24. A (50ºN, 40ºW) and B(50ºN, 80ºE) lie on the surface of the earth. (take R=6370km, $\pi=\frac{22}{7}$)

(a) Calculate the distance along the small circle between A and B, giving your answer in

i) Nautical miles (2 marks)

ii) Kilometers (2 marks)

(b) Find the distance between A and B along a great circle (over North pole) in

i) Nautical miles (2 marks)

ii) Kilometres (2 marks)

b) Find the time at A if the local time at B is 12:00 noon (2 marks)

NAME:.. INDEX NO:................................

SCHOOL .... SIGNATURE ....

DATE ....

INSTRUCTIONS TO CANDIDATE

1. Write your name and admission number in the spaces provided above.
2. Sign and write the date of examination in the spaces provided.
3. The paper contains two sections: Section **I** and **II**.
4. Answer all questions in section **I** and **only five** questions from section **II**.
5. All answers and working must be written on the question paper in the spaces provided below each question.
6. Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.
7. Marks may be given for correct working even if the answer is wrong.
8. Non-programmable silent electronic calculators and KNEC mathematical tables may be used except where stated otherwise.

FOR EXAMINER'S USE ONLY

SECTION A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION B

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL	

This paper consists of **16** printed pages.
Candidates should ensure that all pages are printed as indicated and no questions are missing.

SECTION A
Answer all the questions in this section.

1. Use logarithms to evaluate (4 marks)

$$\sqrt[3]{\frac{0.01369 \times 396.5}{64.11 - 0.001912}}$$

2. Solve the equations (3 marks)

$$x^2 + 2xy = 25 - y^2$$
$$x = y$$

3. A mathematics textbook which is actually 24.3cm long is measured as 24.5cm. Calculate the percentage error in this measurement giving your answer correct to 3 s.f (3 marks)

23. Use a rule and compasses only for all the constructions in this question.

(a) Construct a triangle ABC such that angle BAC = 30º, AB=4cm and AC = 6cm (3 marks)

b) Construct a circle passing through points A, B and C (3 marks)

c) On the opposite side of point C, locate point on the circumference such that $\angle ACD = \angle BCD$. Measure length CD (4 marks)

22. The probability that the school team wins a match is 0.6. The probability that the team looses is 0.3 and the probability that the team ties is 0.1. The team plays two games.

a) Draw a tree diagram to represent this information (2 marks)

b) What is the probability that the team

i) Wins two matches? (2 marks)

ii) Either wins all the matches or looses all the matches (2 marks)

iii) Wins one match and looses one (2 marks)

iv) Looses all the matches or ties all the matches (2 marks)

4. Make P the subject of the formula (3 marks)

$$D = \sqrt[3]{\frac{p}{q - p}}$$

5. Given sin θ= $\frac{\sqrt{5}}{3}$, find cos (90 - θ) without using mathematical tables or calculators. (2 marks)

6. A businessman deposited Ksh 120000 in a bank account which was to earn him interest at the rate of 12% p.a. compounded quarterly. Calculate his total amount after two years to the nearest Kenya shillings (3 marks)

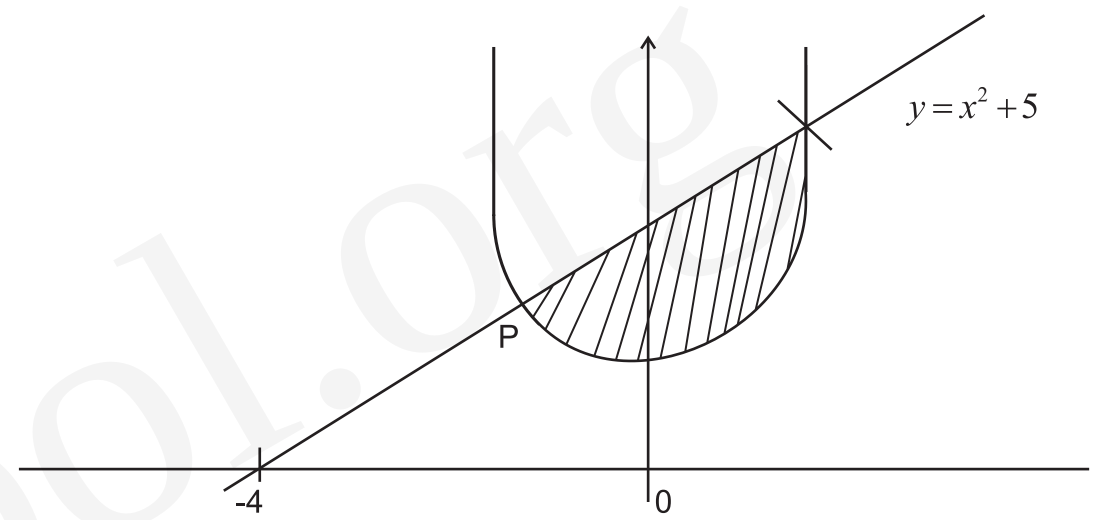
7. Solve for x given that 2 sin x = tax x for $0^{o} \leq x \leq 360^{0}$ (3 marks)

8. A point T divides line AB externally in the ratio 8:2. Given that the position vectors of A and B are $3i - 4l + k \; and \; -1 + j - 3k$ respectively. Find the position vector of T in unit vector form. (3 marks)

9. (a) Expand and simplify $\left(2 - \frac{1}{2}x\right)^5$ (2 marks)

(b) Use the expansion in part (a) above up to the term in x^2 to approximate the value of $(1.96)^5$ correct to 4 significant figures. (2 marks)

21. The sketch below represents the curve $y=x^2+5$ and a straight line PQ which meets the x-axis and y-axis at the points (-4, 0) and (0, 8) respectively. The line intersects the curve at point P and Q as shown.

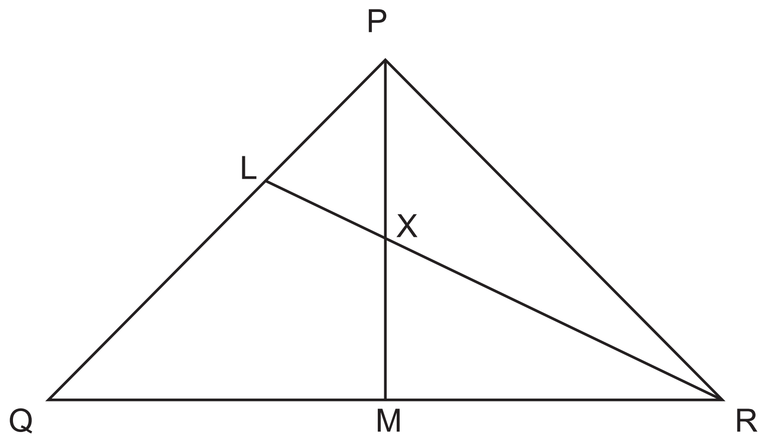


a) Find the equation of the line in the form y = mx + c (3 marks)

b) Determine the co-ordinates of P and Q (3 marks)

c) Calculate the area of the shaded region (4 marks)

20. In the triangle PQR below, L and M are points on PQ and QR respectively such that PL:LQ = 1:3 and QM:MR=1:2. PM and RL intersect at x. Given that PQ=$\vec{b}$ and PR=$\vec{c}$



a) Express the following vectors in terms of $\vec{b}$ and $\vec{c}$

i) $\overrightarrow{QR}$ (1 mark)

ii) $\overrightarrow{PM}$ (1 mark)

iii) $\overrightarrow{RL}$ (1 mark)

b) By taking $\overrightarrow{PX}=h\overrightarrow{pm}$ and $\overrightarrow{Rx}=k\overrightarrow{RL}$ where h and k are constants, find two expression of $\overrightarrow{Px}$ in terms of $\vec{h}$, $\vec{k}$, $\vec{b}$ and $\vec{c}$. Hence determine the values of the constants h and k (6 marks)

c) State the ratio LX:XR (1 mark)

10. y varies partly as the square root of x and partly varies inversely as the cube of x. When y=2, x=4 and when y=3, x=1. Express y in terms of x. (4 marks)

11. Two grades of coffee A and B are costing sh 80 per kg and 180 per kg respectively are to be mixed in order to produce a blend worth sh 120 per kg. In what ratio should they be mixed? (3 marks)

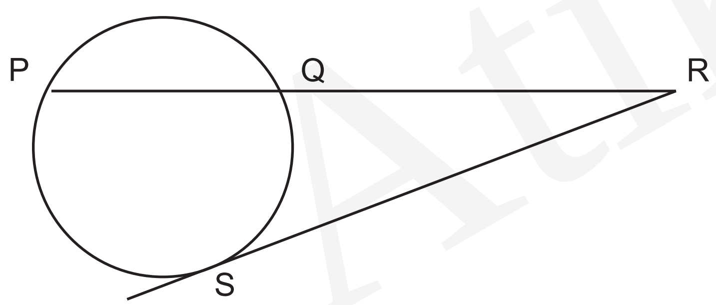
12. Solve for x given $\log_x 3 + \log_3 x = 2$ (2 marks)

13. Given that $A = \begin{pmatrix} 3 & -2 \\ 7 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ find

(a) A + B (1 mark)

(b) AB^{-1} (2 marks)

14. In the figure below SR is a tangent to the circle, PQR is a straight line and PQ: QR = 1:1.
Find PR and QR given that SR = $8\sqrt{2}$ cm (3 marks)



a) Hence or otherwise, determine:

i) The ratio of areas of quadrilateral OABC to quadrilateral $O_1A_1B_1C_1$ (3 marks)

ii) The two successive transformations which maps quadrilateral OABC onto quadrilateral $O_1A_1B_1C_1$ and their respective matrices (4 marks)

19. The transformation T is represented by the matrix $\begin{pmatrix} 2.4 & -1.8 \\ 1.8 & 2.4 \end{pmatrix}$. T maps quadrilateral OABC whose vertices are 0(0,0), A(5,0), B(5, 5) and C(0,5) onto quadrilateral $O_1A_1B_1C_1$

a) Determine the coordinates of $O_1A_1B_1C_1$ and plot quadrilaterals OABC and $O_1A_1B_1C_1$ on the same graph (3 marks)

15. Determine the quartile deviation from the data below. (2 marks)

42, 1, 12, 6, 14, 20, 24

16. The first three terms of a G.P. are the first, fourth and tenth terms of an A.P. Given that the first term is 6 and that all the terms of the G.P. are different, find the common ratio. (4 marks)

SECTION II (50 Marks)

Answer only FIVE questions in this section in the spaces provided

17. (a) Given that $y = \frac{2}{x^2}$ where x=0, complete the table below for the range $-3 \le x \le 3$ correct to 2 d.p. (2 marks)

x	-3	-2,5	-2	-1.5	-1	-0.5	0.5	1	1.5	2	2.5	3
y	0.22			0.89	2.00		8.00	2.00				0.22

(b) On the grid provided draw the graph of $y = \frac{2}{x^2}$ using the values from the table from part (a) above. (4 marks)

(b) Hence, by drawing a suitable straight line on the graph drawn in part (b) above, solve the equation $\frac{-2}{x^2} + x + 4 = 0$ (4 marks)

18. A quantity A varies jointly as the cube of y and inversely as the square root of x. If A = 7 when y=2 and x=25,

(a) Write an equation convecting A, x and y (3 marks)

(b) Find the value of y to three significant figures, when A=12 and x=36 (3 marks)

(c) Find the percentage change in A when y is decreased by 11% and x is increased by 21%. (4 marks)

<table>
<tr><td>1</td><td>Solution
64.110000
<u>- 0.001912</u>
64,108088

No | log
1.369×10^{-2} | 2.1364
3.965×10^{2} | 2.5983
| 0.7347
6.411×10^{1} | 1.8070
$\bar{2}.9277$ | $\bar{2}.89277$
$= \frac{\bar{3}}{3} + \frac{1.9277}{3}$

4.391×10^{-1}
=0.4391</td><td>M1

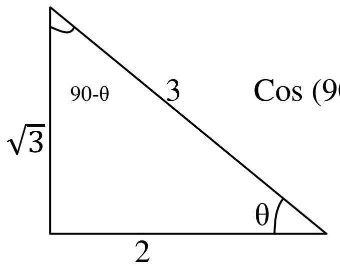
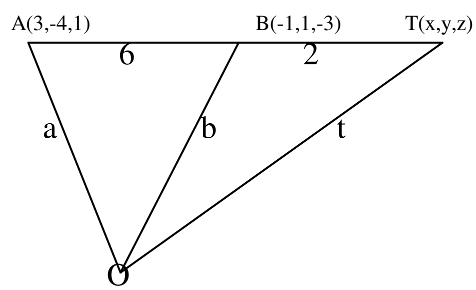
M1

M1
<u>A1</u>
4</td><td></td></tr>
<tr><td>2</td><td>From x=y then
$x^2+2x(x)= 25-x^2$
$x^2+2x^2+x2-25=0$
$4x^2-25=0$
$(2x-5)(2x+5)=0$
Either $2x-5=0 \longrightarrow 2x=5$
$x=\frac{5}{2}$
Or $(2x+5)=0 \longrightarrow 2x=-5$
$x=\frac{-5}{2}$
When $x=\frac{5}{2}$, $y=\frac{5}{2}$
And $x=\frac{-5}{2}$, $y=\frac{-5}{2}$</td><td></td><td></td></tr>
<tr><td>3</td><td>AE=24.5-24.3=0.2cm
$RE=\frac{0.2}{24.3}$
% error$=\frac{0.2}{24.3} x100$
=0.823%</td><td>M1

M1
<u>A1</u>
3</td><td></td></tr>
<tr><td>4</td><td>$d^3= \left(\sqrt[3]{\frac{P}{Q-P}}\right)^3$
$d^3=\frac{P}{Q-P}$
$Qd^3-Pd^3=P$
$Qd^3=P+Pd^3$
$P = \frac{Qd^3}{1+d^3}$</td><td>M1

M1

<u>A1</u>
3</td><td>Collecting term with P on one side</td></tr>
</table>

5	 Cos $(90\text{-}\theta)^0 = \frac{\sqrt{3}}{3}$	B1 B1 2	For $\sqrt{4}$ = 2
6	A = $P(1 + 0.03)^8$ = $120000\ (1.03)^8$ = 152012.00	B1 M1 <u>A1</u> 3	For digits 8 as power
7	2sin$\theta = \frac{-sin\theta}{cos\theta}$ $\frac{cos\theta}{sin\theta}\ x\ 2sin\theta = \frac{sin\theta}{cos\theta}\ x\ \frac{cos\theta}{sin\theta}$ 2cosθ=-1 Cosθ= $\frac{-1}{2}$ $\theta = cos^{-1}\frac{1}{2} \longrightarrow \theta = 60^0$ $\theta = 120^0\ \ or\ 240^0$	M1 A1 <u>B1</u> 3	
8	 OB= OA + AB =a+$\frac{6}{8}AT$ $\begin{pmatrix}-1\\1\\3\end{pmatrix} = \begin{pmatrix}3\\-1\\4\end{pmatrix} + \frac{3}{4}\begin{pmatrix}x-3\\y+4\\z-1\end{pmatrix}$ -1=3 + $\frac{3}{4}(x-3)$ 1=-1+$\frac{3}{4}(y=4)$ a=4+$\frac{3}{4}(z-1)$ x= $\frac{-1}{3}$, y=$\frac{8}{3}$, z=$\frac{-13}{3}$ OT=$\frac{-7}{3}i + \frac{8}{3}j - \frac{13}{3}k$	M1 M1 <u>A1</u> 3	

	alternatively, using the ratio theorem AT:TB=8: -2 OT= $\frac{-2}{6}a + \frac{8}{6}b$ $= \frac{-1}{3}\begin{pmatrix}3\\-4\\1\end{pmatrix} + \frac{4}{3}\begin{pmatrix}-1\\1\\-3\end{pmatrix}$ $= \begin{pmatrix}-1\\\frac{4}{3}\\-\frac{1}{3}\end{pmatrix} + \begin{pmatrix}-\frac{4}{3}\\\frac{1}{3}\\-4\end{pmatrix}$ $= \begin{pmatrix}-\frac{7}{3}\\\frac{8}{3}\\-\frac{13}{3}\end{pmatrix}$ OT=$\left(\frac{-7}{3}i + \frac{8}{3}j - \frac{13}{3}k\right)$		
9	a) $=1(2)^5(-\frac{1}{2}x)0 + 5(2)^4(-\frac{1}{2}x)^1 + 10(2)^3(-\frac{1}{2}x)^2 + 10(2)^2(-\frac{1}{2}x)^3 + 5(2)^1(-\frac{1}{2}x)^4 + 1(2)^0(-\frac{1}{2}x)^5$ = 32-40x + 2x2-5x3+ $\frac{5}{8}x^4$-$\frac{1}{32}x^5$ b) $(2-\frac{1}{2}x)^5 = (2-0.04)^5 \longrightarrow x=0.08$: $(1.96)^5 = 32-40(0.08)^1 + 2(0.08)^2$ =28.928 =28.93 4sf	M1 A1 M1 A1 4	
10	$y=a\sqrt{x} + \frac{b}{x^3}$ when y=2, 2= 2a+$\frac{b}{64}$ and when y=3 3=a+b a=$\frac{125}{127}$:y=$\frac{125}{127}\sqrt{x} + \frac{256}{127x^3}$	M1 A1 B1 B1 4	
11	Cost of 1kg of mixture = $\frac{180x+180y}{x+y}$:sh 120 = $\frac{80x+180y}{x+y}$ x:y=3:2	M1 M1 A1 3	
12	Solution 1+log3x=a Then $\frac{1}{a} + a = 2$ $1+a^2=2a$ (a-1)(a-1)=0 a=1 log3x=1	 M1 M1 A1	

	x=3	3	
13	a) $\begin{pmatrix} 6 & 3 \\ 8 & 7 \end{pmatrix}$ b) $AB^{-1}=\begin{pmatrix} 3 & -2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$ $= \begin{pmatrix} 8 & -21 \\ 9 & -20 \end{pmatrix}$	B1 B1 (for correct inverse) B1 3	
14	PRx ½ PR= $(8\sqrt{2})^2$ PR=$\sqrt{256}$ = 16cm QR= ½ x16=8cm	M1 M1 A1 3	
15	1,6,12,14,20,24,42 Q1=6 Q3=24 Quartile deviation = ½ (24-6) =9	B1 M1 A1 3	
16	r=$\frac{6+3d}{6} = \frac{6+9d}{6+3d}$ d= 0 or 2 6, 6=3d, 6+9d………………GP When d=0, 6, 6, 6 …………... and when d=2, 6, 12, 24 ………… ; the common ratio of GP=2	M1 M1 M1 A1 4	

17. a) Missing values in table

Y= 0.32, 0.50, 8, 0.89, 0.50, 0.32

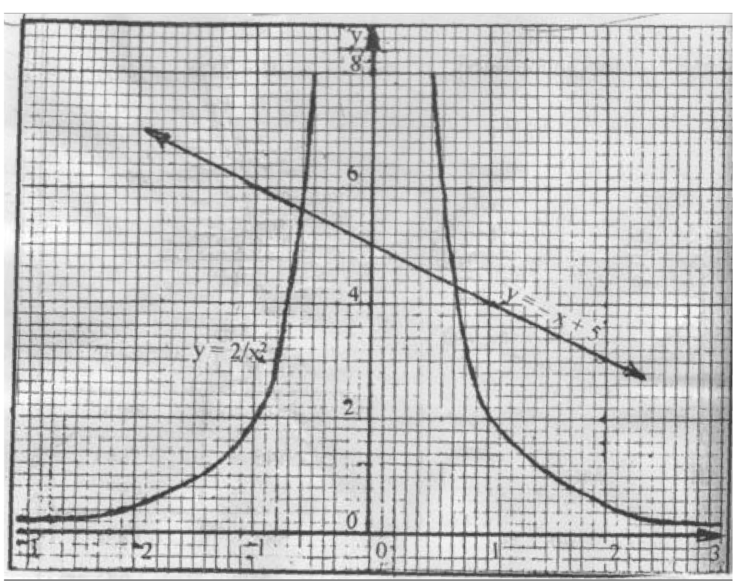
B2 Award B1 if 4 values are correct

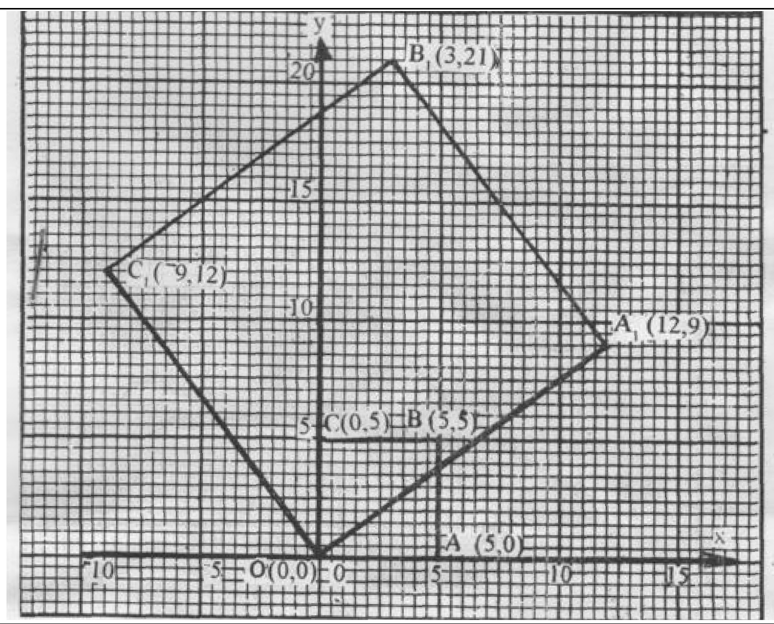
B1 Plotting

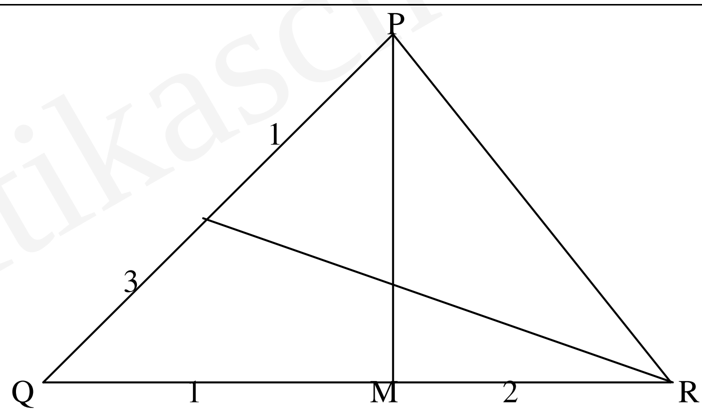
B1 Scale

B2 Smooth curves

b)

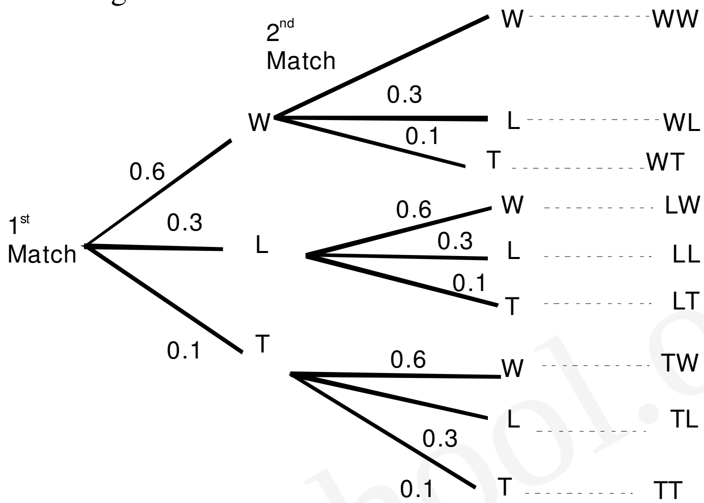


	c) $y=\frac{2}{x^2}$ $0 = \frac{-2}{x^2} + x + 4$ $y= x + 4$	M 1 A1 L1 (line must be seen) B1 (values	
18	a) $A=K\frac{y^3}{\sqrt{x}}$ $7=Kx\frac{8}{5}$ $K=\frac{35}{8}$ or 4.375 $A = 4.375\frac{y^3}{\sqrt{x}}$ b) $y=\sqrt[3]{4.375A\sqrt{x}}$ $= \sqrt[3]{4.375x12\sqrt{36}}$ = 6.804 6.80 (3sf) c) y1=0.89y x1=1.21x $A_1=K\ \frac{(0.89y)3}{\sqrt{1.21x}}$ $=0.64\frac{y^3}{\sqrt{x}}$ =0.64 % change=(0.64A-A)/Ax100 =36% A decrease by 36%	M1 M1 A1 M1 M1 A1 M1 M1 M1 A1 10	
19			

	a) $\begin{pmatrix}2.4 & -1.8\\ 1.8 & 2.4\end{pmatrix}\begin{pmatrix}0 & 5 & 5 & 0\\ 0 & 0 & 5 & 5\end{pmatrix}=\begin{pmatrix}0 & 12 & 3 & -9\\ 0 & 9 & 21 & 12\end{pmatrix}$: 0, (0,0) A1(12,9) B1(3,21) C1(-9.12) b) i) Determine T = (2.4x2.4)-(-1.8 x1.8)=9 Area scale factor= $\sqrt{9} = 3$ $\begin{pmatrix}2.4 & -1.8\\ 1.8 & 2.4\end{pmatrix} = 3\begin{pmatrix}0.8 & -0.6\\ 0.6 & 0.8\end{pmatrix} = \begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}0.8 & -0.6\\ 0.6 & 0.8\end{pmatrix}$ Hence M= $\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$ represents an enlargement about O with lsf=3 and R$\begin{pmatrix}0.8 & -0.6\\ 0.6 & 0.8\end{pmatrix} = \begin{pmatrix}Cos37^0 & -Sin37^0\\ Sin37^0 & Cos37^0\end{pmatrix}$: represents a rotation of 37^0 about O N/B MR(T) = RM(T) ie the 2 transformations are cumulative and we can start with either	B1 B1 B1	
20	 a) QR=QP+PR -b+c PM=PG+QM =b+ $\frac{1}{3}(c-b)$ $\frac{2}{3}b+\frac{1}{3}c$ RL=RP+PL -c+ ¼ b ¼ b-c b) PX=h$\left(\frac{2}{3}b+\frac{1}{3}c\right)$ RX =k$\left(\frac{1}{4}b-c\right)$ ¼ kb-kc PX=PR+RX =c+1/4 kb-kc =c-kc+ ¼ kb		

	=c(1-k)+ ¼ kb PX=PX $\frac{2}{3}$bh+ $\frac{1}{3}hc = c(1-k)+\frac{1}{4}kb$ $\frac{3}{2}\ x\frac{2}{3}bh = \frac{1}{4}kb\ x\frac{3}{2}$ h=$\frac{3}{8}k$ $\frac{1}{3}hc = c(1-k)$ k=1-$\frac{1}{3}\left(\frac{3}{8}k\right)$ k=1-$\frac{1}{8}$k k+$\frac{1}{8}k = 1$ $\frac{9}{8}k = 1$ k=$\frac{8}{9}$ h=$\frac{3}{8}\left(\frac{8}{9}\right) = \frac{1}{3}$ Lx:xR 1-$\frac{8}{9}$:$\frac{8}{9}$ $\frac{1}{9}$:$\frac{8}{9}$ =1:8		
21	a) Find the equation of the line in the form y=mx+c (-4,0) (0,8) Gradient of PQ= $\frac{8-0}{0--4} = \frac{8}{4} = 2$ Equation$\frac{y-8}{x-0} = 2$ Y=2x+8 b) Determine the coordinates of P and Q x^2+5=2x+8 x^2-2x-3=0 x^2+x-3x-3=0 x(x+1)-3(x+1)=0 (x-3) (x+1)=0 x=3 or -1 when x =3, x=-1 y=32 + 5=14 y=(-1)2 + 5=6 P(-1,6) Q(3,14) c) Calculate the area of the shaded region $\int_{-1}^{3}(2x+8)dx - \int_{-1}^{3} 2^2 + 5dx$ $\left[\frac{2x^2}{2} + 8x\right] - \left[\frac{x^3}{3} + 5x\right]$		

<table>
<tr><td></td><td>$= 40-\frac{88}{3} = \frac{120-88}{3} = \frac{32}{3}$</td><td></td><td></td></tr>
<tr><td>22</td><td>Tree diagram



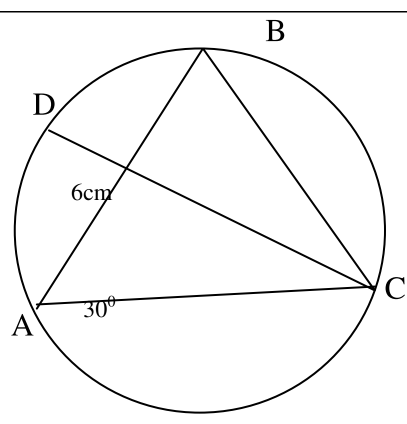
b) What is the probability that the team
i) Win two matches?
P(Wand W)=0.6x0.6
=0.36
ii) Either wins all the matches or loses all the matches
P(WW) or P(LL)
0.6X0.6 + 0.3X0.3=0.36 + 0.09
= 0.45
iii) Wins one match and loses one
P(W& L) or (L& W)
0.6X0.3 + 0..3X0.6
=0.36
iv) Loses all the matches or ties all the matches
P(LL) or P(TT)
0.3X0.3 + 0.1X0.1=0.1</td><td>B2

M1
A1

M1
A1

M1

A1

M1
A1</td><td></td></tr>
<tr><td>23</td><td>


a)
Angle 60^0</td><td>B1</td><td></td></tr>
</table>

	Angle 30^0 Triangle b) bisecting 1st side 2nd side Circle drawn c) Bisecting angle ACD Locating point D Drawing bisector Length CD	B1 B1 B1 B1 B1 B1 B1 B1 B1	
24	Longitude diff is $(80+40)=120^0$ Length of arc AB=60θ Cos 50^0=4628nm Length of arc AB=$\left(\frac{\theta}{360}2\pi RCos50^0\right)$km $=\frac{120}{360}x2x\frac{22}{7}x6370Cos50^0$ 8579km b) Find the distance between A and B along a great circle (over north pole) in; i) Nautical miles ii) Kilometers Angle AOB=80^0 i) Length of arc AB=60x80=4800nm lAB=$\frac{O}{360}2\pi R$ lAB=$\left(\frac{80}{360}x2x\frac{22}{7}x\ 6370\ \right)$km =8897.78km c) Find the time at A if the local time at B is 12.00noon difference in longitude is $(80+40)=120^0$ 1^0 is 4min Difference in time is 120x4=480min=8hrs Local time at A is 8hrs behind that of B ie 4.00a.m.		