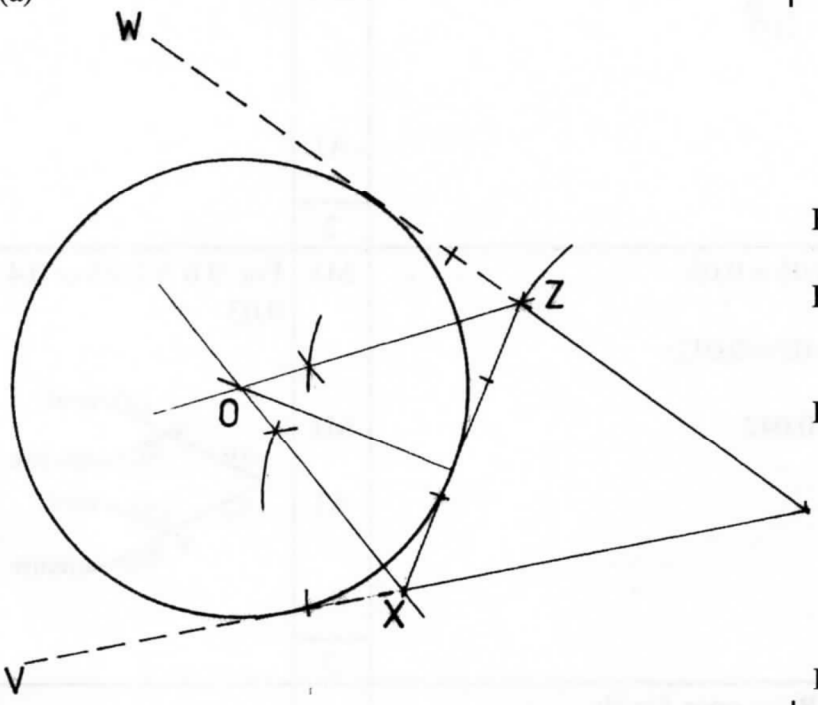
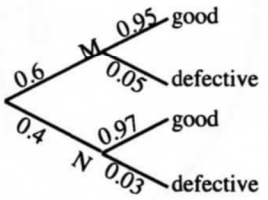
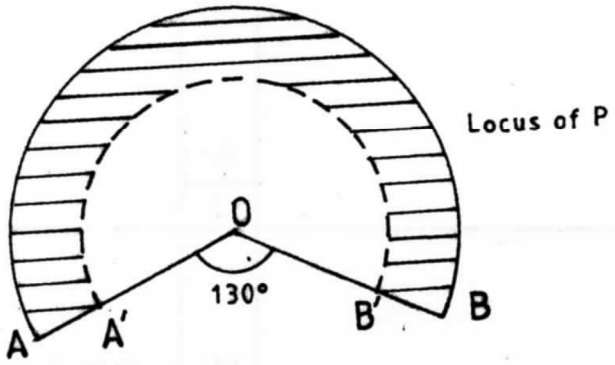
5.1.2 Mathematics Alternative A Paper 2 (121/2)

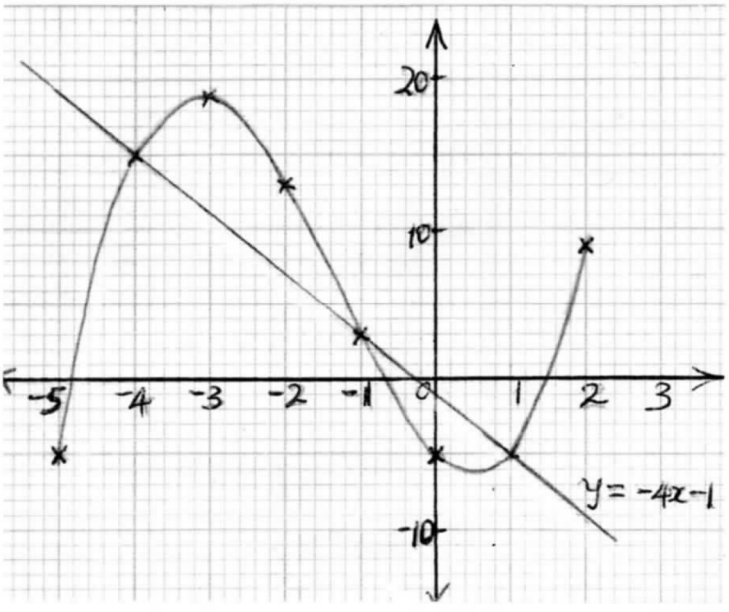
1.	1st term, a = 3; common difference, d = 6 $7500 = \frac{n}{2}\{2 \times 3 + (n-1) \times 6\}$ $3n^2 = 7500$ $n = \sqrt{2500} = 50$	B1 M1 A1 3	
2.	$y = (x + 2)(x - 1)$ $y = x^2 + x - 2$	M1 A1 2	
3.	$P = \frac{1}{2}mn^2 - \frac{qd^2}{n}$ $\frac{qd^2}{n} = \frac{1}{2}mn^2 - P$ $d^2 = \frac{\frac{1}{2}mn^3 - nP}{q}$ $d = \sqrt{\frac{\frac{1}{2}mn^3 - nP}{q}}$	 M1 M1 A1 3	
4.	$Log\left(\frac{x^2}{(x-2)}\right) = \log 3^2$ $\frac{x^2}{x-2} = 9$ $x^2 - 9x + 18 = 0$ $(x - 6)(x - 3) = 0$ $x = 6$ or $x = 3$	M1 M1 A1 3	

5.	(a)  (b) radius = 3.1	B1 B1 B1 B1 4	extending YX and YZ bisecting $\angle s$ VXZ and XZW escribed circle drawn allow ± 0.1
6.	Completing square on L.H.S. $x^2 + 4x + 4 + y^2 - 2y + 1 = 4 + 4 + 1$ $(x + 2)^2 + (y - 1)^2 = 9$ $\therefore$ centre of circle : (-2, 1) radius of circle: 3 units	B1 B1 B1 3	
7.	(a) $(1 - x)^5 = 1 + 5(-x) + 10(-x)^2 + 10(-x)^3 + 5(-x)^4 + (-x)^5$ $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$ (b) $(0.98)^5 = (1 - 0.02)^5 \Rightarrow x = 0.02$ $\therefore (0.98)^5 = 1 - 5(0.02) + 10(0.02)^2 - 10(0.02)^3$ $= 1 - 0.1 + 0.004 - 0.00008$ $= 0.90392$	B1 M1 A1 3	

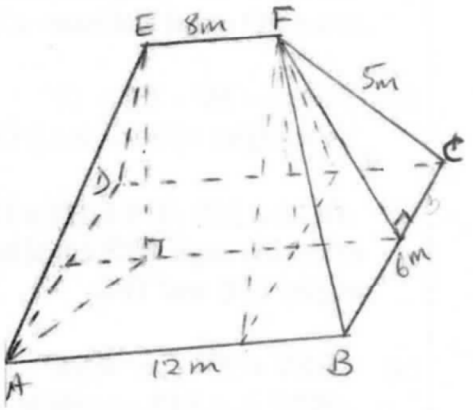
8.	$\underset{\sim}{h} = \frac{-1}{4+(-1)}\underset{\sim}{f} + \frac{4}{4+(-1)}\underset{\sim}{g}$ $= \frac{-1}{3}\underset{\sim}{f} + \frac{4}{3}\underset{\sim}{g}$	M1 A1 2	
9.	P(defective) : M → $0.6 \times 0.05 = 0.03$ N → $0.4 \times 0.03 = 0.012$ P(defective) $0.03 + 0.02 = 0.042$	M1 M1 A1 3	For 0.6×0.05 or 0.4×0.03 
10.	(a) Fraction filled if A and R are open for 5h $5 \times \left(\frac{1}{3} - \frac{1}{6}\right) = \frac{5}{6}$ Fraction of tank still empty = $1 - \frac{5}{6} = \frac{1}{6}$ (b) Fraction filled if A, B and R are open for 1h $\frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$ Time taken to fill the tank = $\frac{1}{6} \div \frac{2}{3} = \frac{1}{6} \times \frac{3}{2}$ $= \frac{1}{4}$h or 15 min	B1 B1 M1 A1 4	
11.	$\frac{\sqrt{48}}{\sqrt{5}+\sqrt{3}} = \frac{4\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$ $= \frac{4\sqrt{3}(\sqrt{5}-\sqrt{3})}{5-3}$ $= 2\sqrt{3}(\sqrt{5}-\sqrt{3})$ $= 2\sqrt{15} - 6$	M1 M1 A1 3	

12.	 $\angle AOB = 130°$ arc AB - solid curve arc A′B′ - broken curve region shown	B1 B1 B1 B1 4	
13.	$9680 \times 0.1 = 968$ 9120×0.15; 9120×0.2; 4580×0.25 $= 1368$ $= 1824$ $= 1145$ Net tax $= (968 + 1368 + 1824 + 1145) - 1056$ $= 4249$	M1 M1 M1 A1 4	
14.	$6(1 - \sin^2 x) + 7 \sin x - 8 = 0$ $6 - 6 \sin^2 x + 7 \sin x - 8 = 0$ $6 \sin^2 x - 7 \sin x + 2 = 0$ $(3 \sin x - 2)(2 \sin x - 1) = 0$ $\sin x = \frac{2}{3}$ *or* $\sin x = \frac{1}{2}$ $x = 41.81°$ or $x = 30°$	M1 M1 M1 A1 4	

15.	Distance between towns K and S $= 2\pi \times 6370 \cos 2° \times \frac{37.4 - 30}{360}$ $= 822.2121281$ $= 822$ km	M1 A1 2	
16.	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 4 & 3 \\ 2 & 2 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 2 & \frac{3}{2} \\ 1 & 1 & 2 \end{pmatrix}$ $a + 2b = \frac{1}{2}$ $4a + 2b = 2$ $3a = \frac{3}{2} \Rightarrow a = \frac{1}{2}$ $\frac{1}{2} + 2b = \frac{1}{2} \Rightarrow b = 0$ $c + 2d = 1$ $4c + 2d = 1$ $3c = 0 \Rightarrow c = 0$ $0 + 2d = 1 \Rightarrow d = \frac{1}{2}$ $\therefore \quad M = \begin{pmatrix} ½ & 0 \\ 0 & ½ \end{pmatrix}$	M1 M1 A1 3	√ formation and solution of simultaneous equations √ formation and solution of simultaneous equations
17.	(a) (i) $\frac{276000 - 60000}{18}$ $= 12\,000$ (ii) 276000×0.9 $= 248400$ (b) 248400×0.95 $= 235980$ 235980×1.2^2 $= 339811.2$ (c) $339811.2 - 276000$ $\frac{63811.2}{276000} \times 100$ $= 23.12\ \%$	M1 A1 M1 A1 M1 M1 A1 M1 M1 A1 10	

18.	(a) $\angle QPR = 90° - 72° = 18°$ $\angle PQR = 90°$ - angle subtended by diameter (b) $\angle PQS = 180° - 2(72) = 36°$ $\angle PSQ = 72°$ - angle subtended at the circumference by chord PQ equal and base $\angle$'s of isosceles $\Delta QPS = 72°$ (c) $\angle OQS = 36° - 18° = 18°$ base angles of isosceles Δ OPQ $= 18°$ (d) $\angle RTS = 180 - (36 + 18) = 126°$ extension angle RTS equal to sum of opposite interior angles TSP and TPS (e) $\angle RSV = 90° - 36° = 54°$ $\angle RSV = \angle RPS$ - angle in alternate segment.	B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 10	 or equivalent
19.	(a) (b)  (c) (i) x = -4.8, -0.7, 1.5 (ii) y = -4x - 1 Solutions x = -4, -1, 1.	B2 S1 P1 C1 B2 P1 L1 B1 10	allow B1 for 4 correct Suitable scale All correctly plotted ±0.1 allow B1 for 2 values √ plotting for line

x	-5	-4	-3	-2	-1	0	1	2
y $=x^3+4x^2-5x-5$	-5	15		13	3		-5	9

20.	(a) = distance of EF from place ABCD slant height from F to BC $= \sqrt{5^2 - 3^2}$ $= 4$ $\therefore$ = distance of EF from plane ABCD $= \sqrt{4^2 - 2^2}$ $= \sqrt{12} = 3.46$ m	M1 M1 A1	
	(b) (i) angle between planes ADE and ABCD $= \tan^{-1}\frac{\sqrt{12}}{2}$ $= 60°$	M1 A1	or equivalent
	(ii) angle between line AE and plane ABCD $= \sin^{-1}\frac{\sqrt{12}}{5}$ $= 43.9°$	M1 A1	or equivalent
	(iii) angle between planes ABFE and DCFE $= 2c\tan^{-1}\frac{3}{\sqrt{12}}$ m $= 81.8°$	M1 M1 A1	$\tan^{-1}\frac{3}{\sqrt{12}}$ or equivalent doubling
		10	

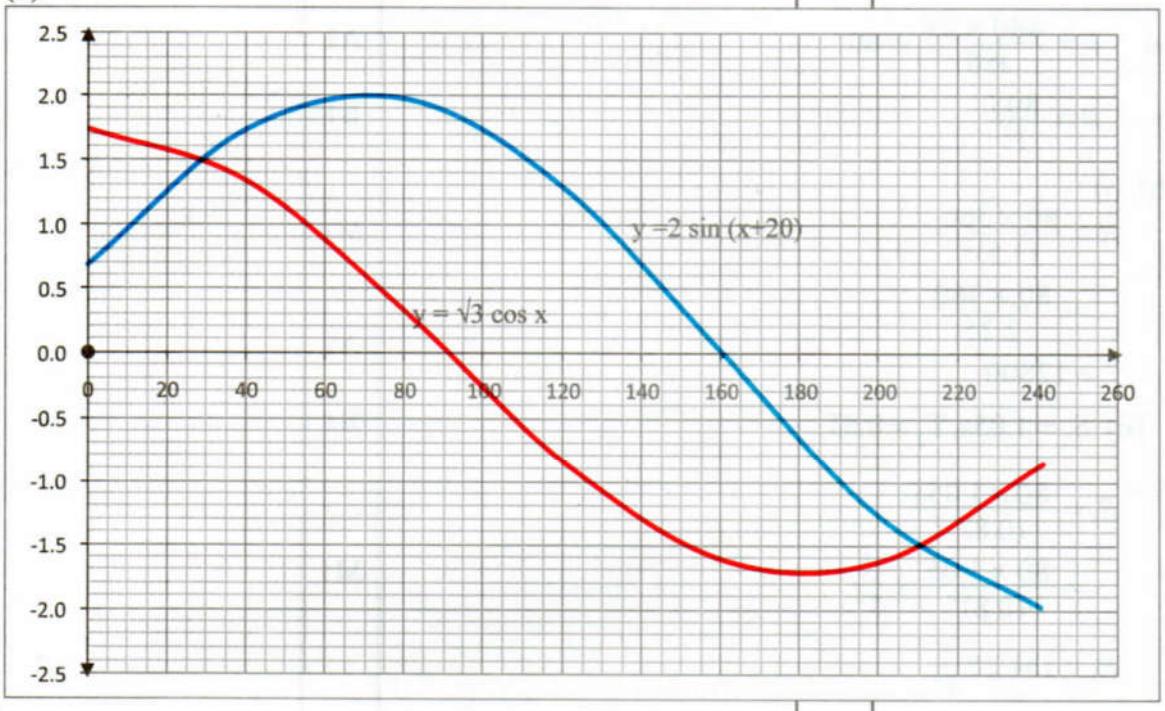
21. (a)

x	0	40	80	120	160	200	240
$y = 2 \sin x + 20$		1.7		1.3		-1.3	
$y = \sqrt{3} \cos x$			0.3		-1.6		-0.9

B1

B1

(b)



(c) (i) $2 \sin (x + 20) = \sqrt{3} \cos x$

$x = 30°$

and $x = 210°$

S1 suitable scale used
P1 plotting 2 Sin (x + 20)
P1 plotting $\sqrt{3}$ cos x
C1 curve for 2 sin x + 20
C1 curve for $\sqrt{3}$ cos x
B1
B1

(ii) amplitude difference

$2 - 1.7 = 0.3$

B1

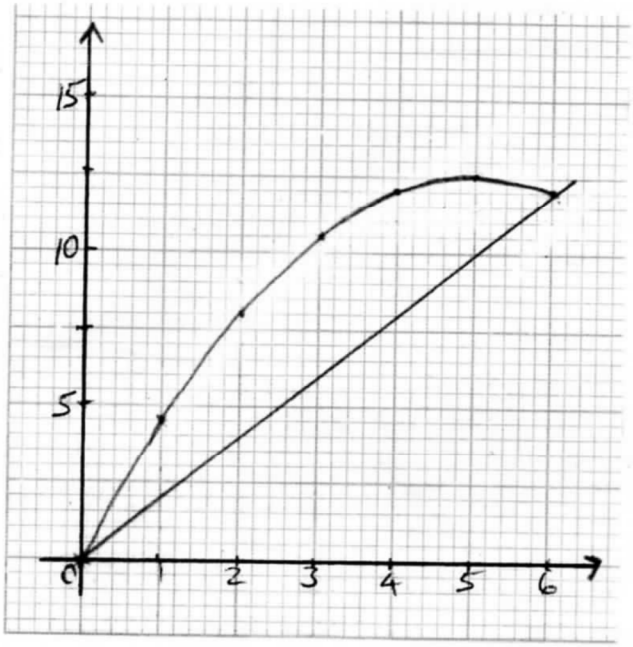
10

22.	(a) $R \propto \frac{S}{T^2} \Rightarrow R = \frac{kS}{T^2}$	B1	
	R = 480 whe n S = 150 and T = 5		
	$\Rightarrow 480 = \frac{k \times 150}{5^2}$	M1	
	$= \frac{150k}{25}$		
	$\Rightarrow \quad k = \frac{480 \times 25}{150} = 80$	A1	
	$\therefore \quad R = \frac{80S}{T^2}$	B1	
	(b) (i) $R = \frac{80 \times 360}{(1.5)^2}$	M1	
	$= \frac{80 \times 360}{2.25}$		
	$= 12800$	A1	
	(ii) $S_2 = 1.05s, T_2 = 0.8T$	B1	
	$R_2 = \frac{80 \times 1.05S}{(0.8T)^2}$		
	$= \frac{80 \times 1.05}{(0.8)^2} \times \frac{S}{T^2}$	M1	
	$R_2 = 131.25\frac{S}{T^2}$		
	$\left(\frac{R_2 - R}{R}\right) \times 100\% = \left(\frac{131.25\frac{S}{T^2} - \frac{80S}{T^2}}{80\frac{S}{T^2}}\right) \times 100\%$	M1	
	$= \frac{S/T^2}{S/T^2}\left(\frac{131.25 - 80}{80}\right) \times 100$		
	$= 64.0625$		
	$= 64.06\ \%$	A1	
		10	

23. (a)

x	0	1	2	3	4	5	6
y $= 5x-\frac{1}{2}x^2$	0	4.5	8	10.5	12	12.5	12

B1 table may be implied



P1 √ plotting

C1 √ curve

(b)

$$\int_0^6 \left(5x - \frac{1}{2}x^2\right) dx$$

$$= \left[\frac{5}{2}x^2 - \frac{1}{2\times 3}x^3\right]_0^6$$

M1 √ integral

$$= \left[\frac{5\times 6^2}{2} - \frac{1}{6}\times 6^3\right] - [0-0]$$

M1 √ substitution

$$= [90-36] - [0] = 54$$

A1

(c) (i) Drawing line y = 2x

L1

(ii) Area of Δ : $\frac{1}{2}\times 6\times 12$

$= 36$

M1
A1

$\therefore$ Bounded area = 54 - 36 = 18

B1

10

24. (a)

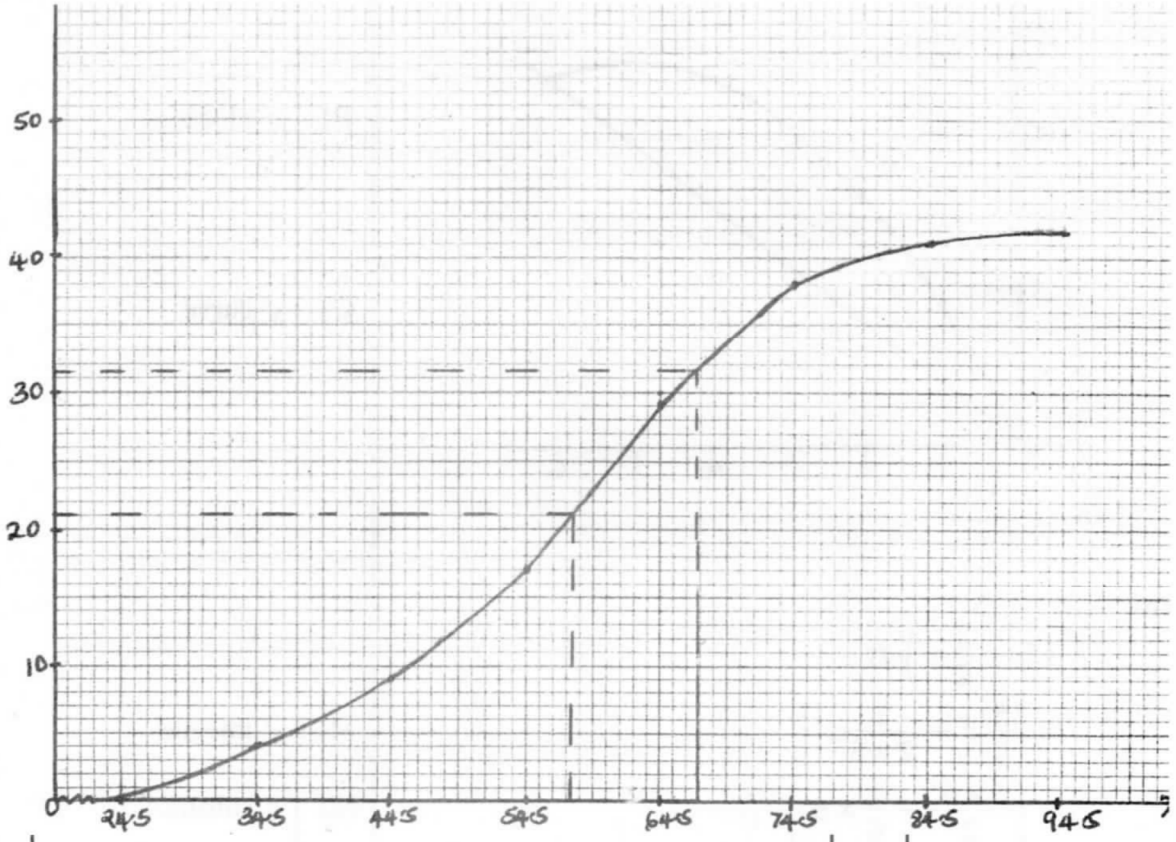
Marks	Frequency	cf	
25-34	4	4	
35-44	5	9	
45-54	8	17	
55-64	12	29	
65-74	9	38	
75-84	3	41	
85-94	1	42	

B1 √ marks class column

B1 √ frequency column

(b) (i) cfs

B1



S1 √ scale
P1 √ plotting
C1 √ curve

(c) (i) Identification of median
= 57.5 ± 0.5

B1
B1

(ii) Identification of upper quartile mark
= 66.5 ± 0.5

B1
B1

10