29.3 MATHEMATICS (121)

29.3.1 Mathematics Paper 1 (121/1)

SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1 Without using mathematical tables or calculators, evaluate $\dfrac{\sqrt{5184}}{6 \times {}^-18 \div 9 + (5 - {}^-3)}$. **(3 marks)**

2 Without using a calculator, evaluate, $\dfrac{2\frac{1}{4} + \frac{3}{5} \div \frac{5}{6} \text{ of } 2\frac{2}{5}}{1\frac{7}{10}}$, leaving the answer as a fraction in its simplest form. **(3 marks)**

3 Given that the ratio $x : y = 2 : 3$, find the ratio $(5x - 2y) : (x + y)$. **(3 marks)**

4 A bus travelling at an average speed of 63 km/h left a station at 8.15 a.m.
A car later left the same station at 9.00 a.m. and caught up with the bus at 10.45 a.m. Find the average speed of the car. **(3 marks)**

5 Without using logarithm tables or calculators, evaluate, $\dfrac{64^{-\frac{1}{2}} \times 27000^{\frac{2}{3}}}{2^{-4} \times 3^0 \times 5^2}$. **(4 marks)**

6 The figure below represents a plot of land ABCD such that AB = 85 m, BC = 75 m, CD = 60 m, DA = 50 m and angle ACB = 90°.
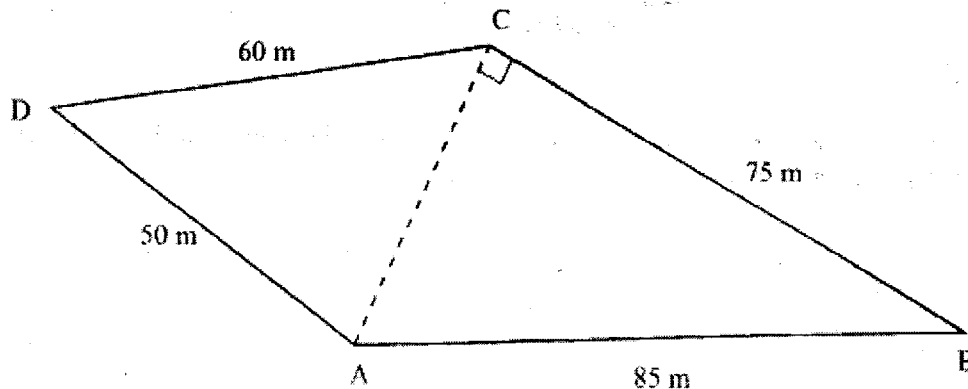

Determine the area of the plot in hectares correct to two decimal places. **(4 marks)**

7 A watch which loses a half-minute every hour was set to read the correct time at 05 45 h on Monday. Determine the time, in the 12-hour system, the watch will show on the following Friday at 19 45 h. **(3 marks)**

8 Simplify the expression $\dfrac{12x^2 + ax - 6a^2}{9x^2 - 4a^2}$. **(3 marks)**

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9   A line which joins the points A (3, k) and B (2, 5) is parallel to another line whose equation is
$5y + 2x = 10$.
Find the value of k.   **(3 marks)**

10   The size of an interior angle of a regular polygon is 6½ times that of its exterior angle.
Determine the number of sides of the polygon.   **(3 marks)**

11   Line AB shown below is a side of a trapezium ABCD in which angle ABC = 105°, BC = 4 cm,
CD = 5 cm and CD is parallel to AB.
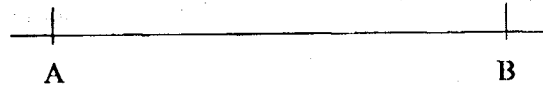
Using a ruler and a pair of compasses only:

(a) complete the trapezium;   **(3 marks)**

(b) locate point T on line AB such that angle ATD = 90°.   **(1 mark)**

12   An electric pole is supported to stand vertically on a level ground by a tight wire. The wire is pegged at
a distance of 6 metres from the foot of the pole as shown.
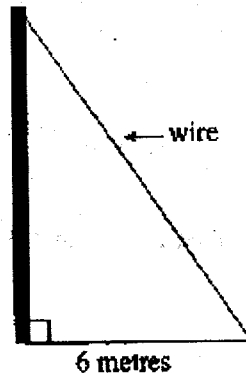
The angle which the wire makes with the ground is three times the angle it makes with the pole.
Calculate the length of the wire to the nearest centimetre.   **(3 marks)**

13   Solve the equation: $\sin(3x + 30°) = \dfrac{\sqrt{3}}{2}$, for $0° \le x \le 90°$.   **(4 marks)**

14   The diagonals of a rhombus PQRS intersect at T. Given that P(2, 2), Q(3, 6) and R(−1, 5):

   (a) draw the rhombus PQRS on the grid provided;   (1 mark)
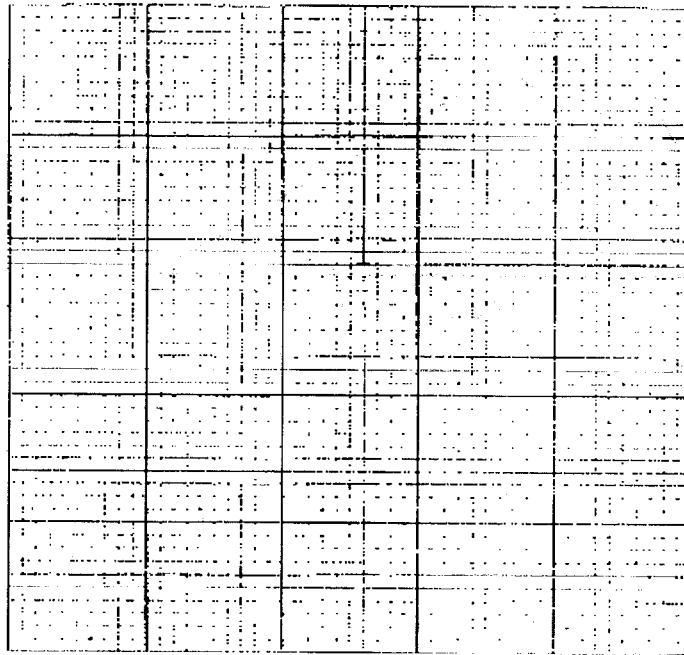
   (b) state the coordinates of T.   (1 mark)

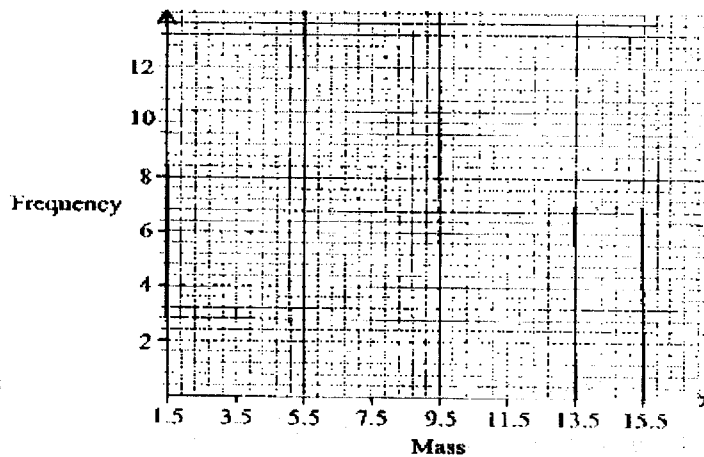15   Abdi sold a radio costing Ksh 3 800 at a profit of 20%. He earned a commission of 22½% on the profit.
     Find the amount he earned.   (2 marks)

16   The following data was obtained for the masses of certain animals.

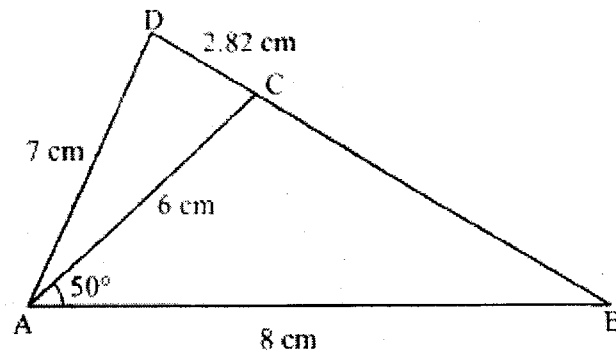| Mass (x kg)	Frequency
$1.5 \leq x < 5.5$	16
$5.5 \leq x < 7.5$	20
$7.5 \leq x < 13.5$	18
$13.5 \leq x < 15.5$	14

Complete the histogram on the grid provided below.   (3 marks)

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17 In the figure below (not drawn to scale). AB = 8 cm, AC = 6 cm, AD = 7 cm, CD = 2.82 cm and angle CAB = 50°.

Calculate, to 2 decimal places:

(a) the length BC; (2 marks)

(b) the size of angle ABC; (3 marks)

(c) the size of angle CAD; (3 marks)

(d) the area of triangle ACD. (2 marks)

18 The marks scored by a group of pupils in a mathematics test were as recorded in the table below.

| Marks	Frequency
0 – 9	1
10 – 19	2
20 – 29	4
30 – 39	7
40 – 49	10
50 – 59	16
60 – 69	20
70 – 79	6
80 – 89	3
90 – 99	1

(a) (i) State the modal class. (1 mark)

(ii) Determine the class in which the median mark lies. (2 marks)

(b) Using an assumed mean of 54.4, calculate the mean mark. (7 marks)

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19 A school planned to buy x calculators for a total cost of Ksh 16 200. The supplier agreed to offer a discount of Ksh 60 per calculator. The school was then able to get three extra calculators for the same amount of money.

(a) Write an expression in terms of x, for the:

(i) original price of each calculator; (1 mark)

(ii) price of each calculator after the discount. (1 mark)

(b) Form an equation in x and hence determine the number of calculators the school bought.

(5 marks)

(c) Calculate the discount offered to the school as a percentage. (3 marks)

20 The position vectors of points A and B with respect to the origin O, are $\begin{pmatrix} -8 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$ respectively. Point M is the midpoint of AB and N is the midpoint of OA.

(a) Find:

(i) the coordinates of N and M; (3 marks)

(ii) the magnitude of NM. (3 marks)

(b) Express vector **NM** in terms of **OB**. (1 mark)

(c) Point P maps onto P' by a translation $\begin{pmatrix} -5 \\ 8 \end{pmatrix}$. Given that

OP = **OM** − 2**MN**, find the coordinates of P'. (3 marks)

21 A glass, in the form of a frustum of a cone, is represented by the diagram below. The glass contains water to a height of 9 cm. The bottom of the glass is a circle of radius 2 cm while the surface of the water is a circle of radius 6 cm.
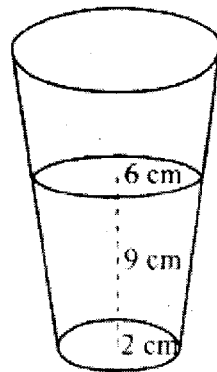
(a) Calculate the volume of the water in the glass. (3 marks)

(b) When a spherical marble is submerged into the water in the glass, the water level rises by 1 cm. Calculate:

(i) the volume of the marble; (4 marks)

(ii) the radius of the marble. (3 marks)

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22   The diagram below shows the speed-time graph for a train travelling between two stations. The train
      starts from rest and accelerates uniformly for 150 seconds. It then travels at a constant speed for 300
   seconds and finally decelerates uniformly for 200 seconds.
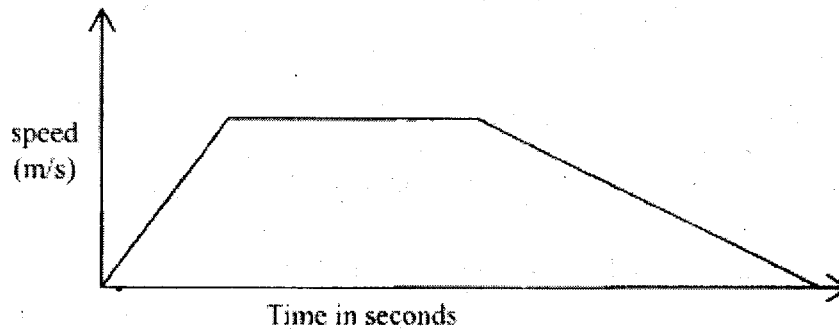
      Given that the distance between the two stations is 10 450 m, calculate the:

      (a) maximum speed, in Km/h, the train attained: (3 marks)

      (b) acceleration: (2 marks)
      (c) distance the train travelled during the last 100 seconds: (2 marks)

      (d) time the train takes to travel the first half of the journey.   (3 marks)

23   Three points P, Q and R are on a level ground. Q is 240 m from P on a bearing of 230°. R is 120 m to
      the east of P.

      (a) Using a scale of 1 cm to represent 40 m, draw a diagram to show the positions of P, Q and R in the
          space provided below.   (2 marks)

      (b) Determine:
          (i) the distance of R from Q;   (2 marks)

          (ii) the bearing of R from Q.   (2 marks)

      (c) A vertical post stands at P and another one at Q. A bird takes 18 seconds to fly directly from the top
          of the post at Q to the top of the post at P.

          Given that the angle of depression of the top of the post at P from the top of the post at Q is 9°,
          calculate:
          (i) the distance, to the nearest metre, the bird covers;   (2 marks)

          (ii) the speed of the bird in Km/h.   (2 marks)

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24   (a) On the grid provided, draw a graph of the function

$y = \frac{1}{2}x^2 - x + 3$ for $0 \le x \le 6$.

(3 marks)
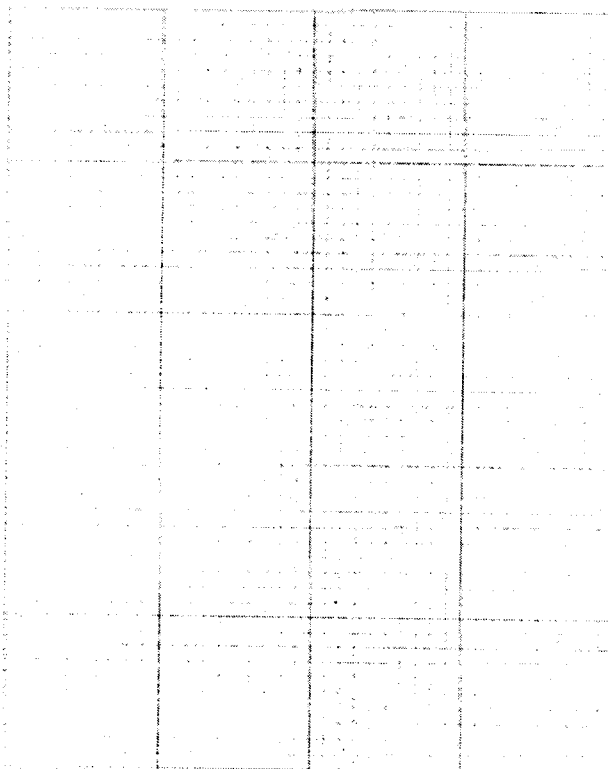
(b) Calculate the mid-ordinates for 5 strips between $x = 1$ and $x = 6$, and hence use the mid-ordinate rule to approximate the area under the curve between $x = 1$, $x = 6$ and the x-axis.   (3 marks)

(c) Assuming that the area determined by intergration to be the actual area, calculate the percentage error in using the mid-ordinate rule.   (4 marks)

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SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

1. A farmer feeds every two cows on 480 Kg of hay for four days. The farmer has 20 160 Kg of hay which is just enough to feed his cows for 6 weeks. Find the number of cows in the farm. (3 marks)

2. Find a quadratic equation whose roots are $1.5 + \sqrt{2}$ and $1.5 - \sqrt{2}$, expressing it in the form $ax^2 + bx + c = 0$, where a, b and c are integers. (3 marks)

3. The mass of a wire m grams(g) is partly a constant and partly varies as the square of its thickness t mm. When $t = 2$ mm, $m = 40$ g and when $t = 3$ mm, $m = 65$ g. Determine the value of m when $t = 4$ mm. (4 marks)

4. In the figure below, O is the centre of the circle and radius ON is perpendicular to the line TS at N.
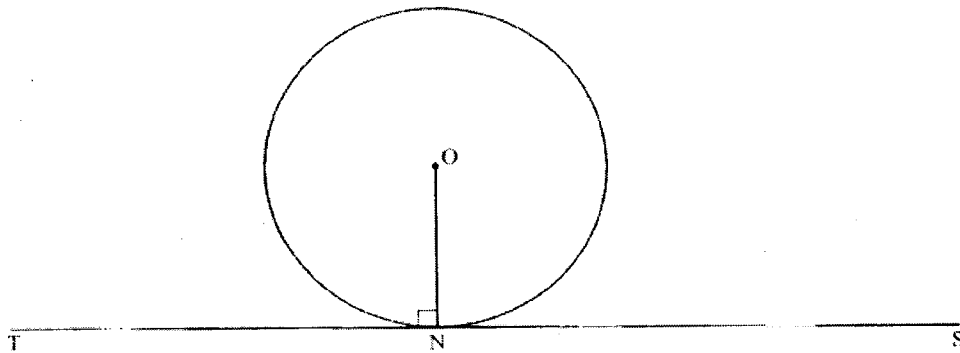

Using a ruler and a pair of compasses only, construct a triangle ABC to inscribe the circle, given that angle ABC = 60°, BC = 12 cm and points B and C are on the line TS. (4 marks)

5. A solution was gently heated, its temperature readings taken at intervals of 1 minute and recorded as shown in the table below.

Time (Min)	0	1	2	3	4	5
Temperature (°C)	4	5.2	8.4	14.3	16.8	17.5

(a) Draw the time-temperature graph on the grid provided. (2 marks)
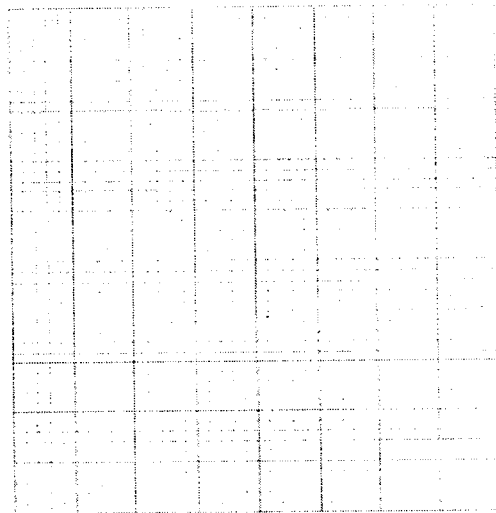
(b) Use the graph to find the average rate of change in temperature between $t = 1.8$ and $t = 3.4$. (2 marks)

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6   Vector $\mathbf{OA} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{OB} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$. Point C is on OB such that CB = 2OC and point D is on AB such that AD = 3DB.

Express **CD** as a column vector.   (3 marks)

7   In a certain commercial bank, customers may withdraw cash through one of the two tellers at the counter. On average, one teller takes 3 minutes while the other teller takes 5 minutes to serve a customer. If the two tellers start to serve the customers at the same time, find the shortest time it takes to serve 200 customers.   (3 marks)

8   (a) Expand and simplify the binomial expression $(2 - x)^7$ in ascending powers of x.   (2 marks)

(b) Use the expansion up to the fourth term to evaluate $(1.97)^7$ correct to 4 decimal places.   (2 marks)

9   The area of triangle FGH is 21 cm². The triangle FGH is transformed using the matrix $\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}$. Calculate the area of the image of triangle FGH.

     (2 marks)

10   Simplify $\dfrac{\sqrt{3}}{\sqrt{3} - \sqrt{2}}$ (2 marks)

11   A circle whose equation is $(x - 1)^2 + (y - k)^2 = 10$ passes through the point $(2, 5)$.

Find the coordinates of the two possible centres of the circle.   (3 marks)

12   On a certain day, the probability that it rains is $\frac{1}{7}$. When it rains the probability that Omondi carries an umbrella is $\frac{2}{3}$. When it does not rain the probability that Omondi carries an umbrella is $\frac{1}{6}$. Find the probability that Omondi carried an umbrella that day.   (2 marks)

13   Point P(40°S, 45°E) and point Q(40°S, 60°W) are on the surface of the Earth.

Calculate the shortest distance along a circle of latitude between the two points.   (3 marks)

14   Solve $4 - 4\cos^2 \alpha - 4\sin \alpha - 1$ for $0° \leq \alpha \leq 360°$.   (4 marks)

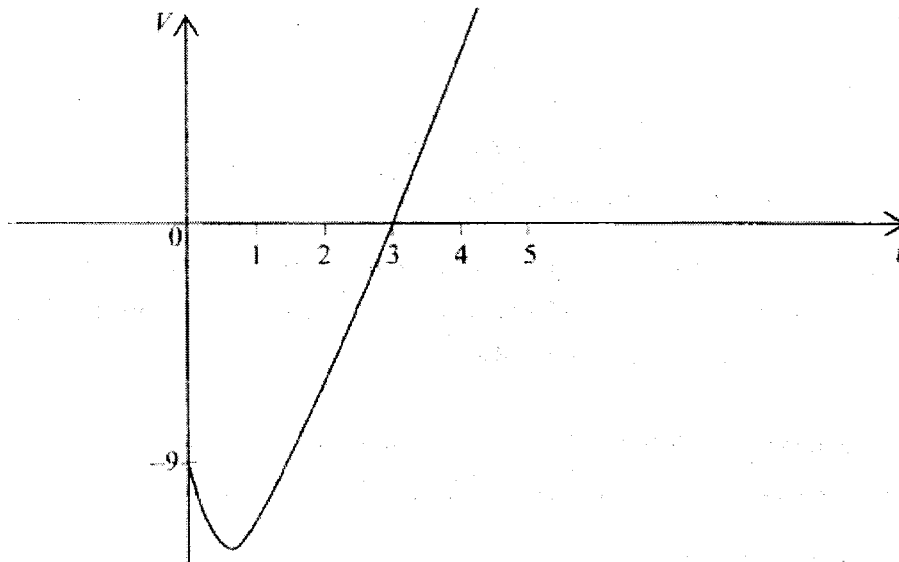15   In the figure below, AT is a tangent to the circle at A. Angle ATB = 48°, BC = 5 cm and CT = 4 cm.
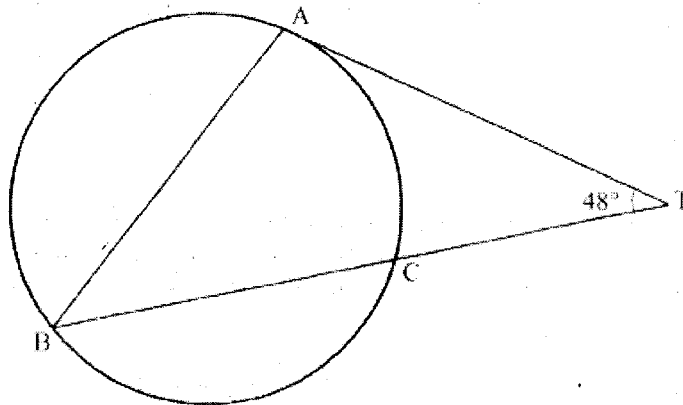
Calculate the length AT.   (2 marks)

16   A particle moves in a straight line with a velocity V ms⁻¹. Its velocity after t seconds is given by $V = 3t^2 - 6t - 9$.

The figure below is a sketch of the velocity-time graph of the particle.

Calculate the distance the particle moves between $t = 1$ and $t = 4$.   (4 marks)

SECTION II (50 marks)

*Answer **only** five questions in this section in the spaces provided.*

17   A water vendor has a tank of capacity 18 900 litres. The tank is being filled with water from two pipes A and B which are closed immediately when the tank is full. Water flows at the rate of 150 000 cm³/minute through pipe A and 120 000 cm³/minute through pipe B.

   (a) If the tank is empty and the two pipes are opened at the same time, calculate the time it takes to fill the tank.   (3 marks)

   (b) On a certain day the vendor opened the two pipes A and B to fill the empty tank. After 25 minutes he opened the outlet tap to supply water to his customers at an average rate of 20 litres per minute.

      (i) Calculate the time it took to fill the tank on that day.   (4 marks)

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(ii) The vendor supplied a total of 542 jerricans, each containing 25 litres of water, on that day. If the water that remained in the tank was 6 300 litres, calculate, in litres, the amount of water that was wasted. **(3 marks)**

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18   At the beginning of the year 1998, Kanyingi bought two houses, one in Thika and the other one in Nairobi, each at Ksh 1 240 000. The value of the house in Thika appreciated at the rate of 12% p.a.

(a) Calculate the value of the house in Thika after 9 years, to the nearest shilling. **(2 marks)**

(b) After n years, the value of the house in Thika was Ksh 2 741 245 while the value of the house in Nairobi was Ksh 2 917 231.

    (i) Find n. **(4 marks)**

    (ii) Find the annual rate of appreciation of the house in Nairobi. **(4 marks)**

19 The table below shows the number of goals scored in handball matches during a tournament.

Number of goals	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
Number of matches	2	14	24	12	8

(a) Draw a cumulative frequency curve on the grid provided.   (5 marks)
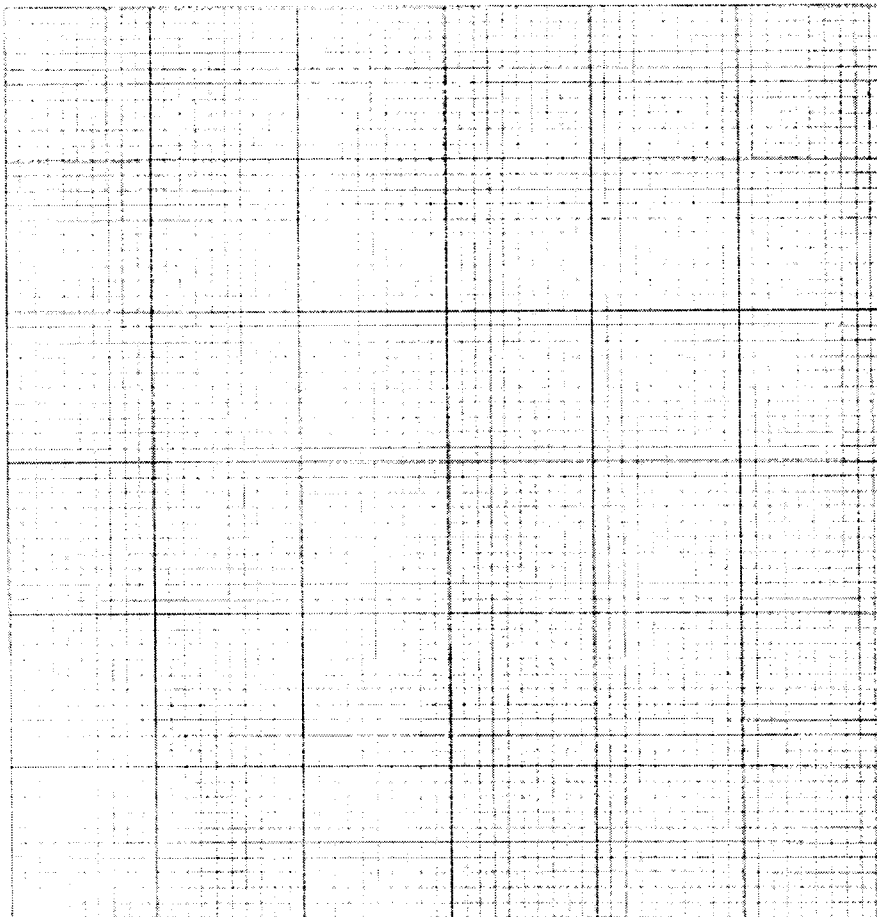
(b) Using the curve drawn in (a) above determine:
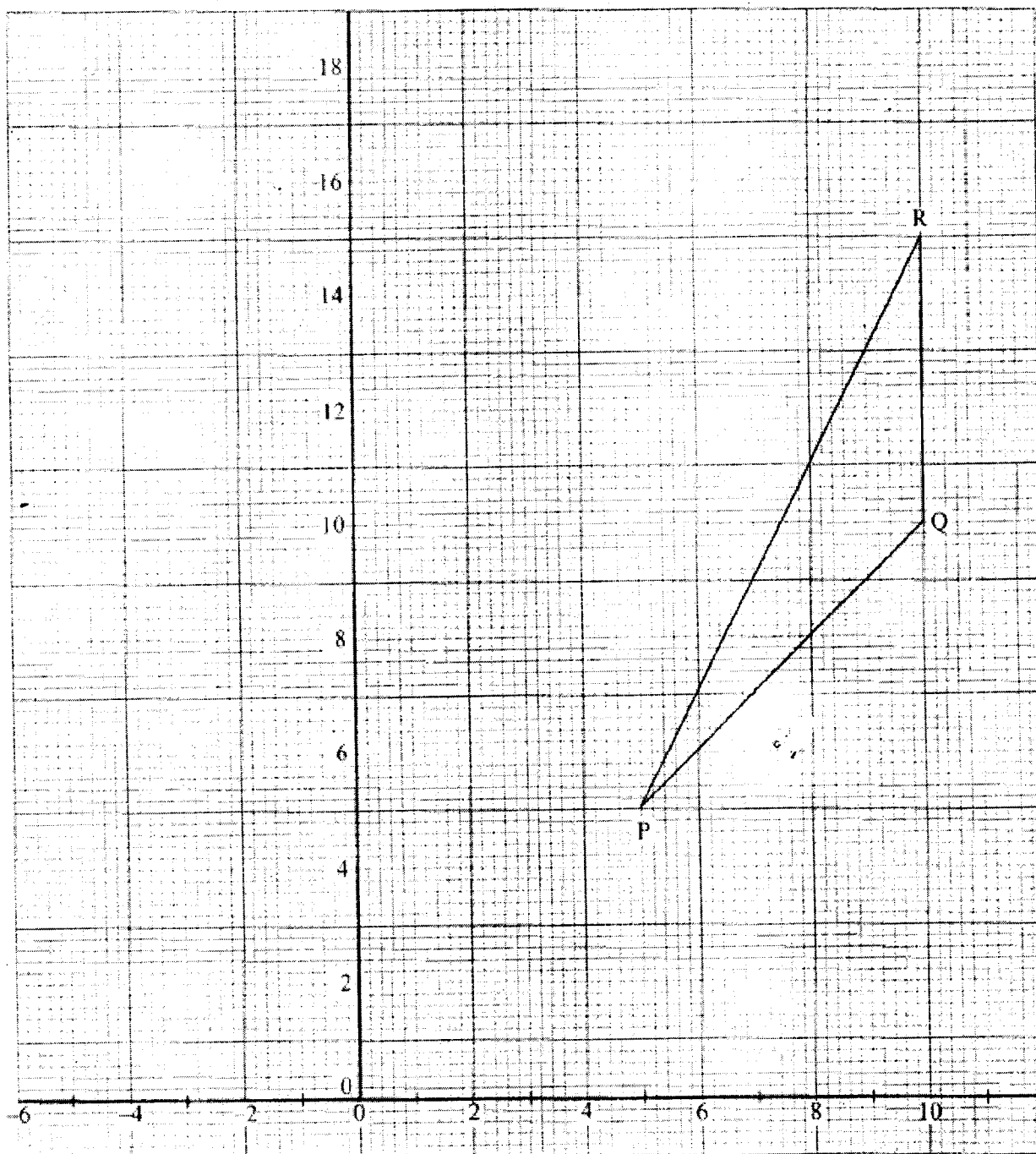   (i) the median;   (1 mark)

   (ii) the number of matches in which goals scored were not more than 37;   (1 mark)

   (iii) the inter-quartile range.   (3 marks)

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20   Triangle PQR shown on the grid has vertices P(5, 5), Q(10, 10) and R(10, 15).

(a) Find the coordinates of the points P′, Q′ and R′, the images of P, Q and R respectively under transformation M whose matrix is

$$\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}$$

(2 marks)

(b) Given that M is a reflection:

    (i) draw triangle P'Q'R' and the mirror line of the reflection;   **(2 marks)**

    (ii) determine the equation of the mirror line of the reflection.   **(1 mark)**

(c) Triangle P"Q"R" is the image of triangle P'Q'R' under reflection N where N is a reflection in the y-axis.

    (i) Draw triangle P"Q"R".   **(1 mark)**

    (ii) Determine a 2 × 2 matrix equivalent to the transformation NM.   **(2 marks)**

    (iii) Describe fully a single transformation that maps triangle PQR onto triangle P"Q"R".

      (2 marks)

21 The table below shows income tax rates.

Monthly income in Kenya shillings (Ksh)	Tax rate percentage (%) in each shilling
Up to 9 680	10
From 9 681 to 18 800	15
From 18 801 to 27 920	20
From 27 921 to 37 040	25
From 37 041 and above	30

In a certain year, Robi's monthly taxable earnings amounted to Ksh 24 200.

(a) Calculate the tax charged on Robi's monthly earnings.   **(4 marks)**

(b) Robi was entitled to the following tax reliefs:

    I: monthly personal relief of Ksh 1 056;

    II: monthly insurance relief at the rate of 15% of the premium paid.

Calculate the tax paid by Robi each month, if she paid a monthly premium of Ksh 2 400 towards her life insurance policy.   **(2 marks)**

(c) During a certain month, Robi received additional earnings which were taxed at 20% in each shilling. Given that she paid 36.3% more tax that month, calculate the percentage increase in her earnings.   **(4 marks)**

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22 The figure below shows a right pyramid mounted onto a cuboid. $AB = BC = 15\sqrt{2}$ cm, $CG = 8$ cm and $VG = 17\sqrt{2}$ cm.
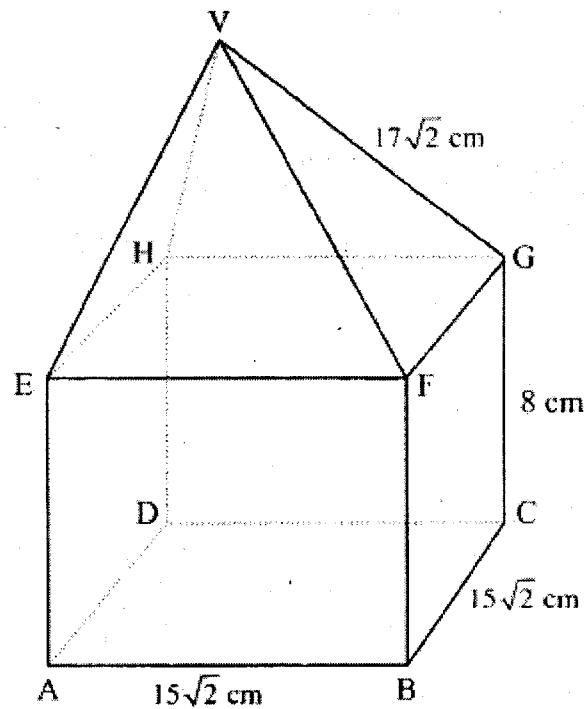

Calculate:

(a) the length of AC; (1 mark)

(b) the angle between the line AG and the plane ABCD; (3 marks)

(c) the vertical height of point V from the plane ABCD; (3 marks)

(d) the angle between the planes EFV and ABCD. (3 marks)

23 (a) The first term of an Arithmetic Progression (AP) is 2. The sum of the first 8 terms of the AP is 156.

(i) Find the common difference of the AP. (2 marks)

(ii) Given that the sum of the first n terms of the AP is 416, find n. (2 marks)

(b) The 3rd, 5th and 8th terms of another AP form the first three terms of a Geometric Progression (GP). If the common difference of the AP is 3, find:

(i) the first term of the GP; (4 marks)

(ii) the sum of the first 9 terms of the GP, to 4 significant figures. (2 marks)

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24 Amina carried out an experiment to determine the average volume of a ball bearing. She started by submerging three ball bearings in water contained in a measuring cylinder. She then added one ball at a time into the cylinder until the balls were nine.

The corresponding readings were recorded as shown in the table below.

Number of ball bearings (x)	3	4	5	6	7	8	9
Measuring cylinder reading (y)	98.0	105.0	123.0	130.5	145.6	156.9	170.0

(a) (i) On the grid provided, plot (x, y) where x is the number of ball bearings and y is the corresponding measuring cylinder reading.   (3 marks)
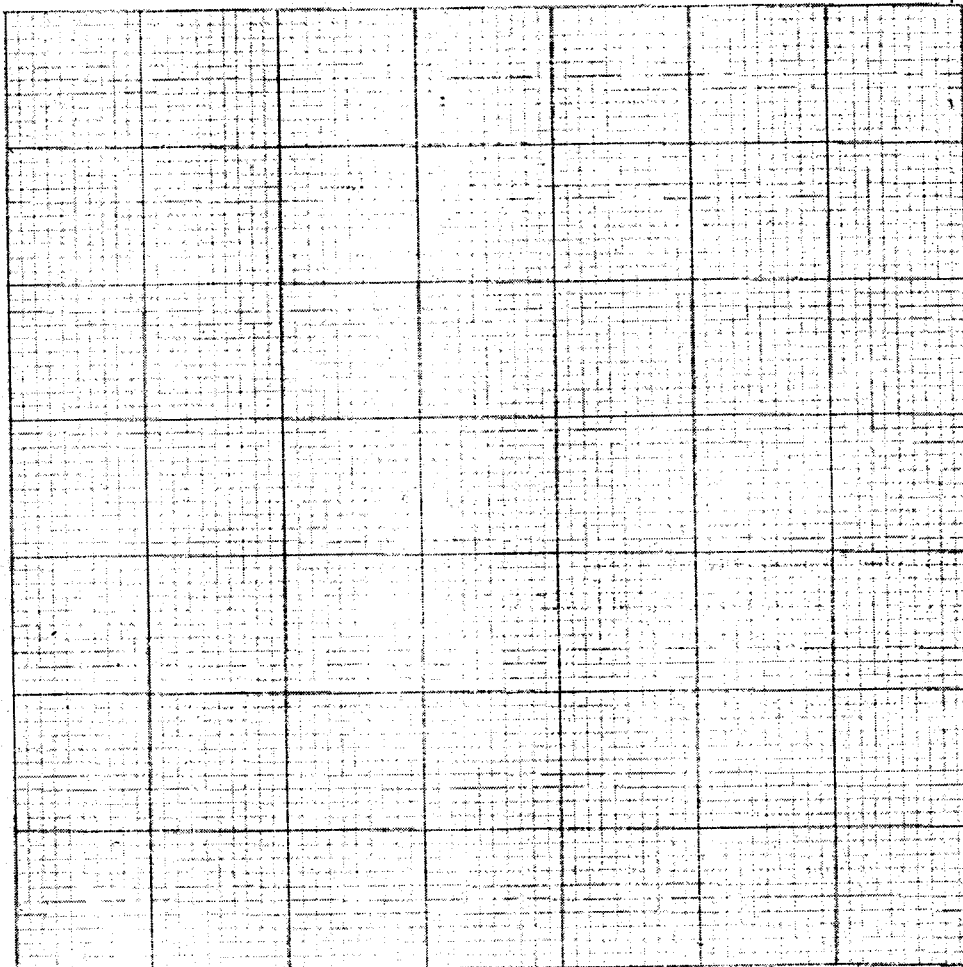
   (ii) Use the plotted points to draw the line of best fit.   (1 mark)

(b) Use the line of best fit to determine:

   (i) the average volume of a ball bearing;   (2 marks)

   (ii) the equation of the line.   (2 marks)

(c) Using the equation of the line in b(ii) above, determine the volume of the water in the cylinder.   — (2 marks)