

SUNSHINE SECONDARY SCHOOL



121/2

MATHEMATICS

Paper 2

END TERM EXAM

Form 3 Term 2 2019

TIME: 2½ HOURS

Name: MAPPING SCHEME. Adm No. _____

Class: _____ Date: _____

Instructions to Candidates

1. Write your name, admission number and class at the top of this paper.
2. The paper contains 2 sections; Section A and Section B.
3. Answer **ALL** the questions in section A and in section B in the spaces provided.
4. Non-Programmable silent electronic calculators **CAN** be used.

For Examiners Use Only.

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	

1. The dimensions of a rectangle are given as 6.7m by 4m to the nearest metre. Calculate the percentage error of the area of a rectangle.

$$L_{\text{max}} = 6.75 \text{ cm}$$

$$L_{\text{min}} = 6.65$$

$$W_{\text{max}} = 4.5$$

$$W_{\text{min}} = 3.5$$

$$\text{A: Area} = 4 \times 6.7 \\ = 26.8$$

$$A_{\text{max}} = 6.75 \times 4.5 \\ = 30.375$$

2. Work out leaving your answer in the form of $a\sqrt{b} - c\sqrt{d}$.

$$A_{\text{min}} = 6.65 \times 3.5 \quad (3 \text{ mks})$$

$$= 23.275$$

$$E_{\text{mr}} = \frac{30.375 - 23.275}{23.275} \\ = 7.1$$

$$\% \text{ error} = \frac{7.1}{26.8} \times 100 \text{ M1}$$

$$= 26.493\% \quad \cancel{M1}$$

$$13.246 \quad (3 \text{ mks})$$

$\cancel{13.25} \quad M1$

$$\frac{7}{\sqrt{2} + \sqrt{3}} = \frac{7}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} \quad \frac{7}{-1} \\ = \frac{7\sqrt{2} - 7\sqrt{3}}{-1} \quad \text{mu} \\ = 7\sqrt{3} - 7\sqrt{2} \quad M1$$

3. Make y the subject of the formula (3 mks)

$$p = \sqrt{\left(\frac{xy^2}{x^2 - y^2}\right)}$$

Squaring both sides yields:

$$p^2 = \frac{xy^2}{x^2 - y^2} \quad \text{mu}$$

$$\underline{xy(x^2 - y^2)} = \underline{xy^2(x^2 - y^2)} \\ x^2y^2$$

$$xy^2 = p^2x^2 - p^2y^2$$

$$p^2x^2 = p^2y^2 + xy^2$$

$$p^2y^2 + xy^2 = p^2x^2$$

$$y^2(p^2 + x) \quad \text{M1} \quad p^2x^2$$

$$y^2 \frac{(p^2 + x)}{(p^2 + x)} = \frac{p^2x^2}{p^2 + x}$$

$$\sqrt{y^2} = \sqrt{\frac{p^2x^2}{p^2 + x}}$$

$$y = \sqrt{\frac{p^2x^2}{p^2 + x}} \quad M1$$

2

4. Calculate the length of line PQ given P (6,8) Q (-5,9) leaving answer to 1d.p (3mks)

$P(6, 8)$ $Q(-5, 9)$ $PQ = \begin{pmatrix} -5 \\ 9 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ $= \begin{pmatrix} -11 \\ 1 \end{pmatrix}$	modulus = $\sqrt{(-11)^2 + (1)^2}$ $= \sqrt{121 + 1}$ $= \sqrt{122}$ $= 11.04$ $= 11.0 \text{ (1.d.p)}$
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5. Solve the following quadratic equation by completing square method (3mks)

$$2x^2 - 5x + 3 = 0$$

$$\frac{2x^2}{2} - \frac{5x}{2} + \frac{3}{2} = 0$$

$$x^2 + \frac{5}{2}x + \frac{3}{2} = 0$$

$$x^2 + \frac{5}{2}x - \left(\frac{5}{4}\right)^2 = -\frac{3}{2} + \frac{25}{16}$$

$$(x - \frac{5}{4})^2 = -\frac{24}{16} + \frac{25}{16} = \frac{1}{16}$$

$$(x - \frac{5}{4}) = \pm \frac{1}{4}$$

$$x = \frac{5}{4} \pm \frac{1}{4}$$

$$x = 1 \text{ or } \frac{3}{2}$$

6. Given that 3θ is an acute angle and $\sin 3\theta = \cos 2\theta$. find the value of θ (3mks)

$$3\theta + 2\theta = 90^\circ$$

$$5\theta = 90^\circ$$

$$\theta = 18^\circ$$

$$\theta = 18^\circ$$

7. Patel bought some articles at sh3060 per dozen and sold them all making a profit of 20% on the cost price. When selling these articles he had allowed his customers a discount of 10% on the marked price of each article. Find the marked price of each article. (3mks)

$$\text{Cost price of each article} = \frac{3060}{12} \\ = \text{sh. } 255$$

$$\text{S.P} = \frac{120}{100} \times 255$$

$$\% \text{ discount} = \frac{\text{M.P} - \text{S.P}}{\text{M.P}} \times 100$$

$$\frac{x-3060}{x} = 10\%$$

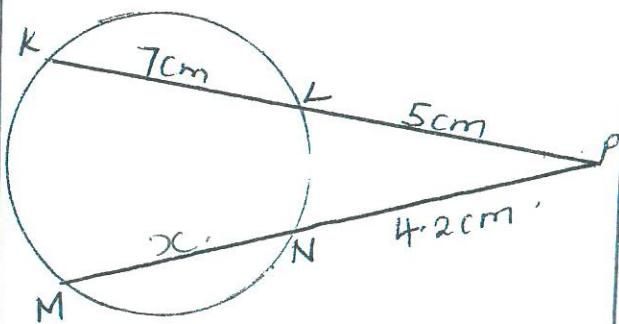
$$10x = x - 3060$$

$$9x = 3060$$

$$x = 340$$

$$\text{M.P} = \text{sh. } 340. \text{ M}$$

8. Chords KL and MN intersect externally at P in the diagram shown below. Given that KL = 7cm, LP = 5cm and NP = 4.2cm, calculate the length of MP (3mks)



$$KP \times LP = MP \times NP$$

$$12 \times 5 = 4.2(4.2 + x)$$

$$60 = 17.64 + 4.2x$$

$$60 - 17.64 = 4.2x$$

$$\frac{42.36}{4.2} = \frac{4.2x}{4.2}$$

$$x = 10.09 \text{ cm M}$$

$$\text{M.P} = 10.09 + 4.2 \quad (3 \text{ mks})$$

$$= 14.29 \text{ cm M}$$

9. Solve for x in the equation

$$\log(x-1) = \log 12 - \log(x-2)$$

$$\log(x-1) = \log\left(\frac{12}{x-2}\right).$$

$$x-1 = \frac{12}{x-2} \text{ M}$$

$$(x-1)(x-2) = 12$$

$$x^2 - 3x + 2 = 12$$

$$x^2 - 3x - 10 = 0 \text{ M}$$

$$x^2 - 2x - 5x - 10 = 0$$

$$x(x+2) - 2(x+2) = 0$$

$$(x-5)(x+2) = 0$$

$$x = -2$$

$$x = 5 \text{ M}$$

10. Use matrices to solve the simultaneous equations

$$4x - 5y = 13$$

$$3x - 2y = 8$$

$$\begin{pmatrix} 4 & -5 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ 8 \end{pmatrix}$$

$$M = \begin{pmatrix} 4 & -5 \\ 3 & -2 \end{pmatrix}$$

$$\det M = (4 \times -2) - (3 \times -5) = 7$$

$$M^{-1} = \frac{1}{7} \begin{pmatrix} -2 & 5 \\ -3 & 4 \end{pmatrix}$$

$$\left| \begin{array}{l} \frac{1}{7} \begin{pmatrix} -2 & 5 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 4 & -5 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -2 & 5 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 13 \\ 8 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -2 \times 13 + 5 \times 8 \\ -3 \times 13 + 4 \times 8 \end{pmatrix} \\ = \frac{1}{7} \begin{pmatrix} -26 + 40 \\ -39 + 32 \end{pmatrix} \\ = \frac{1}{7} \begin{pmatrix} 14 \\ -7 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} : x = 2, y = -1 \end{array} \right. \quad (3 \text{ mks})$$

11. Two similar solid spheres of radius 3.6cm each are melted down and recast into a solid cone of base radius r and height 7.2cm. Calculate to 1 decimal place, the radius r of the cone.

$$\begin{aligned} \text{Vol. of spheres} &= 2 \times \frac{4}{3} \pi \times 3.6^3 \text{ cm}^3 & r^2 &= 4 \times (3.6)^2 \\ \text{Vol. of cone} &= \frac{1}{3} \pi r^2 \times 7.2 \text{ cm}^3 & r &= \sqrt{4 \times (3.6)^2} \\ \frac{1}{3} \pi r^2 \times 7.2 \text{ cm}^3 &= \frac{1}{3} \pi \times 8 \times (3.6)^3 & &= 2 \times 3.6 \\ r^2 &= \frac{8 \times 3.6 \times 3.6 \times 3.6}{7.2} & &= 7.2 \text{ cm} \end{aligned} \quad (3 \text{ mks})$$

12. The first four terms of a G.P are 81, m, n, 3. Find the values of m and n.

(3 mks)

$$G.P = ar^{n-1}$$

$$\therefore 81r^3 = 3$$

$$r^3 = \frac{3}{81} = \frac{1}{27}$$

$$r = \sqrt[3]{\frac{1}{27}}$$

$$= \frac{1}{3} \checkmark$$

$$m = 81 \times \frac{1}{3}$$

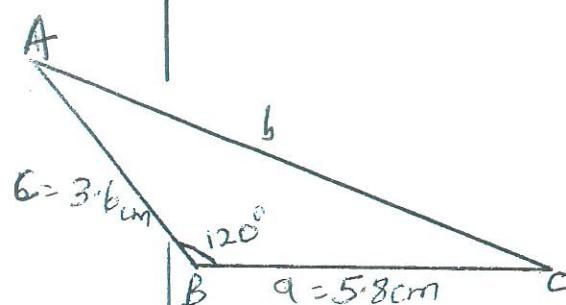
$$= 27 \checkmark$$

$$n = 27 \times \frac{1}{3}$$

$$= 9 \checkmark$$

13. In the figure below $AB = 3.6\text{cm}$, $BC = 5.8\text{cm}$ and $\angle ABC = 120^\circ$, calculate the length of AC

(4mks)



$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$5.8^2 + 3.6^2 - 2 \times 5.8 \times 3.6 \cos 120^\circ$$

$$33.64 + 12.96 - 41.76 \times (-0.5)$$

$$46.6 + 20.88$$

$$b = \sqrt{67.48}$$

$$b = 8.215\text{cm}$$

14. For the past 5 years the value of the car has been depreciating at a constant rate of 12% per annum. The present value of the car is sh316,640. Calculate the value of the car at the beginning of the 5 year period.

(3mks)

$$V = P \left(1 - \frac{r}{100}\right)^n$$

V = present value
 P = original value
 r = rate
 n = 7 years

$$316640 = P \left(1 - \frac{12}{100}\right)^5$$

$$P \left(\frac{100-12}{100}\right)^5 = \left(\frac{88}{100}\right)^5$$

$$P (0.88)^5$$

$$P = \frac{316640}{(0.88)^5}$$

$$= \text{sh. } 600000$$

15. The table below shows the masses of 40 students in a form 4 class

Mass (kg)	Frequency
40 – 44	4
45 – 49	10
50 – 54	15
55 – 59	8
60 – 64	3

- a) State the modal class

50-54 (highest frequency).

(1mk)

- b) Calculate the median class

$$\text{Median} = U_L + \frac{(x_m - f_f)}{f_m} \times w.$$

$$= 49.5 + \frac{(40 - 14)}{15} \times 4$$

$$= 51.5 \text{ kg}$$

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- * 16. A man working on a construction job is paid sh12 an hour for the normal working hours and sh15 for each hour worked overtime. In one week he worked a total of 81 hours and was paid sh~~1701~~¹⁰⁷¹ in wages. Determine the number of hours he worked overtime. (3mks)

Let overtime be hrs = x

Normal hours = y .

$$x+y = 81 \quad \text{--- (i). M}$$

$$15x + 12y = 1071 \quad \text{--- (ii)}$$

Using eqn (i):

$$y = 81 - x \quad \text{M}$$

put $y = 81 - x$ in eqn (ii)

$$15x + 12(81 - x) = 1071$$

$$15x + 972 - 12x = 1071$$

$$3x = 1071 - 972$$

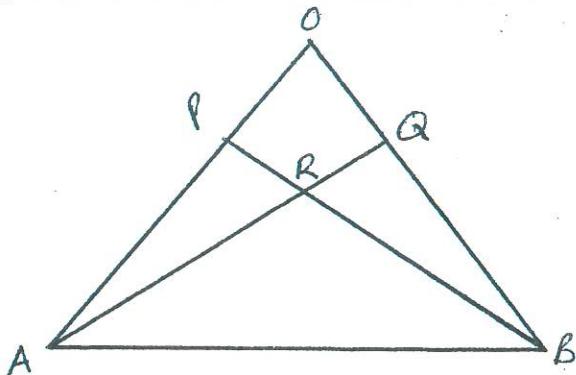
$$3x = 99$$

$$x = 3$$

No. of hours overtime = 3 hours.

SECTION B

17. The figure below shows a triangle OAB in which point P divides line OA in the ratio 1:2 and point Q divides OB in the ratio 1:2 AQ and PB intersect at point R.



a) Given that $OA = 12\vec{a}$ and $OB = 12\vec{b}$, express in terms of \vec{a} and \vec{b}

i) $\vec{AB} = -12\vec{a} + 12\vec{b}$ (1mk)

ii) $\vec{PQ} = \frac{1}{3}\vec{a} + \frac{1}{3}\vec{b}$
 $= 4\vec{a} + 4\vec{b}$ (1mk)

b) Given that $\vec{PR} = \frac{1}{4}\vec{PB}$, express in terms of \vec{a} and \vec{b}
i) $\vec{PB} = 4\vec{a} - 12\vec{b}$ (1mk)

ii) $\vec{AR} = -12\vec{a} + 12\vec{b} + \frac{3}{4}(4\vec{a} - 12\vec{b})$
 $= 12\vec{a} + 12\vec{b} + 3\vec{a} - 9\vec{b}$ (2mks)

iii) $\vec{QR} = \frac{1}{3}\vec{a} + \frac{1}{3}\vec{b}$
 $= 8\vec{a} + 3\vec{b} - 9\vec{b} = 3\vec{a} - \vec{b}$ (2mks)

c) Show that points Q, R and A lie in a straight line. (3mks)

$$\vec{QR} = 3\vec{a} - \vec{b}$$

$$\vec{AR} = -9\vec{a} + 3\vec{b}$$

$$\frac{\vec{QR}}{\vec{AR}} = v$$

$$\frac{3\vec{a} - \vec{b}}{-9\vec{a}} = \frac{-\vec{b}}{3\vec{b}} = -\frac{1}{3}$$

$$\vec{QR} = \frac{1}{3}\vec{AR}$$

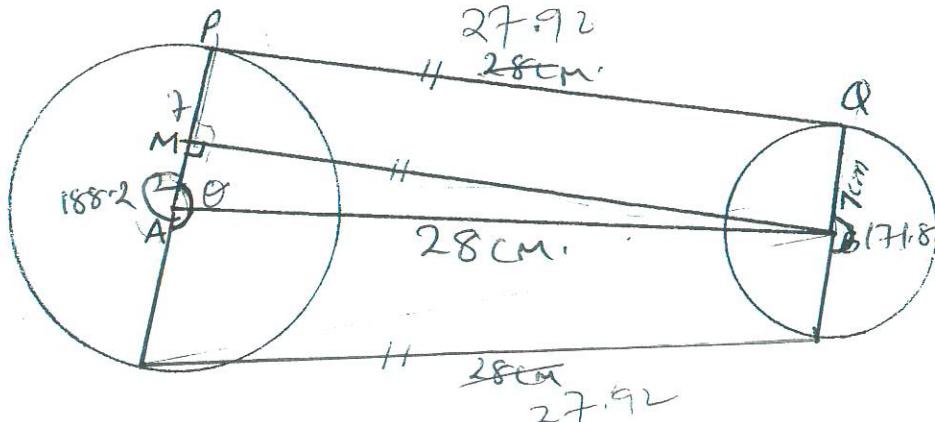
\vec{QR} is parallel to \vec{AR} .

R is common point

$\therefore Q, R$ and A are

collinear. B1

18. The figure below shows a pulley system with two wheels of radii 9cm and 7cm and centres A and B respectively. A continuous belt goes round the wheels.



Calculate to 1 d.p

i) The length MB (3mks)

$$MB = \sqrt{28^2 - 2^2} = \sqrt{780} = 27.9 \text{ cm.}$$

(3mks)

ii) The angle PAB
 $\cos \theta = \frac{2}{28}$ | $\theta = \cos^{-1}(2/28)$
 $\cos \theta = 0.0714$ | $\theta = 85.9^\circ$

(2mks)

iii) The length of the belt (5mks)

$$\frac{\theta}{360} 2\pi R + \frac{\theta}{360} 2\pi r + L + L.$$

$$\left(\frac{188.2}{360} \times 2 \times 3.142 \times 9 \right) + \left(\frac{171.8}{360} \times 2 \times 3.142 \times 7 \right) + 27.92 + 27.92$$

$$= 29.566 + 20.99 + 56.$$

$$= 106.556$$

$$= 106.6 \text{ cm (1d.p.)}$$

106.44

19. Given the arithmetic sequence

$$4, 11, 18 \dots \text{find the next three terms}$$

(1mk)

$$25, 32, 39 \checkmark$$

i) Common difference

$$\begin{aligned} d &= 11 - 4 \\ &= 7 \end{aligned}$$

(1mk)

ii) The 15th term

$$\begin{aligned} n^{\text{th}} &= a + (n-1)d \\ &= 4 + (15-1)7 \\ &= 4 + 98 \\ &= \underline{102} \end{aligned}$$

(2mks)

iv) The sum of the first twenty terms

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ S_{20} &= \frac{20}{2}[2(4) + (20-1)7] \\ &= 10[8 + 133] \\ &= 10[141] = 1410 \end{aligned}$$

(3mks)

c) The fourth term of a geometric sequence is 192. If the first term of the sequence is 3, find the common ratio.

(3mks)

$$n^{\text{th}} = ar^{n-1}$$

$$r^3 = 64$$

$$\begin{aligned} n^{\text{th}} &= 4 \\ a &= 3 \end{aligned}$$

$$r = 4 \checkmark$$

$$3r^{4-1} = 192$$

$$\frac{3r^3}{3} = \frac{192}{3}$$

20. a) P varies directly as Q. If P = 8 and Q = 48, find the equation connecting P and Q.

$$P \propto Q$$

$$P = kQ$$

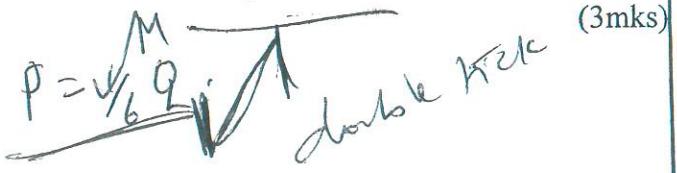
$$P = 8$$

$$Q = 48$$

$$\therefore 8 = 48k$$

$$k = \frac{8}{48}$$

$$k = \frac{1}{6}$$



(3mks)

b) Given that $y \propto \frac{1}{x}$ and that when $y = 6$, $x = 44$, find y when $x = 2$

(3mks)

$$y \propto \frac{1}{x}$$

$$y = k \frac{1}{x}$$

$$6 = \frac{1}{44} k$$

$$k = 6 \times 44$$

$$= 264$$

$$y = \frac{264}{x}$$

$$y \text{ when } x = 2$$

$$y = \frac{264}{2}$$

$$= 132$$

c) A varies directly as B and inversely as the square root of C. Find the percentage change in A when B is decreased by 10% and C increased by 21%

(4mks)

$$A \propto \frac{B}{\sqrt{C}}$$

$$A = k \frac{B}{\sqrt{C}}$$

$$\text{New } B = \frac{90}{100}$$

$$= \frac{9}{10}$$

$$\text{New } C = \frac{121}{100}$$

$$= 1.21$$

$$\begin{aligned} \text{New } A &= \frac{\frac{9}{10}}{\sqrt{1.21}} = \frac{9}{11} \\ &= \frac{9}{11} \end{aligned}$$

% change :

$$\frac{A_{\text{new}} - A}{A} \times 100\%$$

$$\frac{\frac{9}{10} - 1}{1} \times 100\%$$

$$= -\frac{1}{10} \times 100\%$$

$$= -18.1818\% \text{ or}$$

$$-18\frac{2}{11}\%$$

Decreased by $-18\frac{2}{11}\%$

21. The table below shows some values of the curves $y = 2 \cos x$ and $y = 3 \sin x$.

a) Complete the table for the values of $y = 2 \cos x$ and $y = 3 \sin x$, correct to 1 decimal place

X°	0°	30	60	90	120	150	180	210	240	270	300	330	360
$Y=2\cos x$	2	1.7	1	0	-1	-1.7	-2	-1.7	-1	0	1	1.7	2
$Y=3\sin x$	0	1.5	2.6	3	2.6	1.5	0	-1.5	-2.6	-3	-2.6	-1.5	0

b) On the grid provided draw the graph of $y=2\cos x$ and $y=3\sin x$ for $0^\circ \leq x \leq 360^\circ$ on the same axes. (5mks)

Help

c) Use the graph to find the values of x when $2\cos x = 3\sin x$ (2mks)

$$x = 33^\circ \text{ or } 216^\circ$$

d) Use the graph to find the value of y when $2\cos x = 3\sin x$. (1mk)

$$y = 1.6 \text{ or } -1.6$$

