SUNSHINE SECONDARY SCHOOL

MATHEMATICS
END TERM EXAM
Form 3 Term 2 2019
PAPER 1
TIME: 2½HOURS

Name: _____M/ SCHEME._____ Adm No._____

Class:_____ Date: _____

Instructions to Candidates

1. Write your name, admission number and class at the top of this paper.
2. The paper contains **2** sections; Section **A** and Section **B**.
3. Answer **ALL** the questions in section A and in section B in the spaces provided.
4. Non-Programmable silent electronic calculators **CAN** be used .

For Examiners Use Only.

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	

1. Without using a mathematical table or a calculator, evaluate leaving your answer as a fraction in its simplest form. (3mks)

$$\frac{\frac{1}{2} \text{ of } 3\frac{1}{2} + (2\frac{1}{2} - \frac{2}{3})}{\frac{3}{4} \text{ of } 2\frac{1}{2} \div \frac{1}{2}}$$

$\frac{1}{2} \times \frac{7}{2} = \frac{7}{4} + \left(\frac{5}{2} - \frac{2}{3}\right)$

5

$\frac{7}{4} + \frac{15-8}{12} =$

2. $\frac{7}{4} + \frac{7}{12} \quad \frac{21+7}{12}$

$\frac{28}{12} \quad \frac{7}{3}$

$\frac{3}{4} \times \frac{5}{2}$

$\frac{15 \times 2_1}{8_4 \, 4}$

$\frac{15}{4}$

$\frac{7}{3} \times \frac{4}{15} = \frac{28}{45}$

2. Given that sin (2x-30)° - cos (3x -50)° = 0, find the value of x   (3mks)

$\sin(2x-30) = \cos(3x-50)$

$x = 34°$ ✓

$(2x-30) + (3x-50) = 90°$ ✓

$2x-30 + 3x - 50 = 90$

$5x - 80 = 90$

$5x = 170$

3. Simplify the expression $\frac{9a^2-4b^2}{3a^2-5ab+2b^2}$ (3mks)

$9a^2 - 4b^2 = (3a + 2b)(3a - 2b)$

$3a^2 - 5ab + 2b^2$

$3a^2 - 3ab - 2ab + 2b^2$ ✓

$3a(a-b) - 2b(a-b)$

$(3a-2b)(a-b)$ ✓

$\frac{(3a+2b)(3a-2b)}{(3a-2b)(a-b)}$

$= \frac{3a+2b}{a-b}$ ✓

4. Find the value of x in the following equation.   (3mks)

$9^{x+1} + 3^{2x+1} = 36$

$9 \cdot 3^{2x+2} + 3^{2x+1} = 36$

$3^{2x} \times 3^2 + 3^{2x} \times 3^1 = 36$ ✓

let 3^{2x} be V

$9V + 3V = 36$

$12V = 36$

$V = 3$ ✓

$3^{2x} = 3^1$

$2x = 1$

$x = \frac{1}{2}$ ✓

2

5. A Kenyan bank buys and sells Nigerian neira and Canadian dollars at the following rates.

	Buying (ksh)	Selling (ksh)
1 Nigerian neira	32.58	36.42
1 Canadian dollar	91.52	98.99

Mrs Obasanjo a Nigerian arrived in Kenya with 46,000 naira. She exchanged the whole amount to Kenya shillings and spent a total of Kenya shillings 720,000. She later changed the remainder to Canadian dolars on her way to Canada. How much did she receive to the nearest shilling.

(4mks)

If 1 Nawra = Ksh. 32.58
∴ 46,00 Nawa = ?

$$\frac{46,000 \times 32.58}{1}$$

= Ksh. 1,498,680.

Remainder = 1498680 − 720,000
= Sh. 778,680.

If Ksh. 98.99 = 1 dollar
∴ sh 778,680 = ?

$$\frac{778,680 \times 1}{98.99} = 7866.25 \, d_0$$

6. The interior angles of an octagon are 2x°, 1/2x°, (x+40)°, 110°, 135°, 160°, (2x+10)° and 185°. Find the value of x. (2mks)

$$S_n = 180° (n-2)$$
$$= 180 (8-2)$$
$$Sum = 1080°$$

$$2x + \tfrac{1}{2}x + (x+40) + 110° + 135° + 160° + (2x+10)° + 185 =$$

$$2x + \tfrac{1}{2}x + x + 40 + 110 + 135 + 160 + 2x + 10 + 185 = 68$$

$$5.5x + 640 = 1080$$
$$5.5x = 440°$$
$$x = 80°$$

7. Using squares and reciprocals tables find the value of x given that. (4mks)

$$\frac{1}{x} = \frac{1}{3.12^2} + \frac{3}{24.27}$$

$$\frac{1}{x} = \frac{1}{9.734} + \frac{3}{2.427 \times 10^{-1}}$$

$$1(0.1028) + 3(0.4144 \times 10^{-1})$$

$$0.1028 + 3(0.0414 4)$$

$$0.1028 + 0.12432$$

$$0.22712$$

$$\frac{1}{x} = 0.22712$$

$$x = \frac{1}{0.22712}$$

$$= 4.4030$$

$$x = 4.4030$$

3

8. Solve the equation leaving the answer in its simplest form. (3mks)

$$\frac{1}{3x} + \frac{1}{4} = \frac{3}{5x}$$

$\left(\frac{1}{3x}\right)^{60x} + \left(\frac{1}{4}\right)^{60x} = \left(\frac{3}{5x}\right)^{60x}$
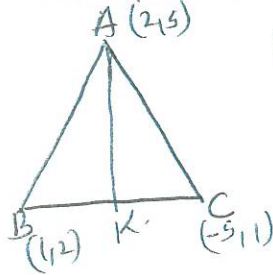
$20 + 15x = 36$

$15x = 36 - 20$

$15x = 16$

$x = \frac{16}{15}$

$x = 1\frac{1}{15}$

9. A triangle has vertices A (2,5) B (1,2) and C (-5,1) determine, leaving answer in $y = mx + c$

i) The equation of line BC (2mks)

$M_1 = \frac{\Delta y}{\Delta x}$

$= \frac{2-1}{1+5}$

$= \frac{1}{6}$

$\frac{1}{6} = \frac{y-1}{x+5}$

$6y - 6 = x + 5$

$6y - x = 11$

$6y = x + 11$

$y = \frac{1}{6}x + \frac{11}{6}$

ii) Line AK is perpendicular to BC, find the equation of line AK. (2mks)

$K = \left(\frac{1-5}{2}, \frac{2+1}{2}\right)$

$(-2, 1.5)$

Gradient of AK = -6 $(M_1 \times M_2 = -1)$

$-6 = \frac{y - 1.5}{x + 2}$

$y - 1\frac{1}{2} = -6x - 12$

$y + 6x = -10\frac{1}{2}$

10. A farmer has 3 containers of capacity of 48 litres, 36 litres and 27 litres. Find the smallest container that can be filled by each of them an exact number of times leaving your answer in standard form. (3mks)
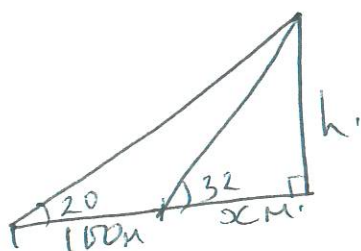
	27	36	48
2	27	18	24
2	27	9	12
2	27	9	6
2	27	9	3
3	9	3	1
3	3	1	
3	1		

$LCM = 2^4 \times 3^3$

$= 432$

$= 4.32 \times 10^2$

4

11. The angle of elevation of the top of a tree from a boy's eye positioned at point A is 20°. The boy moves 100 metres closer to the tree and the angle of elevation becomes 32°. Find the height of the tree (disregard the height of the boy)   (4mks)

$$\tan 20 = \frac{h}{100+x}$$

$$\tan 32 = \frac{h}{x}$$

$h = \tan 20 (x+100)$

$h = \tan 32 (x)$

$\tan 32 x = \tan 20 (100+x)$

$0.6249 = 0.3640 (100+x)$

$0.6249 = 36.4 + 0.3640 x$

$x = \dfrac{35.78}{0.3640}$

$x = 98.30$

$h = \tan 32 (98.30)$

$h = 61.42 M$

12. Given that **a** = 4i - 4j , **b** = 2i + 3j and **c** = 16i + 26j find the scalars m and n such that ma + nb = c.   (4mks)

$$M \begin{pmatrix} 4 \\ -4 \end{pmatrix} + n \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 16 \\ 26 \end{pmatrix}$$

$4M + 2n = 16 \quad -(i)$

$-4M + 3n = 26 \quad -(ii) \quad +$

.Type equation here.
_____

$5n = 42$

$n = 8\tfrac{2}{5} \text{ or } 8.4$

$M = \dfrac{16 - 2n}{4}$

$M = \dfrac{16 - 16.8}{4}$

$M = -\tfrac{1}{5} \text{ or } -0.2$

13. Solve the simultaneous inequalities and represent the solution on a number line.   (3mks)

$4 - 2x < 8$

$2 - 3x \geq -7$

$4 - 2x < 8$

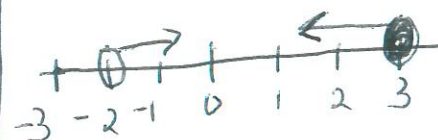$-2x < 8 - 4$

$\dfrac{-2x}{-2} < \dfrac{4}{-2}$

$x > -2$

$2 - 3x \geq -7$

$-3x \geq -7 - 2$

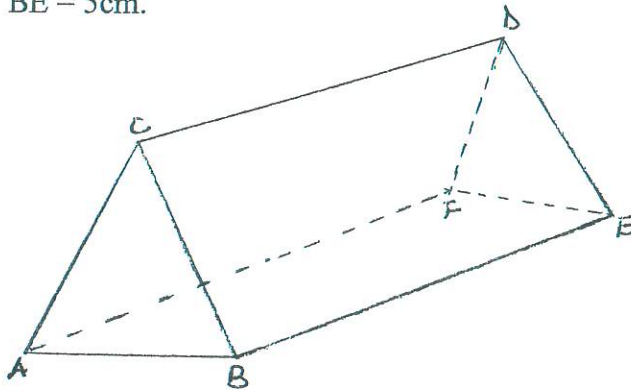$\dfrac{-3x}{-3} \geq \dfrac{-9}{-3}$

$x \leq 3$

$-2 < x \leq 3$

5

14. The ratios of the base areas of two cones are 9:16. If the larger cone has a volume of 125cm³, find the volume of the smaller cone. (3mks)

$$ASF = \frac{9}{16}$$

$$LSF = \sqrt{\frac{9}{16}}$$

$$= \frac{3}{4}$$

$$VSF = \left(\frac{3}{4}\right)^3$$

$$= \frac{27}{64}$$

$$\frac{27}{64} = \frac{V_s}{125}$$

$$V_s = \frac{27 \times 125}{64}$$

$$= 52.73 \, cm^3$$

15. The figure below is a prism whose cross-section is an equilateral triangle AB = 3cm, BE = 5cm.

Draw the net of the prism. (2mks)

- Evidence of arc B1
- Correct dimensions B1

5cm

3cm

3cm

3cm

5cm

3cm

6
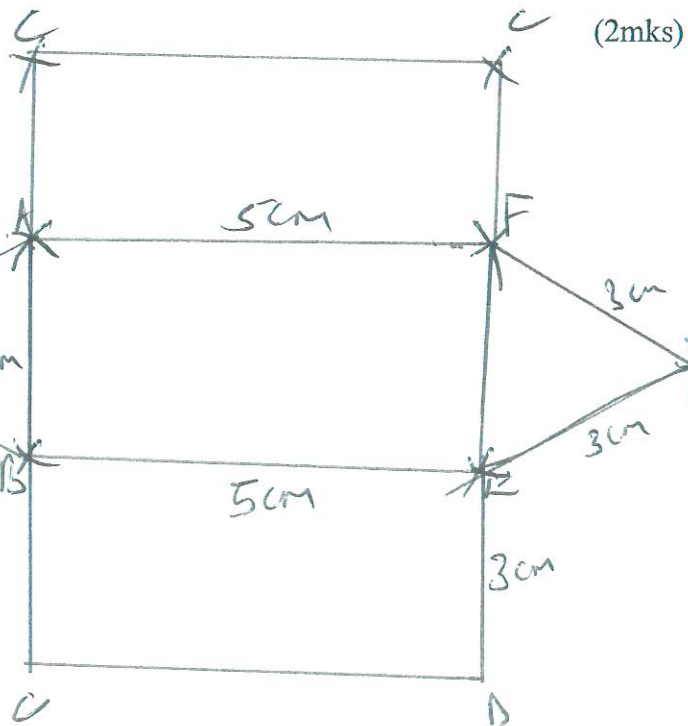
16. A number m is formed by writing all the prime numbers between 0 and 10 in an ascending order. Another number n is formed by writing all the square numbers between 0 and 10 in an ascending order.
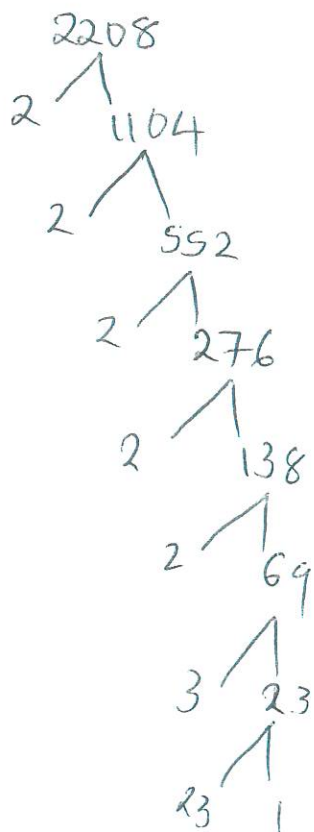
a) Find m − n (2mks)

$M = 2357$

$n = 149$

$2357 - 149$

2208

b) Express (m-n) as a product of its prime factors. (1mk)

$$2208$$

$2 \diagup 1104$

$2 \diagup 552$

$2 \diagup 276$

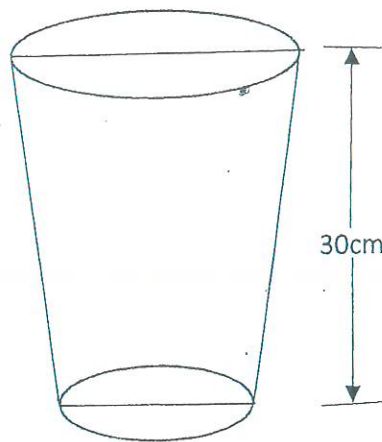$2 \diagup 138$

$2 \diagup 69$

$3 \diagup 23$

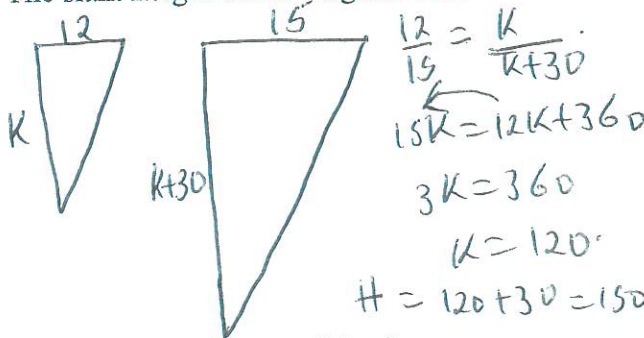$23 \diagup 1$

$2^5 \times 3 \times 23$

SECTION II

17. The diagram below shows a frustum with an open end diameter of 30cm and bottom diameter 24cm. The frustum is 30cm deep as below

30cm

Calculate to d.p

i) The slant height of the original cone (3mks)

$$\frac{12}{15} = \frac{K}{K+30}$$

$$15K = 12K + 360$$

$$3K = 360$$

$$K = 120$$

$$H = 120 + 30 = 150$$

$$L = \sqrt{150^2 + 15^2}$$

$$= \sqrt{22500 + 225}$$

$$= \sqrt{22725}$$

$$= 150.75 \text{ cm} \quad \underline{150.8}$$

ii) The total surface area of the frustum (3mks)

$$l = \sqrt{120^2 + 12^2}$$

$$= \sqrt{14544}$$

$$= 120.60$$

$$\pi RL - \pi r l$$

$$\left(\frac{22}{7} \times 15 \times 150.75\right) - \left(\frac{22}{7} \times 12 \times 120.6\right)$$

7106.79 − 4548.34

2558.45

Area of bottom:

$$\frac{22}{7} \times 12 \times 12$$

$$= 452.57$$

$$A_f = 2558.45 + 452.57$$

$$= 3011.02 \text{ cm}^2$$

$$\underline{3011.0}$$

iii) The volume of the frustum (3mks)

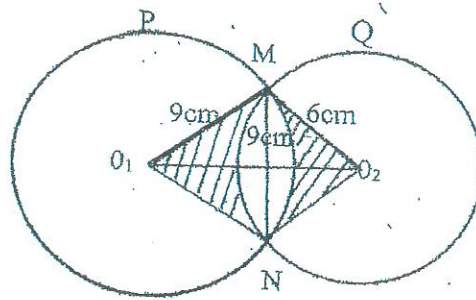$$V_f = \left(\frac{1}{3}\pi R^2 H\right) - \left(\frac{1}{3}\pi r^2 h\right)$$

$$= \frac{1}{3} \times 3.142 \times 15^2 \times 150 - \frac{1}{3} \times 3.142 \times 12^2 \times 120$$

$$= 35347.5 - 18097.92$$

$$= 17249.58 \text{ cm}^3$$

$$\underline{17249.6}$$

18. The following shows two circles P and Q with centre O_1, and O_2 respectively and their radii are 9cm and 6cm respectively. The common chord MN is 9cm long. (Not drawn to scale)

a) Find the value of

i) Angle MO_1N (2mks)

$$\sin \theta = \frac{4.5}{9}$$
$$\theta = \sin^{-1} 0.5$$
$$= 30^0$$
$$2\theta = \underline{\underline{60^0}}$$

ii) Angle MO_2N (2mks)

$$\sin \alpha = \frac{4.5}{6}$$
$$\alpha = \sin^{-1} 0.75$$
$$= 48.59$$
$$2\alpha = 97.18^0$$

b) Find the area of (i) triangle MO_1N (1mk)

$$A = \frac{1}{2} ab \sin \theta$$
$$= \frac{1}{2} \times 9 \times 9 \sin 60 \quad \Big| = 35.07 cm^2$$

ii) triangle MO_2N

$$A = \frac{1}{2} ab \sin \alpha \quad \Big| \frac{1}{2} \times 6 \times 6 \sin 97.18$$
$$= 17.86 cm^2$$

c) find the area of the shaded region (4mks)

$$\frac{\theta}{360} \pi r^2 - \frac{1}{2} ab \sin \theta$$

$$\frac{60}{360} \times 3.142 \times 9^2 - \frac{1}{2} \times 9 \times 9 \sin 60$$
$$42.42 - 35.07$$
$$= 7.35 cm^2$$

$$\frac{97.18}{360} \times 3.142 \times 6^2 - \frac{1}{2} \times 6 \times 6 \sin 97.18$$
$$30.53 - 17.86$$
$$= 12.67 cm^2$$

$$A_{shaded} = (35.07 + 17.86) - (7.35 + 12.67)$$
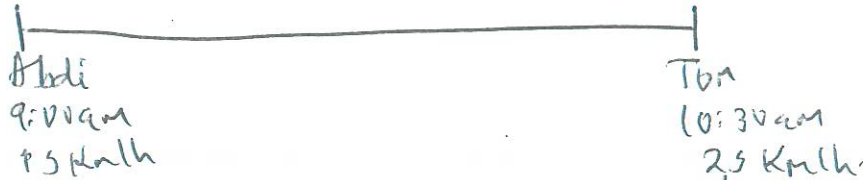$$52.93 - 20.02$$
$$\underline{\underline{32.91 cm^2}}$$

9

19. Two friends Abdi and Tom live 40km apart. One day Abdi left his house at 9.00a.m and cycled towards Tom's house at an average speed of 15km/hr. Tom left at 10.30a.m on the same day and cycled towards Abdi's house at an average speed of 25km/hr

a) Determine

i) The distance from Abdi's house, where the two friends met   (4mks)

Abdi
9:00am
15 km/h

Tom
10:30am
25 Km/h.

Dist covered by Abdi before Tom left:

15 × 1½
= 22·5 km
Dist left: 40 − 22·5
         = 17·5km.

R·S = 15+25
    = 40km/h.
R·T = $\frac{17·5 \text{ Km/h}}{40 \text{ Km/h}}$ = 23·75 mins
    ≈ 24 Mins.
Dist by Abdi: (2mks)
15 × $\frac{24}{60}$ = 6 km
22·5 + 6
= 28·5km.

ii) The time they met

Time of meeting:
10:30am + 24 mins
≈ 10:54 am.

iii) How far Abdi was from Tom's house when they met   (2mks)

Dist = S×T
     = 15 × $\frac{19}{10}$
     = 28·5 KM

40 − 28·5
= 11·5KM

b) The two friends took 10 minutes at the meeting point and they cycled to Tom's house at an average speed of 12km/hr. Find the time they arrived at Tom's house.   (2mks)

Dist left to Tom's house.
11·5KM
T = D/S
  = $\frac{11·5 \text{ km}}{12 \text{ km/h}}$

$\frac{23}{24}$ hrs
= 57·5 min ≈ 58 min+10min
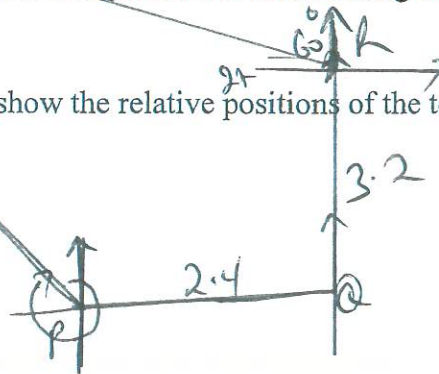10:54
1:08
12:02
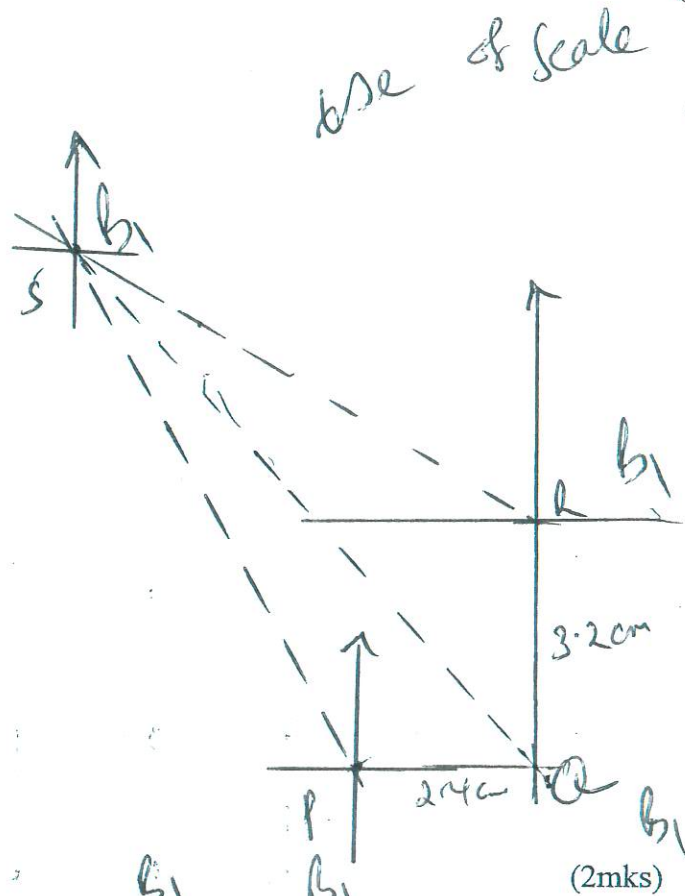12:02 PM

10

20. Four towns P, Q, R and S are such that town Q is 120km due East of town P town R is 160km due North of town Q town S is on a bearing of 330° from P and on a bearing of 300° from R

a) i) draw a sketch to show the relative positions of the town.   (1mk)

ii) Using a ruler and a pair of compasses only, show the relative positions of towns P, Q R and S. Take a scale of 1 cm rep 50km.   (5mks)

b) determine
   i)   the distance SP in km   (2mks)

   7.7 ± 0.1 385 ± 5 km

   ii)   the bearing of S from Q   (2mks)

   $318 \pm 1°$

   or $S42° W$

11

21. a) On the grid provided, draw the square whose vertices are A (2, -2) B (4, -2) C (4,0) and D (2,0) (2mks)

b) On the same grid draw

   i)   A`B`C`D` the image and ABCD under a reflection in the line x = 0 and state the coordinates of A`B`C`D` (3mks)

A^1 (-2,-2) B^1 (-4,-2) C^1 (-4,0) D(-2,0)

   ii) A"B"C"D" the image of A`B`C`D` under a rotation of + 90º about the origin. (O,O) and state the coordinates A"B"C"D" (3mks)

A^{11} (2,-2) B^{11} (2,-2) C^{11} (0,-4) D^{11} (2,-4)

   iii) Describe a single transformation that maps ABCD onto A"B"C"D (2mks)

Reflection in the line y =