NAME: Marking Scheme ..... INDEX NO: .....

CANDIDATE'S SIGNATURE: .....

DATE: .....

121/2
MATHEMATICS
JULY 2019
PAPER 2
TIME: 2 ½ HOURS

END OF TERM 2 2019 EVALUATION

INSTRUCTIONS TO CANDIDATES:

(a) Write your name and index number in the spaces provided above
(b) Sign and write the date of examination in the spaces provided above.
(c) This paper consists of ***TWO*** sections: ***Section I*** and ***Section II.***
(d) Answer ***ALL*** the questions in ***section I*** and only five from ***Section II***
(e) All answers and working must be written on the question paper in the spaces provided below each question.
(f) ***Show all the steps in your calculations, giving your answers at each stage in the spaces below each question.***
(g) Marks may be given for correct working even if the answer is wrong.
(h) ***Non-programmable*** silent electronic calculators and KNEC Mathematical tables may be used except where stated otherwise.

FOR EXAMINER'S USE ONLY

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total

This paper consists of 14 printed pages.
Candidates must check to ascertain that all pages are printed as indicated and that no question(s) is/are missing.

SECTION I (50 MARKS)

1. Evaluate without using Mathematical tables or a calculator. (3mks)

$2\log 5 - \frac{1}{2}\log 16 + 2\log 40$

$\log 5^2 - \log 16^{\frac{1}{2}} + \log 40^2$ ✓M1

$\log \frac{25 \times 1600}{4}$ ✓M1

$\log 10,000$

$\log 10^4$

$4 \log 10$

$= 4$ ✓A1

2. Solve for x given that the following is a singular matrix $\begin{pmatrix} 1 & 2 \\ x & x-3 \end{pmatrix}$ (2mks)

$(x-3) - 2x = 0$ ✓M1

$x - 3 - 2x = 0$

$-x - 3 = 0$

$-x = 3$

$x = -3$ ✓A1

3. Make b the subject of the formula $a = \frac{bd}{\sqrt{b^2 - d}}$ (3 mks)

$a^2 = \frac{b^2 d^2}{b^2 - d}$ ✓M1

$a^2(b^2 - d) = b^2 d^2$

$a^2 b^2 - a^2 d = b^2 d^2$

$a^2 b^2 - b^2 d^2 = a^2 d$ ✓M1

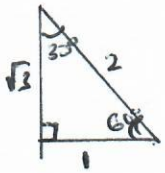
$b^2(a^2 - d^2) = a^2 d$

$b^2 = \frac{a^2 d}{a^2 - d^2}$

$b = \pm\sqrt{\frac{a^2 d}{a^2 - d^2}}$ A1

4. Without using mathematical tables or calculators express in surd form and simplify

$\frac{1 + Cos\,30^\circ}{1 - Sin\,60^\circ}$ (3 mks)



$\frac{2+\sqrt{3}}{2}$

$\frac{2-\sqrt{3}}{2}$

$Cos\,30 = \sqrt{3}/2$

$Sin\,60 = \sqrt{3}/2$

$\frac{1 + \sqrt{3}/2}{1 - \sqrt{3}/2}$ ✓M1

$\frac{2+\sqrt{3}}{2} \div \frac{2-\sqrt{3}}{2}$

$\frac{2+\sqrt{3}}{2} \times \frac{2}{2-\sqrt{3}}$ (2)

$\frac{2+\sqrt{3}}{2-\sqrt{3}} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$ =

$\frac{4 + 2\sqrt{3} + 2\sqrt{3} + 3}{4 - 3}$ ✓M1

$7 + 4\sqrt{3}$ A1

5. Agotho has a rectangular plot that was measured to the nearest meter and found to be 80m in length and 60m in width. Determine the percentage error in its perimeter. (3 marks)

Length 80 ± 0.5

width 60 ± 0.5

w. value $= (80+60)2$

$= 280$m.

max $P = 2(80.5 + 60.5)$

$= 282$m.

min $P = 2(79.5 + 59.5)$

$= 278$m

abs error $= \frac{282-278}{2}$ ✓ M_1

$= 2$

% error $= \frac{2}{280} \times 100$ ✓ M_1 $= 0.7143\%$ ✓ A_1

6. Peter operates a printing firm and the cost of printing a book is partly constant and partly varies as the number as pages. If a book has 200 pages, the cost in sh 400 and if it has 100 pages, the cost is sh 240. Find the cost of printing a book with 400 pages. (4 mks)

$C = K + CN$

$400 = K + 200C$ ✓ M_1

$240 = K + 100C$

$\frac{160}{100} = \frac{100C}{100}$

$C = 1.6$

$400 = K + 200C$

$K = 400 - 320$

$K = 80$ ✓ A_1 for both values of K and C.

$C = 80 + 1.6N$

$C = 80 + 1.6 \times 400$ ✓ M_1

$=$ Shs 720 ✓ A_1

7. A body starts from rest and after t seconds its velocity in ~~ms-1~~ ms^{-1} was recorded as shown below;

T in (sec)	0	1	2	3	4	5	6
Velocity	0	0.29	5.4	7.7	9.7	11.4	12.7

Use the trapezoidal rule to estimate the distance covered by the body between 1 and 6 seconds (2 mks)

$A = \frac{1}{2} h$(sum of ends + 2(sum of middles)

$= \frac{1}{2} \times 1 [(0.29 + 12.7) + 2(5.4 + 7.7 + 9.7 + 11.4)]$ ✓ M_1

$= \frac{1}{2} (12.99 + 2(34.2)]$

$= 40.695$ m ✓ A_1

8. 14 people can build 10 huts in 30 days. Find the number of people working at the same rate that will build 18 similar huts in 27 days. (3mks)

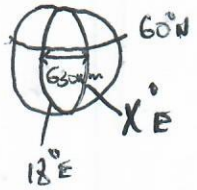
P	H	D
14	10	30
	18	27

$\frac{10}{18} \times \frac{30}{27} \times 14$ ✓ $M_1 M_1$

$= 28$ people ✓ A_1

9. A point M (60 °N, 18 °E) is on the surface of the earth. Another point N is situated at a distance of 630 nautical miles east of M.
Find:

(a) the longitude difference between M and N; (2 marks)



$630 = 60 \times \cos\theta$ ✓

$\frac{630}{60 \cos 60} = \frac{60 \times \cos 60}{60 \cos 60}$

$M_1 \quad x = 21^\circ$ ✓ A_1

(b) The position of N. (1 mark)

$X - 18 = 21$

$X = 39^\circ$

$N\ (60^\circ N, 39^\circ E)$ B_1

10. (a) Expand $(x - 0.2)^5$ in ascending powers of x. (2mks)

$1 \cdot (x)^5(-0.2)^0 + 5 \cdot x^4(-0.2)^1 + 10 \cdot x^3(-0.2)^2 + 10 \cdot x^2(-0.2)^3 + 5 \cdot x^1(-0.2)^4 + 1 \cdot x^0(-0.2)^5$ ✓ M_1

$x^5 - x^4 + 0.4x^3 - 0.08x^2 + 0.008x - 0.00032$

$x^5 - x^4 + 0.4x^3 - 0.08x^2 + 0.008x - 0.00032$ ✓ A_1

(b) Use your expansion up to the fourth term to evaluate 9.8^5. (2mks)

$x - 0.2 = 9.8$

$x = 9.8 + 0.2$

$= 10.0$

$10^5 - 10^4 + 0.4(10)^3 - 0.08(10)^2$ ✓ M_1

$90,392$ ✓ A_1

11. The equation of a circle center (a, b) is $x^2 - y^2 - 6x - 10y + 30 = 0$.
Find the values of a and b. (3 mks)

$x^2 - 6x + (-3)^2 + y^2 - 10y + (-5)^2 = -30 + 25 + 9$ ✓M1

$(x-3)^2 + (y-5)^2 = 4$

Centre $(3, 5)$ ✓A1

$a = 3$ ✓B1

$b = 5$.

12. Solve for x in the equation $\sqrt{3} \tan (x - 20)^0 = -1$, for $0^0 \leq x \leq 360^0$ (3mks)

$\tan (x-20) = -\frac{1}{\sqrt{3}}$ ✓M1

$\tan (x-20) = \tan 150, \tan 330$ ✓M1

$x - 20 = 150$

$x = 170^\circ$

$x - 20 = 330$

$x = 350^\circ$ ✓A1 for both 170° and 350°.

13. Find the equation of the ~~normal~~ tangent to the curve $2x^2 - 8y = 0$ at the point (12,18). (3mks)

$\frac{dy}{dx} = \frac{4x}{}$

$\frac{8y}{8} = \frac{2x^2}{8}$

$y = \frac{1}{4}x^2$

$\frac{dy}{dx} = \frac{1}{2}x$ ✓M1

gradient $= \frac{1}{2} \times 12 = 6$

$\frac{y-18}{x-12} = 6$ ✓M1

$y - 18 = 6x - 72$

$y = 6x - 54$. ✓A1

14. Transformations M and N are represented by the matrices; $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 \\ 1 & 3 \end{pmatrix}$ respectively. Point R has co-ordinates (3, -2), find the co-ordinates of R^1 the image of R under transformation represented by N followed by M. (3 mks)

N followed by M $\Rightarrow$ MN

$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = R^1$ ✓M1

$\begin{pmatrix} 6 & 0 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 18 \\ 6-12 \end{pmatrix}$ ✓M1

$R^1 (18, -6)$ ✓A1

15. A coffee dealer mixes two brands of coffee, x and y to obtain 40kg of the mixture worth Ksh. 2,600. If brand x is valued at Ksh. 70 per kg and brand y is valued at Ksh. 55 per kg. Calculate the ratio in its simplest form in which brands x and y are mixed. (4mks)

Cost per kg $\frac{2600}{40}$

$= 65$

$\frac{70x}{x+y} + \frac{55y}{x+y} = 65$

$70x + 55y = 65x + 65y$

$70x - 65x = 65y - 55y$

$\frac{5x}{5y} = \frac{10y}{5y}$

$\frac{x}{y} = 2$

$x : y = 2 : 1$

ALTERNATIVELY

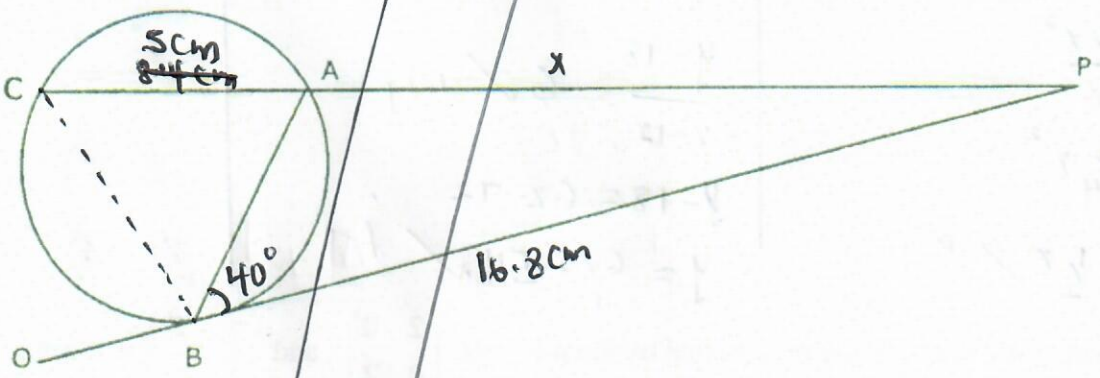
70 — 55

65

10 : 5

2 : 1

16. In the figure below ~~PQ~~ is a tangent to the circle at B. CA produced meets ~~QP~~ OP at P. if AC=8.4cm, PB=16.8cm and angle ABP=40^0 find:



i) Length AP (2 marks)

$BP^2 = CP \cdot AP$

$16.8^2 = x(8.4+x)$

$282.24 = 8.4x + x^2$

ii) Angle ~~ACB~~ ACP (2 marks)

15. A coffee dealer mixes two brands of coffee, x and y to obtain 40kg of the mixture worth Ksh. 2,600. If brand x is valued at Ksh. 70 per kg and brand y is valued at Ksh. 55 per kg. Calculate the ratio in its simplest form in which brands x and y are mixed. (4mks)

Cost per kg $= \frac{2600}{40}$

$= 65$ B1

$\frac{70x}{x+y} + \frac{55y}{x+y} = 65$ ✓M1

$70x - 55y = 65x + 65y$

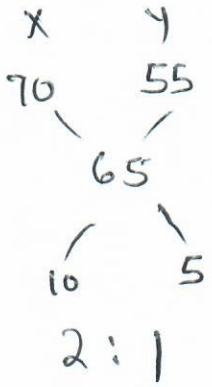
$70x - 65x = 65y - 55y$ ✓M1

$\frac{5x}{5y} = \frac{10y}{5y}$

$\frac{x}{y} = 2$

$x : y = 2 : 1$ A1

ALTERNATIVELY



2 : 1

16. Given that $y = 3 \sin(\frac{2}{5}x + 30)^\circ$ for $0^\circ \leq x \leq 360^\circ$. Determine:

a) Amplitude of the curve. (1 mk)

Amplitude = 3 B1

b) Phase angle of the curve (1 mk)

Phase angle $= 30^\circ$ B1

c) Period of the curve. (2 mks)

Period $= \frac{360}{2/5}$

$= 360 \times \frac{5}{2}$ ✓M1

$= 900^\circ$ ✓A1

SECTION II (50 MARKS)

Answer only five questions in this section

17. (a Hellen's earnings are as follows:
Basic salary sh. 38000 per month
House allowance sh. 14000 per months
Travelling allowance sh.8500 per month and
Medical allowance Ksh.3300 per month.
She is given a personal relief of Ksh. 12672 P.9

The table for payable tax is shown below

Income in K£ p.a	Payable tax rate in Kshs per K£
0-6000	2
6001-12000	3
12001-18000	4
18001 -24000	5
24001-30000	6
30001-36000	7
36001-42000	8
42001-48000	9
Over 48000	10

Calculate

(i) Hellen's taxable income in K£ p.a (2mks)

$\left(\frac{38000+14000+8500+3300}{20}\right) \times 12 = 38,280$ K£ P.9. ✓M1 ✓A1

(ii) Her P.A.Y.E (5mks)

6000 X2 = 12000
6000 x3 = 18000 } M1
6000 x4 = 24000
6000 x 5 = 30000 } M1
6000 x 6 = 36000
6000 x 7 = 42000
2280 x 8 = 18240 ✓M1

Gross tax P.9 = 180,240 ✓M1
Less relief = 12672

P.A.Y.E = ~~22176 P.9~~
167,568 P.9
= ~~2348 P.m~~
= 13,964 P.m A1

Hellen is deducted the following items per month

NHIF	Ksh.320
Cooperative shares	Ksh.2000
Loan repayment	Ksh5000

allow for any either per annum or per month.

Determine her net salary per month (3mks)

total ded = (13968 + 320 + 2000 + 5000) ✓M1
= Kshs 21,284

Net Salary = 63800 - 21284 ✓M1
= Kshs 42,516 ✓A1

18. The data below represent the heights taken to the nearest centimeters of 40 lemon trees in a garden. (NB: A = Assumed mean)

Height (cm)	f	x	d = x - A	fd	d^2	fd^2
131 – 140	3	135.5	-30	-90	900	2700
141 – 150	4	145.5	-20	-80	400	1600
151 – 160	7	155.5	-10	-70	100	700
161 – 170	11	165.5	0	0	0	0
171 – 180	9	175.5	10	90	100	900
181 – 190	5	185.5	20	100	400	2000
191 – 200	1	195.5	30	30	900	900
				$\Sigma fd = -20$		$\Sigma fd^2 = 8800$

a) Complete the table. (6 mks)

b) Using 165.5 as the assumed mean, calculate the mean height. (2 mks)

$$\bar{x} = \frac{\Sigma fd}{\Sigma f} + A$$

$$= \frac{-20}{40} + 165.5 \quad M1$$

$$= 165 \quad A1$$

c) Calculate the standard deviation of the distribution. (2 mks)

$$S.d = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{8800}{40} - \left(\frac{-20}{40}\right)^2} \quad M1$$

$$= \sqrt{220 - 0.25}$$

$$\sqrt{219.75}$$

$$= 14.8240 \quad A1$$

19. An arithmetic progression (AP) has the first term a and the common difference d.

(a) Write down the third, ninth and twenty fifth terms of the AP in terms of a and d. (1mk)

$T_3 = a+2d$ $T_{25} = a+24d$.

$T_9 = a+8d$

B1 for all Correct.

(b) The AP above is increasing and the third, ninth and twenty fifth terms form the first three consecutive terms of a Geometric Progression (G.P) The sum of the seventh and twice the sixth terms of the AP is 78. Calculate:-

(i) the first term and common difference of the AP. (5mks)

$a+2d,\ a+8d,\ a+24d$

$\frac{a+8d}{a+2d} = \frac{a+24d}{a+8d}$ M1

$(a+8d)(a+8d) = (a+2d)(a+24d)$

$a^2 + 16ad + 64d^2 = a^2 + 26ad + 48d^2$

$64d^2 - 48d^2 = 26ad - 16ad$ M1

$\frac{16d^2}{2d} = \frac{10ad}{2d}$

$8d = 5a \Rightarrow 16d = 10a$

$a+6d + 2(a+5d) = 78$ ✓

$a+6d+2a+10d = 78$

$3a+16d = 78$

$3a+10a = 78$

$13a = 78$

$a = 6$ A1

M1 $8d = 5 \times 6$

$8d = 30$

$d = 3.75$ B1

(ii) the sum of the first nine terms of the AP. (2mks)

$S_9 = \frac{9}{2}(2a+(n-1)d)$

$= \frac{9}{2}(2\times 6 + (8)3.75)$ ✓ M1

$= \frac{9}{2}(12+30)$

$= 189.$ ✓ A1

(iii) The difference between the fourth and the seventh terms of an increasing AP. (2mks)

$T_4 = a+3d$

$= 6 + 3\times 3.75$

$= 17.25$

$T_7 = a+6d$

$= 6+22.5$

$= 28.5$

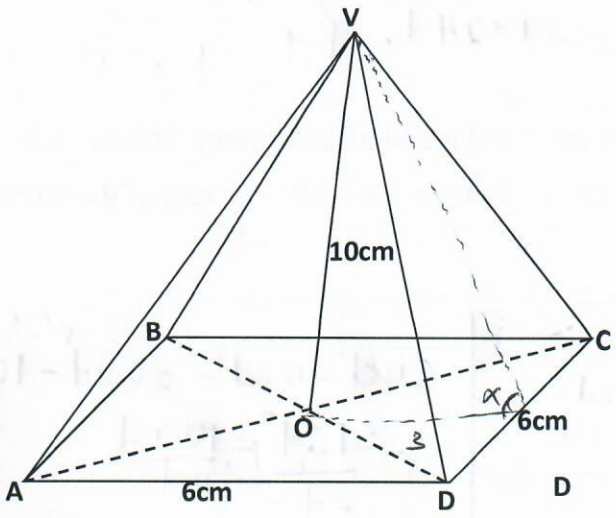
B1 for both T_4 and T_7 Correct

diff

$28.5 - 17.25$

$= 11.25$ ✓ B1

20. The figure below is a square based pyramid ABCDV with AD = DC = 6cm, and height VO = 10 cm.



(a) State the projection of VA on the base ABCD. (1 mk)

AO B1

(b) Find

(i) The length of VA (3 mks)

$AC = \sqrt{6^2+6^2}$

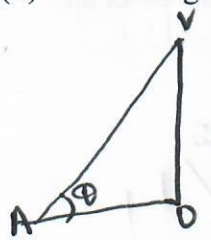
$= 8.4853$

$AO = 4.2426$ B1

$VA = \sqrt{4.2426^2+10^2}$ M1

$= 10.8628\,cm$ A1

(ii) The angle between VA and ABCD (2 mks)



$\sin\theta = \frac{VO}{VA}$

$= \frac{10}{10.8628}$ M1

$\theta = 67.01^\circ$ A1

(iii) The angle between the planes VDC and ABCD (2 mks)

$\tan x = \frac{10}{3}$ M1

$x = 73.30^\circ$ A1

(iv) Volume of the pyramid M1 (2 mks)

$\frac{1}{3} \times 36 \times 10 = 120\,cm^3$ A1

21. (a) Complete the table below for the function $y = x^2 + 3$ (2mks)

x	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
y	4	5.25	7	9.25	12	15.25	19	23.25	27	33.25	39

B2 for 9 correct
B1 for 4 correct
B0 if any 2 wrong.

(b) Use the mid-ordinate rule with five strips to estimate the area bounded by the curve, the line $x = 1$ and the line $x = 6$. (2mks)

$A = h$ (Sum of the mid-ordinates)

$= 1(5.25 + 9.25 + 15.25 + 23.25 + 33.25)$ ✓M1

$= 86.25$ Sq units. ✓A1

(c) Use integration to find the exact area in (b) above. (3mks)

$$\int_1^6 x^2 + 3 \, dx$$

$$\left[\frac{x^3}{3} + 3x\right]_1^6$$ ✓M1

$$\left[\frac{6^3}{3} + 3(6)\right] - \left[\frac{1^3}{3} + 3(1)\right]$$ ✓M1

$(72 + 18) - \left(\frac{1}{3} + 3\right)$

$86\frac{2}{3}$ Sq units ✓A1

do not allow for decimals.

(d) Calculate the percentage error arising from the use of mid-ordinate rule. (3mks)

$\frac{5\frac{1}{12}}{86\frac{2}{3}} \times 100\%$ ✓B1 M1

0.48076923% A1

or $\frac{25}{52}\%$

22. A contractor applied for contracts

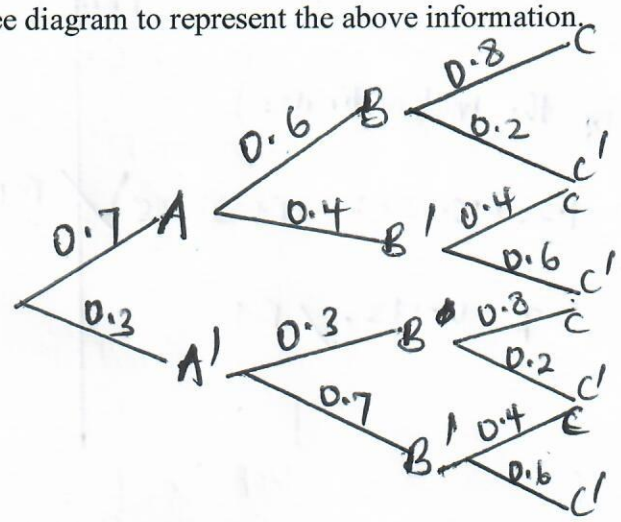
A - Building a classroom block

B - Constructing school dining hall

C - Putting up a dormitory block

The probability of getting A is 0.7. The probability of getting B is 0.6 If A is obtained and only 0.3 if A is not obtained. The probability of getting C is 0.8 if B is obtained and only 0.4 if B is not obtained.

a) Draw a tree diagram to represent the above information. (2 marks)



B2

b) Find the probability of getting

i) The three contracts (2 marks)

$P(ABC) = 0.7 \times 0.6 \times 0.8$ M1

$= 0.336$ A1

ii) Only one contract (2 marks)

$P(AB'C') \text{ or } P(A'BC') \text{ or } P(A'B'C)$

$(0.7 \times 0.4 \times 0.6) + (0.3 \times 0.3 \times 0.2) + (0.3 \times 0.7 \times 0.4)$ M1

$0.168 + 0.018 + 0.084 = 0.27$ A1

iii) At least one contract (2 marks)

$1 - \text{None}$

$1 - (0.3 \times 0.7 \times 0.6)$ M1

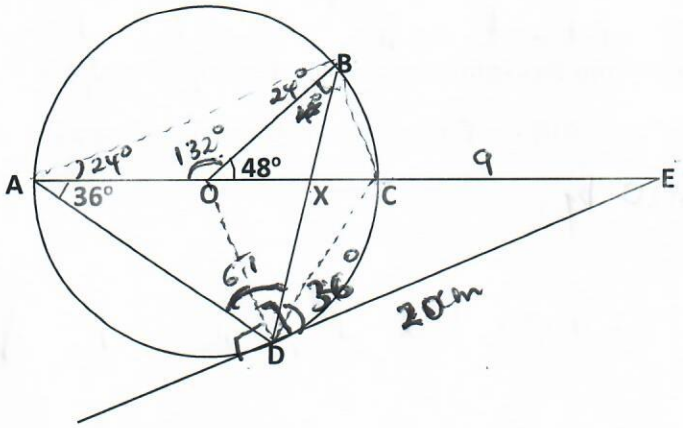
0.874 A1

iv) None of the contract (2 marks)

$0.3 \times 0.7 \times 0.6$ M1

$= 0.126.$ A1

23. In the figure below, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. A, O, X and C are points on a straight line. DE is a tangent to the circle at D. Angle BOC= 48^0 and angle CAD = 36^0.



(a) Giving reasons or otherwise, find the value of the following angles:-

(i) Angle CBA (1 mk)

90° → angle subtended by the diameter at the circumference B1

(ii) Angle BDE (2 mks)

36° B1 – Alternate Segment theorem. B1

(iii) Angle CED (3 mks)

180 − (36+36+61) M1

180 − 133°

= 47° A1 Angles in a triangle add upto 180° B1

(b) It is also given that AX = 12 cm, XC = 4 cm, DB = 14 cm and DE ~~= 25 cm.~~ 20 Cm

Calculate:

DX = x

XB = (14 − x)

(i) DX (2 mks)

AX · XC = DX · XB

12 × 4 = x(14 − x) M1

48 = 14x − x^2

$x^2 - 14x + 48 = 0$

$(x-8)(x-6) = 0$

x = 8 or x = 6 cm. A1 for both

(ii) AE (2 mks)

CE = 9

$AE \cdot CE = DE^2$

$20^2 = 16(16+9)$ M1

400 ~~625~~ = 256 + 169

144 = 169

9 = 9 cm A1

24. A tailoring business makes two types of garments A and B. Garment A requires 3 metres of material while garment B requires 2 ½ metres of material. The business uses not more than 600 metres of material daily in making both garments. It must make not more than 100 garments of type A and nor less than 80 of type B each day.

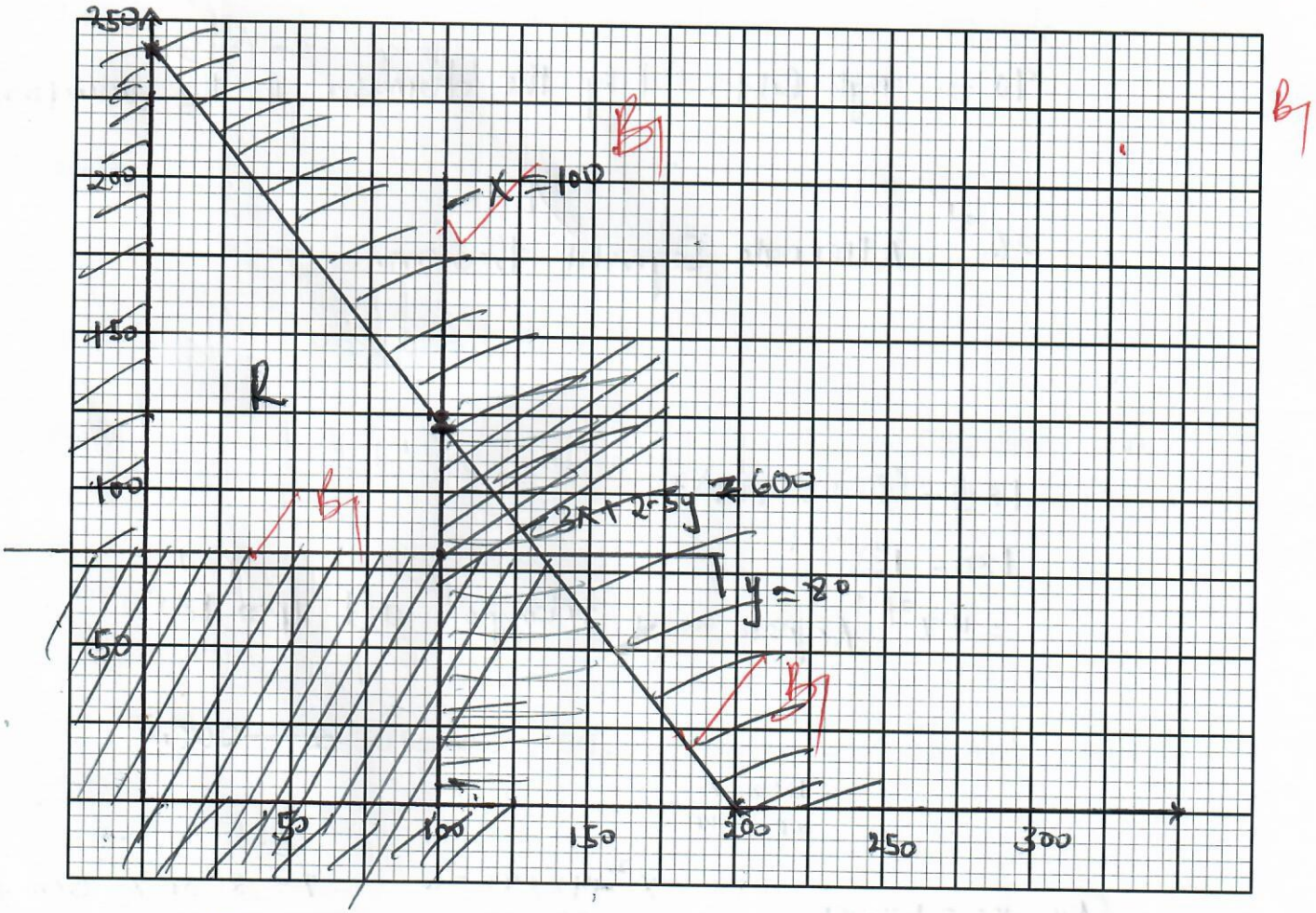
(a) Write down three inequalities from this information other than $x \geq 0$ and $x \geq y$, where x is the number of garments of type A and y the number of garments of type B. (3mks)

$3x + 2.5y \leq 600$ B1

$x \leq 100$ B1

$y \geq 80$ B1

(b) Graph these inequalities. (3mks)



(c) If the business makes a profit of sh 80 on garment A and a profit of sh. 60 on garment B, how many garments of each type must it make in order to maximize the profit and what is the total profit? (4mks)

(100, 80) (100, 120) B1

(0, 250) B1

(0, 80)

100 type A B1

120 type B.

Profit = 100 X 80 + 120 X 60 M1

= Kshs 15,200 A1