23.3 MATHEMATICS (121)

23.3.1 Mathematics Paper 1(121/1)

Name .... Index Number .... / ....

121/1
MATHEMATICS
Paper 1
Oct./Nov. 2007
2½ hours

Candidate's Signature ....

Date ....

THE KENYA NATIONAL EXAMINATIONS COUNCIL

Kenya Certificate of Secondary Education

MATHEMATICS

Paper 1

2½ hours

Instructions to candidates

1. Write your name and index number in the spaces provided above.
2. Sign and write the date of examination in the spaces provided above.
3. The paper contains TWO sections: Section I and Section II.
4. Answer ALL the questions in Section I and any five questions from Section II.
5. All answers and working must be written on the question paper in the spaces provided below each question.
6. Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.
7. Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.

For Examiner's use only.

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

| 17	18	19	20	21	22	23	24	Total

Grand Total

This paper consists of 19 printed pages

**Candidates should check the question paper to ascertain
that all the pages are printed as indicated and no questions are missing.**

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*Answer **all** the questions in this section.*

1   Evaluate without using Mathematical tables or a calculator, $\dfrac{0.0084 \times 1.23 \times 3.5}{2.87 \times 0.056}$, expressing the answer as a fraction in its simplest form.

(2 marks)

2   The size of an interior angle of a regular polygon is $3x°$ while its exterior angle is $(x - 20)°$.
Find the number of sides of the polygon.

(3 marks)

3   Expand the expression: $(x^2 - y^2)(x^2 + y^2)(x^4 - y^4)$.

(2 marks)

4   A Kenyan businessman bought goods from Japan worth 2 950 000 Japanese Yen. On arrival in Kenya, custom duty of 20% was charged on the value of the goods.
If the exchange rates were as follows:

   1 US dollar = 118 Japanese Yen
   1 US dollar = 76 Kenya shillings

Calculate the duty paid in Kenya shillings.

(3 marks)

5   The gradient of the tangent to the curve $y = ax^3 + bx$ at the point $(1,1)$ is -5.
Calculate the values of a and b.

(4 marks)

6   Simplify the expression: $\dfrac{15a^2b - 10ab^2}{3a^2 - 5ab + 2b^2}$.

(3 marks)

7   A square brass plate is 2 mm thick and has a mass of 1.05 kg. The density of the brass is 8.4 g/cm³.
Calculate the length of the plate in centimetres.

(3 marks)

8   Given that x is an acute angle and $\cos x = \dfrac{2\sqrt{5}}{5}$, find without using Mathematical tables or a calculator,
$\tan (90 - x)°$.

(2 marks)

9   A cylindrical solid of radius 5 cm and length 12 cm floats lengthwise in water to a depth of 2.5 cm as shown in the figure below.
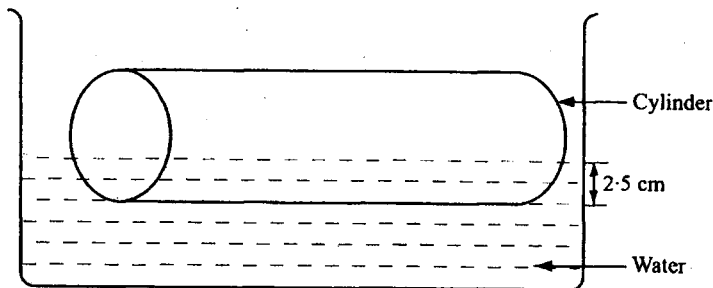
Calculate the area of the curved surface of the solid in contact with water, correct to 4 significant figures.

(4 marks)

10   In the figure below ∠A = 62°, ∠B = 41°, BC = 8.4 cm and CN is the bisector of ∠ACB.
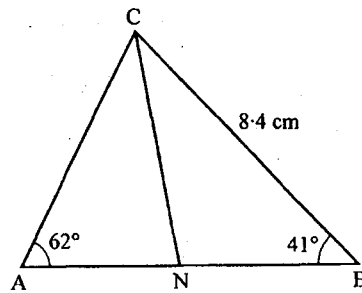
Calculate the length of CN to 1 decimal place.   (3 marks)

11   In fourteen years time, a mother will be twice as old as her son. Four years ago, the sum of their ages was 30 years. Find how old the mother was, when the son was born.   (4 marks)

12.   (a) Draw a regular pentagon of side 4 cm.   (1 mark)

      (b) On the diagram drawn, construct a circle which touches all the sides of the pentagon.   (2 marks)

13   The sum of two numbers x and y is 40. Write down an expression, in terms of x, for the sum of the squares of the two numbers.
      Hence determine the minimum value of $x^2 + y^2$.   (4 marks)

14   In the figure below, PQR is an equilateral triangle of side 6 cm. Arcs QR, PR and PQ are arcs of circles with centres at P, Q and R respectively.
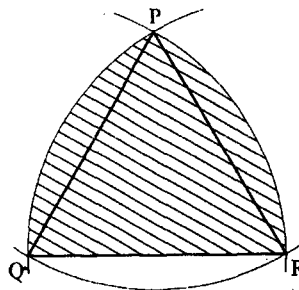
Calculate the area of the shaded region to 4 significant figures.   (4 marks)

15   Points L and M are equidistant from another point K. The bearing of L from K is 330°. The bearing of M from K is 220°.
      Calculate the bearing of M from L.   (3 marks)

16   A rally car travelled for 2 hours 40 minutes at an average speed of 120 Km/h. The car consumes an average of 1 litre of fuel for every 4 kilometres.
      A litre of the fuel costs Ksh 59.
   Calculate the amount of money spent on the fuel.   (3 marks)

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Answer any five questions in this section.

17   Three business partners: Asha, Nangila and Cherop contributed Ksh 60 000, Ksh 85 000 and Ksh 105 000 respectively. They agreed to put 25% of the profit back into business each year. They also agreed to put aside 40% of the remaining profit to cater for taxes and insurance. The rest of the profit would then be shared among the partners in the ratio of their contributions. At the end of the first year, the business realized a gross profit of Ksh 225 000.

(a) Calculate the amount of money Cherop received more than Asha at the end of the first year.
(5 marks)

(b) Nangila further invested Ksh 25 000 into the business at the beginning of the second year. Given that the gross profit at the end of the second year increased in the ratio 10 : 9, calculate Nangila's share of the profit at the end of the second year.   (5 marks)

18   In the diagram below, PA represents an electricity post of height 9.6 m. QB and RC represent two storey buildings of heights 15.4 m and 33.4 m respectively. The angle of depression of A from B is 5.5° while the angle of elevation of C from B is 30.5° and BC = 35 m.
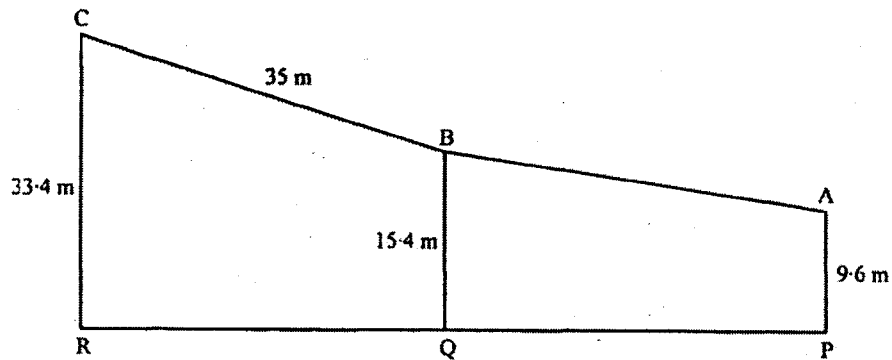
(a) Calculate, to the nearest metre, the distance AB.   (2 marks)

(b) By scale drawing find,
        (i) the distance AC in metres.   (5 marks)

    (ii) ∠BCA and hence determine the angle of depression of A from C.   (3 marks)

19   A frequency distribution of marks obtained by 120 candidates is to be represented in a histogram. The table below shows the grouped marks, frequencies for all the groups and also the area and height of the rectangle for the group 30–60 marks.

Marks	0–10	10–30	30–60	60–70	70–100
Frequency	12	40	36	8	24
Area of rectangle			180		
Height of rectangle			6		

(a) (i) Complete the table.   .(4 marks)
     (ii) On the grid provided below, draw the histogram.   (2 marks)
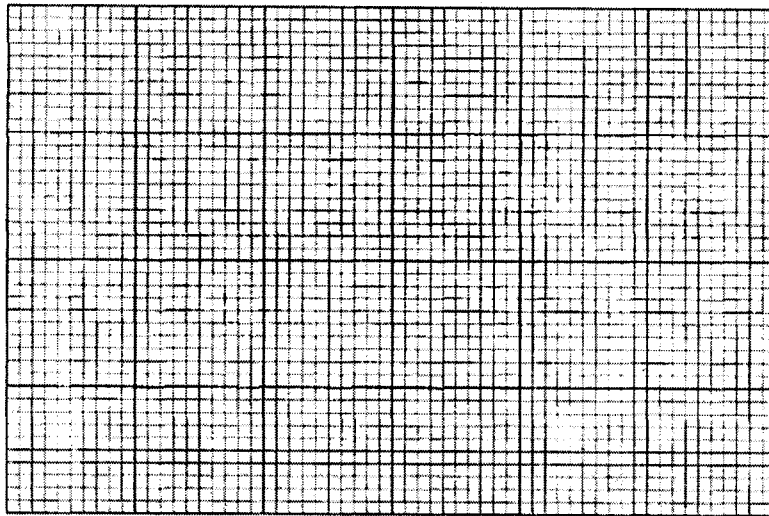
(b) (i) State the group in which the median mark lies.   (1 mark)

(ii) A vertical line drawn through the median mark divides the total area of the histogram into two equal parts.
Using this information or otherwise, estimate the median mark.   (3 marks)

20   A retailer planned to buy some computers from a wholesaler for a total of sh 1 800 000. Before the retailer could buy the computers the price per unit was reduced by sh 4 000. This reduction in price enabled the retailer to buy five more computers using the same amount of money as originally planned.

(a) Determine the number of computers the retailer bought.   (6 marks)

(b) Two of the computers purchased got damaged while in store, the rest were sold and the retailer made a 15% profit.
Calculate the profit made by the retailer on each computer sold.   (4 marks)

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21 In the figure below, **OQ** = q and **OR** = r. Point X divides OQ in the ratio 1 : 2 and Y divides OR in the ratio 3 : 4. Lines XR and YQ intersect at E.
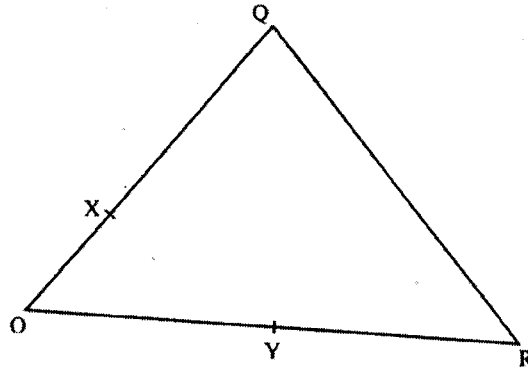
(a) Express in terms of **q** and **r**:

(i) **XR** (1 mark)

(ii) **YQ**. (1 mark)

(b) If **XE** = m **XR** and **YE** = n **YQ**, express **OE** in terms of:

(i) r, q and m (1 mark)

(ii) r, q and n. (1 mark)

(c) Using the results in (b) above, find the values of m and n. (6 marks)

22 Two cylindrical containers are similar. The larger one has internal cross-section area of 45 cm² and can hold 0.945 litres of liquid when full. The smaller container has internal cross-section area of 20 cm².

(a) Calculate the capacity of the smaller container. (4 marks)

(b) The larger container is filled with juice to a height of 13 cm. Juice is then drawn from it and emptied into the smaller container until the depths of the juice in both containers are equal. Calculate the depth of juice in each container. (2 marks)

(c) One fifth of the juice in the larger container in part (b) above is further drawn and emptied into the smaller container. Find the difference in the depths of the juice in the two containers. (4 marks)

23 (a) Find the inverse of the matrix $\begin{pmatrix} 9 & 8 \\ 7 & 6 \end{pmatrix}$. (2 marks)

(b) In a certain week a businessman bought 36 bicycles and 32 radios for a total of Ksh 227 280. In the following week, he bought 28 bicycles and 24 radios for a total of Ksh 174 960. Using matrix method, find the price of each bicycle and each radio that he bought. (4 marks)

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(c) In the third week, the price of each bicycle was reduced by 10% while the price of each radio was raised by 10%. The businessman bought as many bicycles and as many radios as he had bought in the first two weeks.
Find by matrix method, the total cost of the bicycles and radios that the businessman bought in the third week. (4 marks)

24 The diagram on the grid below represents an extract of a survey map showing two adjacent plots belonging to Kazungu and Ndoe.
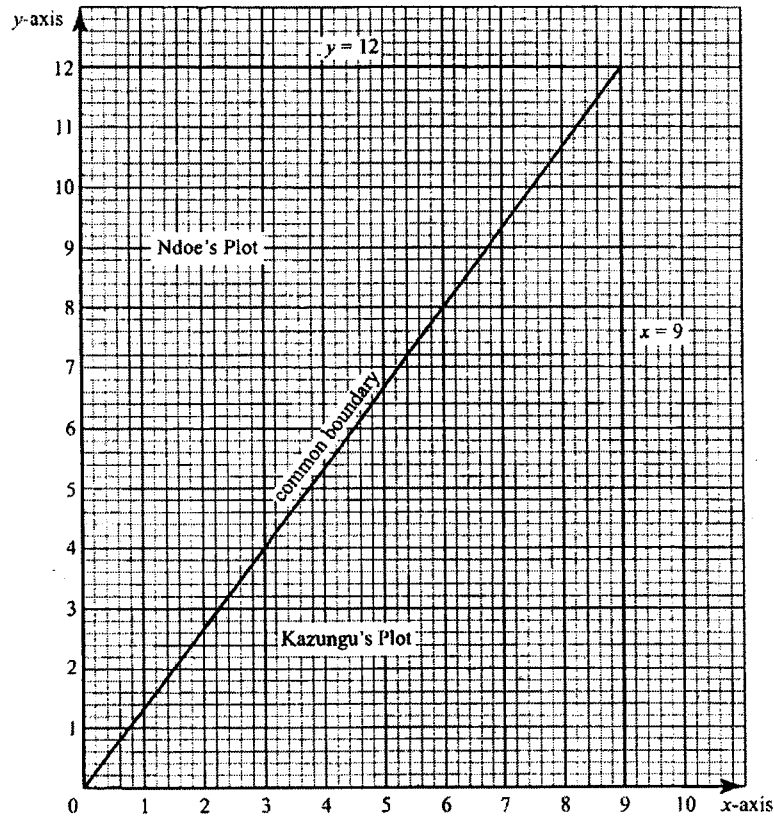
The two dispute the common boundary with each claiming boundary along different smooth curves . Coordinates (x, y_1) and (x, y_2) in the table below, represent points on the boundaries as claimed by Kazungu and Ndoe respectively.

x	0	1	2	3	4	5	6	7	8	9
y_1	0	4	5.7	6.9	8	9	9.8	10.6	11.3	12.
y_2	0	0.2	0.6	1.3	2.4	3.7	5.3	7.3	9.5	12

(a) On the grid provided above draw and label the boundaries as claimed by Kazungu and Ndoe.
(2 marks)

(b) (i) Use the trapezium rule with 9 strips to estimate the area of the section of the land in dispute.
(5 marks)

(ii) Express the area found in b(i) above, in hectares, given that 1 unit on each axis represents 20 metres.
(3 marks)

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23.3.2 Mathematics Paper 2(121/2)

Name .... Index Number .... / ....

121/2
MATHEMATICS
Paper 2
Oct./Nov. 2007
2½ hours

Candidate's Signature ....

Date ....

THE KENYA NATIONAL EXAMINATIONS COUNCIL

Kenya Certificate of Secondary Education

MATHEMATICS

Paper 2

2½ hours

Instructions to candidates

1. *Write your name and index number in the spaces provided above.*

2. *Sign and write the date of examination in the spaces provided above.*

3. *The paper contains two sections: Section I and Section II.*

4. *Answer all the questions in Section I and any five questions from Section II.*

5. *All answers and working must be written on the question paper in the spaces provided below each question.*

6. *Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.*

7. *Non-programmable silent electronic calculators and KNEC Mathematical tables may be used, except where stated otherwise.*

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Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total	

This paper consists of 17 printed pages

Candidates should check the question paper to ascertain
that all the pages are printed as indicated and no questions are missing.

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Turn over

SECTION I (50 marks)

*Answer **all** the questions in this section.*

1   Using logarithm tables, evaluate $\left(\dfrac{0.032 \times 14.26}{0.006}\right)^{\frac{2}{3}}$.   (3 marks)

2   Given that $y = \dfrac{2x - z}{x + 3z}$, express x in terms of y and z.   (3 marks)

3   Solve the equation $3 \cos x = 2 \sin^2 x$, where $0° \leq x \leq 360°$.   (4 marks)

4   (a) Expand the expression $\left(1 + \dfrac{1}{2}x\right)^5$ in ascending powers of x, leaving the coefficients as fractions in their simplest form.   (2 marks)

   (b) Use the first three terms of the expansion in (a) above to estimate the value of $\left(1\dfrac{1}{20}\right)^5$. (2 marks)

5   A particle moves in a straight line through a point P. Its velocity v m/s is given by $v = 2 - t$, where t is time in seconds, after passing P. The distance s of the particle from P when $t = 2$ is 5 metres.

   Find the expression for s in terms of t.   (3 marks)

6   The cash price of a T.V. set is Ksh 13 800. A customer opts to buy the set on Hire Purchase terms by paying a deposit of Ksh 2 280.

   If Simple Interest of 20% p.a. is charged on the balance and the customer is required to repay by 24 equal monthly instalments, calculate the amount of each instalment.   (3 marks)

7   Find the equation of a straight line which is equidistant from the points (2,3) and (6,1), expressing it in the form $ax + by = c$ where a, b and c are constants.   (4 marks)

8   A rectangular block has a square base whose side is exactly 8 cm. Its height, measured to the nearest millimetre, is 3.1 cm.

   Find in cubic centimetres, the greatest possible error in calculating its volume.   (2 marks)

9   Water and milk are mixed such that the ratio of the volume of water to that of milk is 4:1.

   Taking the density of water as 1 g/cm³ and that of milk as 1.2 g/cm³, find the mass, in grams of 2.5 litres of the mixture.   (3 marks)

10 A carpenter wishes to make a ladder with 15 cross-pieces. The cross-pieces are to diminish uniformly in lengths from 67 cm at the bottom to 32 cm at the top.

   Calculate the length, in cm, of the seventh cross-piece from the bottom.   (3 marks)

11 In the figure below AB is a diameter of the circle. Chord PQ intersects AB at N. A tangent to the circle at B meets PQ produced at R.
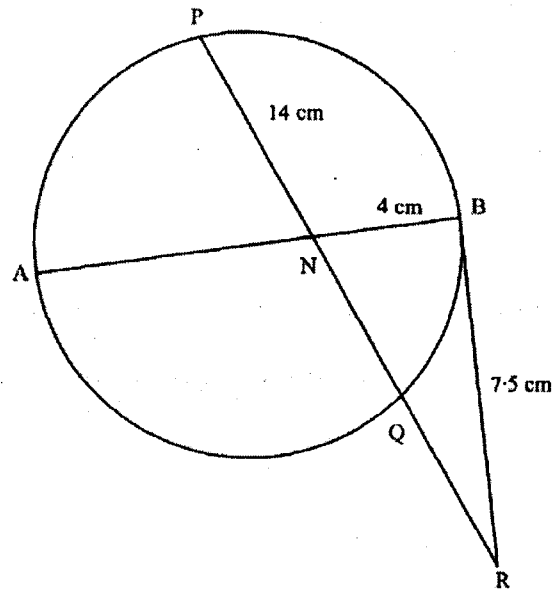
Given that PN = 14 cm, NB = 4 cm and BR = 7.5 cm, calculate the length of:

(a) NR   (1 mark)

(b) AN.   (3 marks)

12 Vector **q** has a magnitude of 7 and is parallel to vector **p**. Given that **p** = 3 **i** – **j** + 1 ½ **k**, express vector **q** in terms of **i**, **j** and **k**.   (2 marks)

13 Two places A and B are on the same circle of latitude north of the equator. The longitude of A is 118°W and the longitude of B is 133°E. The shorter distance between A and B measured along the circle of latitude is 5422 nautical miles.

Find, to the nearest degree, the latitude on which A and B lie.   (3 marks)

14 The figure below is a sketch of the graph of the quadratic function $y = k(x + 1)(x - 2)$.
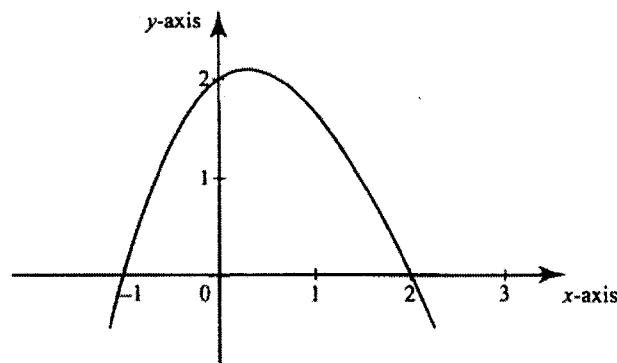
Find the value of k.   (2 marks)

15 Simplify $\dfrac{3}{\sqrt{5} - 2} + \dfrac{1}{\sqrt{5}}$ leaving the answer in the form $a + b\sqrt{c}$, where a, b and c are rational numbers.

(3 marks)

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16 Find the radius and the coordinates of the centre of the circle whose equation is
$2x^2 + 2y^2 - 3x + 2y + \frac{1}{2} = 0$. (4 marks)

SECTION II (50 marks)

Answer any five questions in this section.

17 A tank has two inlet taps P and Q and an outlet tap R. When empty, the tank can be filled by tap P alone in 4½ hours or by tap Q alone in 3 hours. When full, the tank can be emptied in 2 hours by tap R.

(a) The tank is initially empty. Find how long it would take to fill up the tank:

(i) if tap R is closed and taps P and Q are opened at the same time (2 marks)

(ii) if all the three taps are opened at the same time. (2 marks)

(b) The tank is initially empty and the three taps are opened as follows:

P at 8.00 a.m.
Q at 8.45 a.m.
R at 9.00 a.m.

(i) Find the fraction of the tank that would be filled by 9.00 a.m. (3 marks)

(ii) Find the time the tank would be fully filled up. (3 marks)

18 Given that y is inversely proportional to x^n and k as the constant of proportionality;

(a) (i) Write down a formula connecting y, x, n and k. (1 mark)

(ii) If $x = 2$ when $y = 12$ and $x = 4$ when $y = 3$, write down two expressions for k in terms of n.

Hence, find the value of n and k. (7 marks)

(b) Using the value of n obtained in (a) (ii) above, find y when $x = 5\frac{1}{3}$. (2 marks)

19 (a) Given that $y = 8 \sin 2x - 6 \cos x$, complete the table below for the missing values of y, correct to 1 decimal place. (2 marks)
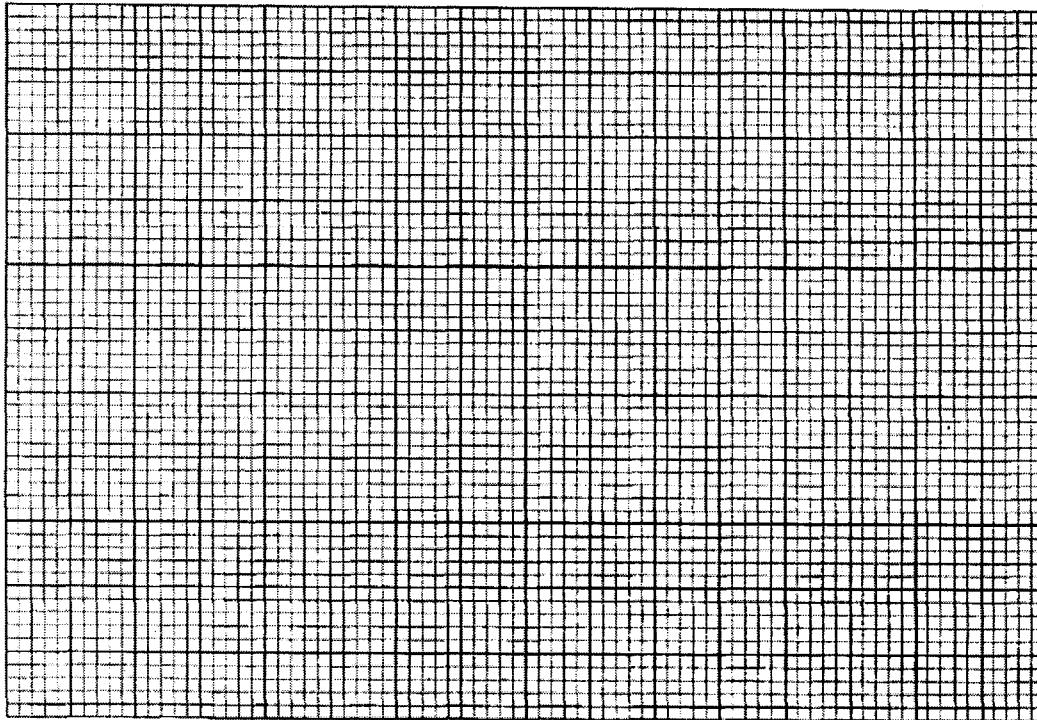
x	0°	15°	30°	45°	60°	75°	90°	105°	120°
$y = 8 \sin 2x - 6 \cos x$	–6	–1.8		3.8	3.9	2.4	0		–3.9

(b) On the grid provided below, draw the graph of $y = 8 \sin 2x - 6 \cos x$ for

$0° \leq x \leq 120°$.

Take the scale: 2 cm for 15° on the x-axis
2 cm for 2 units on the y-axis. (4 marks)

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(c) Use the graph to estimate:
    (i) the maximum value of y,   (1 mark)

        (ii) the value of x for which $4 \sin 2x - 3\cos x = 1$.   (3 marks)

20 The gradient function of a curve is given by the expression $2x + 1$. If the curve passes through the point $(-4, 6)$;

(a) Find:
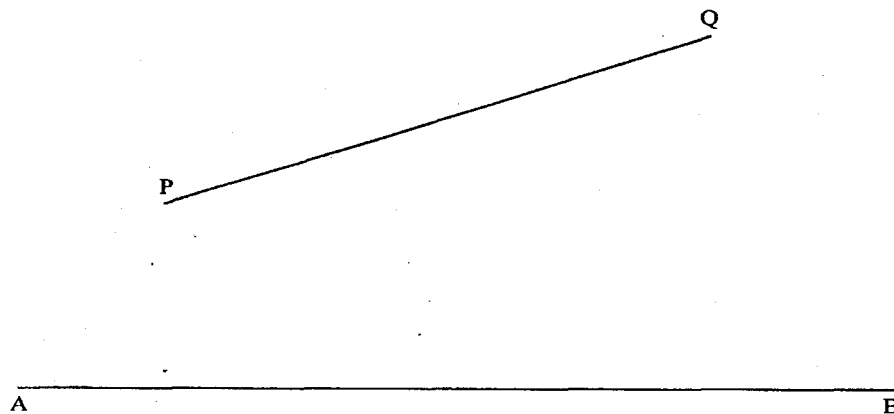   (i) the equation of the curve,   (3 marks)

    (ii) the values of x at which the curve cuts the x-axis.   (3 marks)

(b) Determine the area enclosed by the curve and the x-axis.   (4 marks)

21 *In this question use a ruler and a pair of compasses only.*
In the figure below, AB and PQ are straight lines.

(a) Use the figure to:
  (i) find a point R on AB such that R is equidistant from P and Q (1 mark)
  (ii) complete a polygon PQRST with AB as its line of symmetry and hence measure the distance of R from TS. (5 marks)

(b) Shade the region within the polygon in which a variable point X must lie given that X satisfies the following conditions:

  I: X is nearer to PT than to PQ
  II: RX is not more than 4.5 cm
 III: $\angle PXT > 90°$ (4 marks)

22 A company is considering installing two types of machines, A and B. The information about each type of machine is given in the table below.

Machine type	Number of operators	Floor space	Daily profit
A	2	5m²	Ksh 1 500
B	5	8m²	Ksh 2 500

The company decided to install x machines of type A and y machines of type B.

(a) Write down the inequalities that express the following conditions:
  I: the number of operators available is 40
  II: the floor space available is 80m²
  III: the company is to install not less than 3 of type A machines
  IV: the number of type B machines must be more than one third the number of type A machines. (4 marks)

(b) On the grid provided, draw the inequalities in part (a) above and shade the unwanted region. (4 marks)
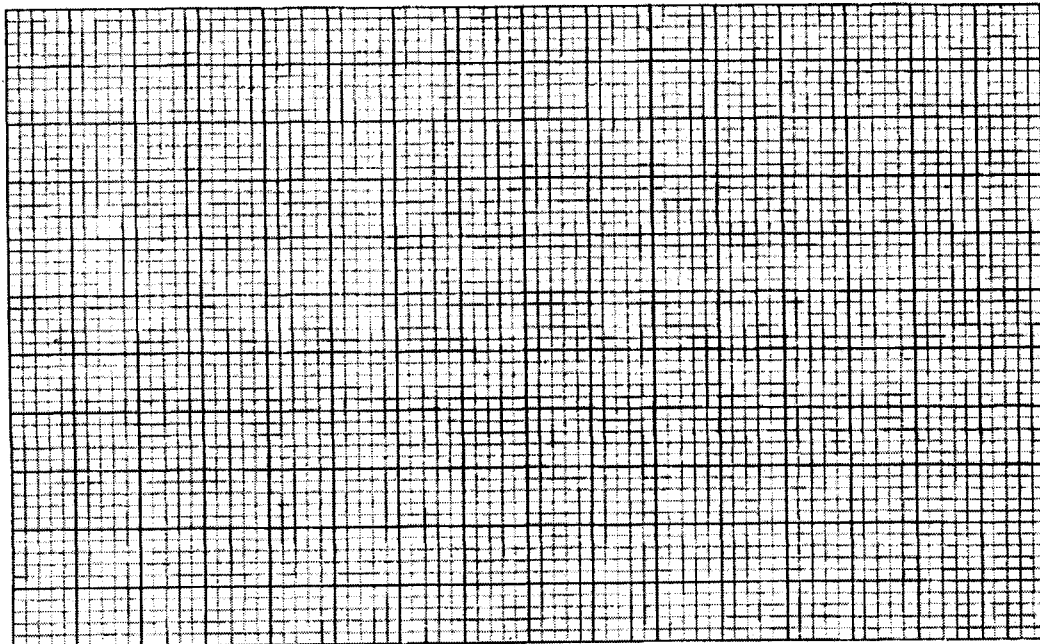
(c) Draw a search line and use it to determine the number of machines of each type that should be installed to maximize the daily profit. (2 marks)

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23 The table below shows the values of the length X (in metres) of a pendulum and the corresponding values of the period T (in seconds) of its oscillations obtained in an experiment.

X (metres)	0.4	1.0	1.2	1.4	1.6
T (seconds)	1.25	2.01	2.19	2.37	2.53

(a) Construct a table of values of log X and corresponding values of log T, correcting each value to 2 decimal places. (2 marks)

(b) Given that the relation between the values of log X and log T approximate to a linear law of the form $m \log T = b \log X + \log a$ where a and b are constants;

(i) Use the axes on the grid provided to draw the line of best fit for the graph of log T against log X. (2 marks)
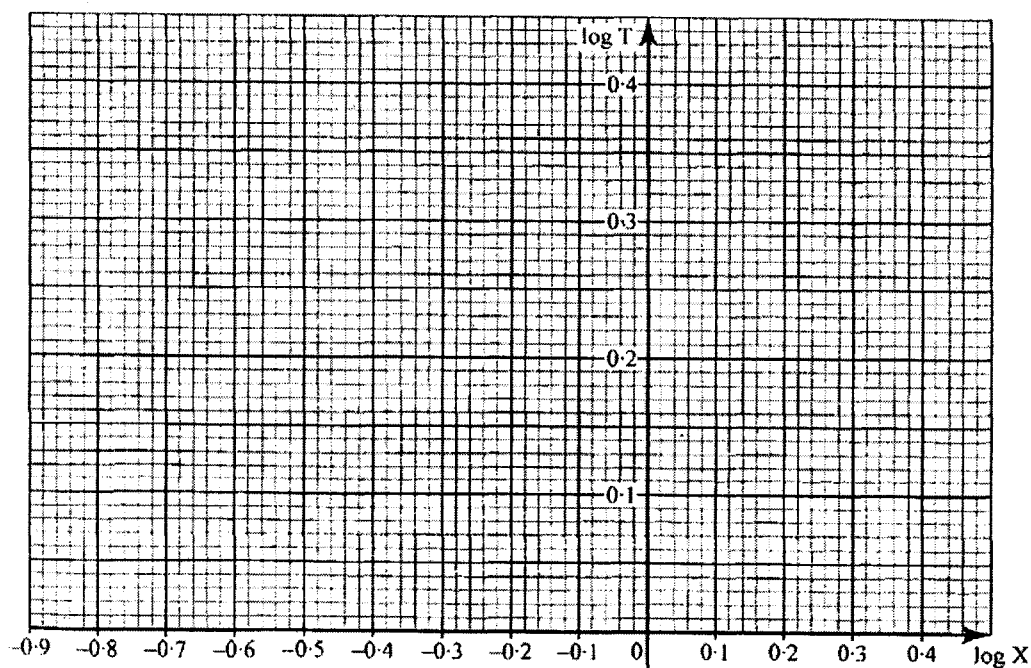
(ii) Use the graph to estimate the values of a and b. (3 marks)

(c) Find, to 2 decimal places, the length of the pendulum whose period is 1 second. (3 marks)

24 Two bags A and B contain identical balls except for the colours. Bag A contains 4 red balls and 2 yellow balls. Bag B contains 2 red balls and 3 yellow balls.

(a) If a ball is drawn at random from each bag, find the probability that both balls are of the same colour. (4 marks)

(b) If two balls are drawn at random from each bag, one ball at a time without replacement, find the probability that;

(i) the two balls drawn from bag A or bag B are red, (4 marks)

(ii) all the four balls drawn are red. (2 marks)

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