $3s, \frac{d^2s}{dt} = 18 - 12 = 6$ At (3,5) Mim turning point.	M 1 M 1 A1 M 1 A1 A1 M 1 M 1 A1 10