## K.C.S.E MATHEMATICS PAPER 12007

## Answer all the questions in this section.

1. Evaluate without using mathematical tables or a calculator $0.0084 \times 1.23 \times 3.5$, $2.87 \times 0.056$
Expressing the answer as a fraction in its simplest form
2. The size of an interior angle of a regular polygon is $3 x^{0}$ while its exterior angle is $(x-20)^{0}$. Find the number of sides of the polygon
( 3 marks)
3. Expand the expression $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)\left(x^{4}-y^{4}\right)$
( 2 marks)
4. A Kenyan businessman bought goods from Japan worth 2, 950000 Japanese yen. On arrival in Kenya custom duty of $20 \%$ was charged on the value of the goods. If the exchange rates were as follows

1 US dollar = 118 Japanese Yen
1 US dollar $=76$ Kenya shillings
Calculate the duty paid in Kenya shillings
( 3 marks)
5. The gradient of the tangent to the curve $y=a x^{3}+b x$ at the point $(1,1)$ is -5 Calculate the values of $a$ and $b$
6. Simplify the expression $\frac{15 a^{2} b-10 a b^{2}}{3 a^{2}-5 a b+2 b^{2}}$
7. A square brass plate is 2 mm thick and has a mass of 1.05 kg . The density of the brass is $8.4 \mathrm{~g} / \mathrm{cm}^{3}$. Calculate the length of the plate in centimeters ( 3 marks)
8. Given that $x$ is an acute angle and $\cos x=\frac{2 \sqrt{5}}{5}$, find without using mathematical tables or a calculator, $\tan (90-x)^{0}$.
9. A cylindrical solid of radius 5 cm and length 12 cm floats lengthwise in water to a depth of 2.5 cm as shown in the figure below.


Calculate the area of the curved surface of the solid in contact with water, correct to 4 significant figures
( 4 marks)
10. In the figure below $\angle \mathrm{A}=62^{\circ}, \angle \mathrm{B}=41^{\circ}, \mathrm{BC}=8.4 \mathrm{~cm}$ and CN is the bisector of $\angle$ ACB.


Calculate the length of CN to 1 decimal place.
( 3 marks)
11. In fourteen years time, a mother will be twice as old as her son. Four years ago, the sum of their ages was 30 years. Find how old the mother was, when the son was born.
12. (a) Draw a regular pentagon of side 4 cm
( 1 mark)
(b) On the diagram drawn, construct a circle which touches all the sides of the pentagon
( 2 marks)
13. The sum of two numbers $x$ and $y$ is 40 . write down an expression, in terms of $x$, for the sum of the squares of the two numbers.
Hence determine the minimum value of $x^{2}+y^{2}$
14. In the figure below, $P Q R$ is an equilateral triangle of side 6 cm . Arcs $Q R, P R$ and PQ arcs of circles with centers at $\mathrm{P}, \mathrm{Q}$ and R respectively.


Calculate the area of the shaded region to 4 significant figures
( 4 marks)
15. Points $L$ and $M$ are equidistant from another point $K$. The bearing of $L$ from $K$ is $330^{\circ}$. The bearing of M from K is $220^{\circ}$.
Calculate the bearing of M from L
16. A rally car traveled for 2 hours 40 minutes at an average speed of $120 \mathrm{~km} / \mathrm{h}$. The car consumes an average of 1 litre of fuel for every 4 kilometers.

A litre of the fuel costs Kshs 59
Calculate the amount of money spent on fuel

## SECTION II (50 marks)

## Answer any five questions in this section

17. Three business partners: Asha Nangila and Cherop contributed Kshs 60,000, Kshs 85,000 and Kshs 105000 respectively. They agreed to put $25 \%$ of the profit back into business each year. Thay also agreed to put aside $40 \%$ of the remaining profit to cater for taxes and insurance. The rest of the profit would then be shared among the partners in the ration of their contributions. At the end of the first year, the business realized a gross profit of Kshs 225000
(a) Calculate the amount of money Cherop received more than Asha at the end of the first year
( 5 marks)
(b) Nangila further invested Kshs 25,000 into the business at the beginning of the second year. Given that the gross profit at the end of the second year increased in the ratio 10: 9, calculate Nangila's share of the profit at the end of the second year.
( 5 marks)
18. In the diagram below PA represents an electricity post of height $9.6 \mathrm{~m} . \mathrm{BB}$ and RC represents two storey buildings of heights 15.4 m and 33.4 m respectively. The angle of depression of A from B is $5.5^{0}$ While the angle of elevation of C from $B$ is $30.5^{0}$ and $B C=35 \mathrm{~m}$.

(a) Calculate, to the nearest metre, the distance $A B$
( 2 marks)
(b) By scale drawing find,
(i) The distance AC in metres
( 5 marks)
(ii) $\angle \mathrm{BCA}$ and hence determine the angle of depression of A from C
( 3 marks)
19. A frequency distribution of marks obtained by 120 candidates is to be represented in a histogram. The table below shows the grouped marks. Frequencies for all the groups and also the area and height of the rectangle for the group 30-60 marks.

| Marks | $0-10$ | $10-30$ | $30-60$ | $60-70$ | $70-100$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 12 | 40 | 36 | 8 | 24 |
| Area of rectangle |  |  | 180 |  |  |
| Height of rectangle |  |  | 6 |  |  |

(a) (i) Complete the table
( 4 marks)
(ii) On the grid provided below, draw the histogram
( 2 marks)

(b) (i) State the group in which the median mark lies
( 1 mark)
(ii) A vertical line drawn through the median mark divides the total area of the histogram into two equal parts
Using this information or otherwise, estimate the median mark
20. A retailer planned to buy some computers form a wholesaler for a total of Kshs $1,800,000$. Before the retailer could buy the computers the price per unit was reduced by Kshs 4,000 . This reduction in price enabled the retailer to buy five more computers using the same amount of money as originally planned.
(a) Determine the number of computers the retailer bought
( 6 marks)
(b) Two of the computers purchased got damaged while in store, the rest were sold and the retailer made a $15 \%$ profit
Calculate the profit made by the retailer on each computer sold ( 4 marks)
21. In the figure below, $\mathrm{OQ}=\mathrm{q}$ and $\mathrm{OR}=\mathrm{r}$. Point X divides OQ in the ratio $1: 2$ and Y divides OR in the ratio 3: 4 lines XR and YQ intersect at E .
(a) Express in terms of $q$ and $r$
(i) XR

(ii) YQ
(b) If $\mathrm{XE}=\mathrm{m}$ XR and $\mathrm{YE}=\mathrm{n} Y \mathrm{Y}$, express OE in terms of: ( 1 mark )
(i) $r, q$ and $m$
(ii) $\mathrm{r}, \mathrm{q}$ and n
(c) Using the results in (b) above, find the values of $m$ and $n . \quad$ ( 6 marks)
22. Two cylindrical containers are similar. The larger one has internal cross- section area of $45 \mathrm{~cm}^{2}$ and can hold 0.945 litres of liquid when full. The smaller container has internal cross- section area of $20 \mathrm{~cm}^{2}$
(a) Calculate the capacity of the smaller container
(b) The larger container is filled with juice to a height of 13 cm . Juice is then drawn from is and emptied into the smaller container until the depths of the juice in both containers are equal.
Calculate the depths of juice in each container. (2marks)
(c) On fifth of the juice in the larger container in part (b) above is further drawn and emptied into the smaller container. Find the difference in the depths of the juice in the two containers.
( 4 marks)
23. (a) Find the inverse of the matrix $\left(\begin{array}{ll}9 & 8 \\ 7 & 6\end{array}\right)$
( 2 marks)
(b) In a certain week a businessman bought 36 bicycles and 32 radios for total of Kshs 227 280. In the following week, he bought 28 bicycles and 24 radios for a total of Kshs 174960
Using matrix method, find the price of each bicycle and each radio that he bought ( 4 marks)
(c) In the third week, the price of each bicycle was reduced by $10 \%$ while the price of each radio was raised by $10 \%$. The businessman bought as many bicycles and as many radios as he had bought in the first two weeks.
Find by matrix method, the total cost of the bicycles and radios that the businessman bought in the third week.
24. The diagram on the grid below represents as extract of a survey map showing two adjacent plots belonging to Kazungu and Ndoe.


| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{y}_{1}$ | 0 | 4 | 5.7 | 6.9 | 8 | 9 | 9.8 | 10.6 | 11.3 | 12 |
| $\mathrm{y}_{2}$ | 0 | 0.2 | 0.6 | 1.3 | 2.4 | 3.7 | 5.3 | 7.3 | 9.5 | 12 |

(a) On the grid provided above draw and label the boundaries as claimed by Kazungu and Ndoe

## K.C.S.E MATHEMATICS PAPER 22007 <br> SECTION 1 (50 Marks) <br> Answer all the questions in this section

1. Using logarithm tables, evaluate $\left(\frac{0.032 \times 14.26}{0.006}\right)^{2 / 3} \quad$ ( 3 marks)
2. Given that $\mathrm{y}=\underline{2 \mathrm{x}-\mathrm{z}}$, express x in terms of y and z ( 3 marks)
3. Solve the equation $3 \cos x=2 \sin 2 x$, where $0^{\circ} \leq x \leq 360^{\circ} \quad$ ( 4 marks)
4. (a) Expand the expression $(1+1 / 2 x)^{5}$ in ascending powers of $x$, leaving the coefficients as fractions in their simplest form ( 2 marks)
(b) Use the first three terms of the expansion in (a) above to estimate the value of $\left(1 \frac{1}{20}\right)^{5}$
( 2 marks)
5. A particle moves in a straight line through a point $P$. Its velocity $\mathrm{v} \mathrm{m} / \mathrm{s}$ is given by $v=2-1$, where $t$ is time in seconds, after passing $P$. The distance s of the particle from P when $\mathrm{t}=2$ is 5 metres. Find the expression for s in terms of $t$.
( 3 marks)
6. The cash price of a T.V set is Kshs 13,800 . A customer opts to buy the set on Hire purchase terms by paying a deposit of Kshs 2, 280
If simple interest of $20 \%$ p.a is charged on the balance and the customer is required to repay by 24 equal monthly installments, calculate the amount of each installment.
( 3 marks)
7. Find the equation of a straight line which is equidistant from the points $(2,3)$ and ( 6,1 ), expressing it in the form $\mathrm{ax}+\mathrm{by}=\mathrm{c}$ where $\mathrm{a}, \mathrm{b}$ and c are constants
( 4 marks)
8. A rectangular block has a square base whose side is exactly 8 cm . Its height measured to the nearest millimeter is 3.1 cm
Find in cubic centimeters, the greatest possible error in calculating its volume.
( 2 marks)
9. Water and milk are mixed such that the ratio of the volume of water to that of milk is 4: 1 .
Taking the density of water as $1 \mathrm{~g} / \mathrm{cm}^{3}$ and that of milk as $1.2 \mathrm{~g} / \mathrm{cm}^{3}$, find the mass in grams of 2.5 litres of the mixture.
( 3 marks)
10. A carpenter wishes to make a ladder with 15 cross- pieces. The cross- pieces are to diminish uniformly in length from 67 cm at the bottom to 32 cm at the top.
Calculate the length in cm, of the seventh cross- piece from the bottom
11. In the figure below AB is a diameter of the circle. Chord PQ intersects AB at N. A tangent to the circle at B meets PQ produced at R .


Given that $\mathrm{PN}=14 \mathrm{~cm}, \mathrm{NB}=4 \mathrm{~cm}$ and $\mathrm{BR}=7.5 \mathrm{~cm}$, calculate the length of:
(a) NR
( 1 mark)
(b) AN
( 3 marks)
12. Vector q has a magnitude of 7 and is parallel to vector p . Given that $p=3 \mathrm{i}-\mathrm{j}+1^{1 / 2} \mathrm{k}$, express vector q in terms of $\mathrm{I}, \mathrm{j}$, and k . ( 2 marks)
13. Two places A and B are on the same circle of latitude north of the equator. The longitude of A is $118^{0} \mathrm{~W}$ and the longitude of B is $133^{\circ} \mathrm{E}$. The shorter distance between A and B measured along the circle of latitude is 5422 nautical miles.
Find, to the nearest degree, the latitude on which A and B lie (3 marks)
14. The figure below is a sketch of the graph of the quadratic function $y=k$ ( $\mathrm{x}+1$ ) (x-2)


Find the value of k
15. Simplify $\frac{3}{\sqrt{5}}+\frac{1}{\sqrt{5}}$ leaving the answer in the form $\mathrm{a}+\mathrm{bc}$, where $\mathrm{a}, \mathrm{b}$ and c are rational numbers
16. Find the radius and the coordinate of the centre of the circle whose equation is

$$
2 x^{2}+2 y^{2}-3 x+2 y+1 / 2=0
$$

( 4 marks)

## SECTION 11 ( 50 MARKS)

 Answer any five questions in this section17. A tank has two inlet taps $P$ and $Q$ and an outlet tap R. when empty, the tank can be filled by tap P alone in $41 / 2$ hours or by tap Q alone in 3 hours. When full, the tank can be emptied in 2 hours by tap R.
(a) The tank is initially empty. Find how long it would take to fill up the tank
(i) If tap R is closed and taps P and Q are opened at the same time
( 2 marks)
(ii) If all the three taps are opened at the same time( 2 marks)
(b) The tank is initially empty and the three taps are opened as follows
$P$ at 8.00 a.m
Q at 8.45 a.m
R at 9.00 a.m
(i) Find the fraction of the tank that would be filled by 9.00 a.m
(ii) Find the time the tank would be fully filled up ( 3 marks)
18. Given that y is inversely proportional to $\mathrm{x}^{\mathrm{n}}$ and k as the constant of proportionality;
(a) (i) Write down a formula connecting $\mathrm{y}, \mathrm{x}, \mathrm{n}$ and k
( 1 mark)
(ii) If $x=2$ when $y=12$ and $x=4$ when $y=3$, write down two expressions for $k$ in terms of $n$.
Hence, find the value of $n$ and $k$. ( 7 marks)
(b) Using the value of n obtained in (a) (ii) above, find y when $\mathrm{x}=5^{1 / 3}$
( 2 marks)
19. (a) Given that $y=8 \sin 2 x-6 \cos x$, complete the table below for the missing values of $y$, correct to 1 decimal place.

| X | $0^{0}$ | $15^{0}$ | $30^{\circ}$ | $45^{0}$ | $60^{0}$ | $75^{\circ}$ | $90^{\circ}$ | $105^{0}$ | $120^{0}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}=8 \sin 2 \mathrm{x}-6 \cos \mathrm{x}$ | -6 | -1.8 |  | 3.8 | 3.9 | 2.4 | 0 |  | -3.9 |

(b) On the grid provided, below, draw the graph of $y=8 \sin 2 x-6 \cos$ for $0^{0} \leq \mathrm{x} \leq 120^{0}$
Take the scale 2 cm for $15^{0}$ on the x - axis
2 cm for 2 units on the y - axis
(c) Use the graph to estimate
(i) The maximum value of $y$
( 1 marks)
(ii) The value of x for which $4 \sin 2 \mathrm{x}-3 \cos \mathrm{x}=1$
( 3 marks)
20. The gradient function of a curve is given by the expression $2 x+1$. If the curve passes through the point $(-4,6)$;
(a) Find:
(i) The equation of the curve
(ii) The vales of x , at which the curve cuts the x - axis
(b) Determine the area enclosed by the curve and the $x$ - axis
21. In this question use a ruler and a pair of compasses only In the figure below, AB and PQ are straight lines

(a) Use the figure to:
(i) Find a point R on AB such that R is equidistant from P and Q (1 mark)
(ii) Complete a polygon $\operatorname{PQRST}$ with AB as its line of symmetry and hence measure the distance of R from TS.
(b) Shade the region within the polygon in which a variable point X must lie given that X satisfies the following conditions

I: $\quad \mathrm{X}$ is nearer to PT than to PQ
II: $\quad$ RX is not more than 4.5 cm
III. $\angle \mathrm{PXT}>90^{\circ}$
22. A company is considering installing two types of machines. A and B. The information about each type of machine is given in the table below.

| Machine | Number of operators | Floor space | Daily profit |
| :--- | :--- | :--- | :--- |
| A | 2 | $5 \mathrm{~m}^{2}$ | Kshs 1,500 |
| B | 5 | $8 \mathrm{~m}^{2}$ | Kshs 2,500 |

The company decided to install $x$ machines of types A and y machines of type B
(a) Write down the inequalities that express the following conditions
I. The number of operators available is 40
II. The floor space available is $80 \mathrm{~m}^{2}$
III. The company is to install not less than 3 type of A machine
IV. The number of type B machines must be more than one third the number of type A machines
(b) On the grid provided, draw the inequalities in part ( a) above and shade the unwanted region
( 4 marks)
(c) Draw a search line and use it to determine the number of machines of each type that should be installed to maximize the daily profit.
( 2 marks)
23. The table below shows the values of the length $X$ (in metres ) of a pendulum and the corresponding values of the period T ( in seconds) of its oscillations obtained in an experiment.

| X ( metres) | 0.4 | 1.0 | 1.2 | 1.4 | 1.6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| T ( seconds) | 1.25 | 2.01 | 2.19 | 2.37 | 2.53 |

(a) Construct a table of values of $\log \mathrm{X}$ and corresponding values of $\log \mathrm{T}$, correcting each value to 2 decimal places
( 2 marks)
(b) Given that the relation between the values of $\log \mathrm{X}$ and $\log \mathrm{T}$ approximate to a linear law of the form $\mathrm{m} \log \mathrm{X}+\log \mathrm{a}$ where a and b are constants
(i) Use the axes on the grid provided to draw the line of best fit for the graph of $\log \mathrm{T}$ against $\log \mathrm{X}$.

(ii) Use the graph to estimate the values of $a$ and $b$
(b) Find, to decimal places the length of the pendulum whose period is 1 second
24. Two bags A and B contain identical balls except for the colours. Bag A contains 4 red balls and 2 yellow balls. Bag B contains 2 red balls and 3 yellow balls.
(a) If a ball is drawn at random from each bag, find the probability that both balls are of the same colour.
( 4 marks)
(b) If two balls are drawn at random from each bag, one at a time without replacement, find the probability that:
(i) The two balls drawn from bag A or bag B are red ( 4 marks)
(ii) All the four balls drawn are red
( 2 marks)

